The Collective Behavior of Multi-Agents System for Tracking a Desired Path

Miswanto[a1], I. Pranoto[b], H. Muhammad[c] and D. Mahayana[d]
[a]Department of Mathematics, Airlangga University, Surabaya 60115, Indonesia
[b]Industrial and Financial Mathematics Research Groups, ITB, Bandung 40132, Indonesia
[c]Department of Aeronautics and Astronautics, ITB, Bandung 40132, Indonesia
[d]Control and Computer Systems Research Groups, ITB, Bandung 40132, Indonesia

***Abstract*-- The formation of movement in a colony of animals in nature is as important as the movement itself. In transportation world or multi-agents system (*swarm*), like in military world, the formation of aircrafts or ships is also vital. This paper addresses the tracking situation of multi agents system that models swarming behavior moving along a desired path. This paper tries to model the movement of the swarm agents to follow the desired path in a bounded 2-dimensional region. In this paper, we also study the dynamics of the center of swarm relative to the desired path. We find that the distance between the center of swarm path and the desired path is exponentially stable. Numerical simulations shows that the agents will aggregate then move together around the desired path.**

***Index Term*-- multi-agents systems, swarm center, tracking, numerical simulation.**

I. INTRODUCTION

Collective behavior in multi agents system (*swarm*) is a very interesting phenomenon in mathematical modeling. Swarming behavior or aggregation of organism in groups is abundant in nature. Swarming of organisms in groups can be found in many organisms ranging from simple bacteria to mammals. Examples of swarms include colonies of bacteria, flocks of birds, schools of fish etc. By swarming behavior, they obtain advantages such as avoiding predator and increasing the chance of finding food. The swarming can be used in engineering applications such as optimization, robotics and autonomous air vehicle **Error! Reference source not found.**. The study of collective behavior of multi agents system attract much attention in many fields such as biology, physics and mathematics.

In recent years, coordination of the motion of multi dynamic agents have emerged as a topic of major interests [**Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**, **Error! Reference source not found.**]. In this literatures, some researchers have discussed the coordination and design control problem of the motion of the swarm. In **Error! Reference source not found.**, the authors consider an isotropic swarm model and study aggregation, cohesion and stability properties. Subsequently, Chu, Wang and Chen **Error! Reference source not found.** generalized their model by considering an anisotropic swarm model, and obtained the properties of aggregation, cohesion and complete stability. In **Error! Reference source not found.**, the authors study a stable and decentralized control strategy for multi agent systems to capture a moving target in a specific formation. They use artificial potentials to do tasks of both tracking and formation task. In **Error! Reference source not found.**, the authors consider the aggregation, foraging, and formation control of swarm whose agents are moving in 2-dimensions with unicle agent dynamics. Their approach uses artificial potentials and sliding mode control. In **Error! Reference source not found.**, the authors consider a kinematic model (in a sense) for the capture/intercept and develop a method for that case and they build on the developed method to include general fully actuated vehicle dynamics for the pursuer agent. In **Error! Reference source not found.**, the authors consider the task of tracking a maneuvering target both with a single non-holonomic agent and a swarm of non-holonomic agents.

This paper proposes some dynamics of the swarm agents whose agents are moving to trace a desired path. In this paper, we study the motion of the swarm center relative to the desired path. The organization of this paper is as follows. Section 2 formulates the swarm tracking model. In Section 3, we discuss main result about the swarm center moves to track a desired path, and we obtain the radius of the bounded region around the swarm center. All agents will eventually enter this bounded region then its move to follow together a desired path. In section 4, we show numerical simulations to illustrate our results. The conclusion is given in the last section.

II. SWARM TRACKING MODEL

Consider a swarm tracking model of M agents in an n-dimensional Euclidean space. This model is a generalization of the models in **Error! Reference source not found.**, i.e.

$$\dot{x}_i = \sum_{j=1}^{M} f(x_i - x_j), i = 1, \ldots, M. \quad (1)$$

In **Error! Reference source not found.** they discuss the stationary of the swarm center. They also study the motion of the swarm agents aggregate and approach a bounded region

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around the swarm center. In this paper, we modify the attraction/repulsion function **Error! Reference source not found.**. We discuss the tracking problem of the swarm model. The swarm agents we discuss here move in 2-dimensional space. Of course, this model can be generalized to any dimensional space. The model is given by

$$\dot{x}_i = \sum_{j=1}^{M} f(x_i - x_j) - p(x_i - \gamma) + \dot{\gamma}, i = 1, \ldots, M. \quad (2)$$

Here $\dot{x}_i = \frac{dx_i}{dt}$, where $x_i \in \mathbf{R}^n$ represents the position of the i-th agent. The symbol $f(.)$ is the attraction and repulsion force among members. In **Error! Reference source not found.**, Gazi and Passino present the attraction/repulsion function as

$$f(y) = -y(a - b\exp(-\frac{\|y\|^2}{c})). \quad (3)$$

In this paper, the attraction/repulsion function is given by

$$f(y) = -y(a - \frac{r}{\|y\|^2+1}), \quad (4)$$

where a, r are positive constants with $r > a$ and $\| y \| = \sqrt{y^T y}$ is the Euclidean norm. The parameter a represents the attraction and the term $\frac{r}{\|y\|^2+1}$ represents the repulsion. The function is attractive for large distance and repulsive for small distance. By equating $f(y) = 0$, we can easily obtain that $f(.)$ switches sign at the set of points defined as $Y = \{y = 0$ or $\| y \| = \delta = \sqrt{\frac{r-a}{a}}\}$. The distance δ is the distance at which the attraction and repulsion balance. The term $-p(x_i - \gamma)$ represents the attractor function of the desired path for all agent of the swarm. Here p is a positive constant with $a < p < r$ and γ is the desired path. In the next section, we discuss the behavior of the swarm center tracking the desired path.

III. MAIN RESULTS

In this section, the main results concerning the motion of the swarm center to track the desired path is presented. Let the center of the swarm agents be

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x_i. \quad (5)$$

The following theorem states that the tracking error of the path of the swarm center tracing a desired path is approaching zero.

Theorem 1 For any path $\gamma(t)$ the trajectory $\bar{x}$ in ((2)) exponentially approaches γ, i.e. there exist $T, A, B > 0$ such that $\| \bar{x}(t) - \gamma(t) \| \leq A\exp(-Bt)$ *for* $t > T$.

Proof. Differentiating the swarm center $\bar{x}(t)$ in ((5)) with respect to time, one obtains

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M} \dot{x}_i$$

$$= \frac{1}{M}\sum_{i=1}^{M} \{\sum_{j=1}^{M} f(x_i - x_j) - p(x_i - \gamma) + \dot{\gamma}\}$$

It is clear from the definition of the attraction/repulsion function $f(y)$ that $f(-y) = -f(y)$ for all $y \in \mathbf{R}^n$. Thus, we have

$$\dot{\bar{x}} = \frac{1}{M}\sum_{i=1}^{M} \{-p(x_i - \gamma) + \dot{\gamma}\}$$

$$= -p(\bar{x} - \gamma) + \dot{\gamma}$$

so we have

$$\dot{\bar{x}} - \dot{\gamma} = -p(\bar{x} - \gamma). \quad (6)$$

Let $r(t) = \bar{x}(t) - \gamma(t)$, then differentiating $r(t)$ with respect to time, one obtains

$$\dot{r}(t) = \dot{\bar{x}}(t) - \dot{\gamma}(t)$$

Such that, the equation ((6)) can be write as

$\dot{r}(t) = -pr(t)$. The solution of this equation is

$$r(t) = r(0)\exp(-pt)$$

$$\bar{x}(t) - \gamma(t) = r(0)\exp(-pt)$$

thus

$$\| \bar{x}(t) - \gamma(t) \| = \| r(0) \| \exp(-pt)$$

It means, we can find $A, B, T > 0$ that make $\| \bar{x}(t) - \gamma(t) \| < A\exp(-Bt)$ for $t > T$. ■

The following theorem shows that all agents will aggregate and approach a bounded region around the swarm center then they continuously move to follow together the desired path.

Theorem 2 *If* $x_i(0)$ *in ((2)) for some* $i \in I (I = \{1,2,3,\ldots,n\})$ *is not element* $\Omega_\varepsilon(\gamma(0))$, *where*

$$\Omega_\varepsilon(\gamma(0)) = \{x: \| x - \gamma(0) \| \leq \varepsilon\},$$

and $\varepsilon = \frac{r}{2a}$, then for each $\eta > 0$ there is $T > 0$ such that

$$\| x_i(t) - \gamma(t) \| \leq \varepsilon + \eta,$$

for every $t > T$.

IV. NUMERICAL SIMULATIONS

In this section, some numerical simulations to illustrate model ((2)) are reported. As an illustration, a desired path is the following parametric curve:

$$\gamma_x(t) = -\frac{3}{800}t^4 + \frac{9}{80}t^3$$

$$\gamma_y(t) = \frac{1}{200}t^4 - \frac{3}{20}t^3 + t^2$$

where $t \in [0, \infty)$. Fig. 1-2 in below show the simulation results, where $M = 10, a = 1, r = 50$ and $p = 2.5, 15$ and 30. Fig. 1-2 shows the numerical simulations of the trajectories of the agents of the swarm model. Fig. 1 shows the trajectories of the agents of the swarm and the desired path with the parameter $p = 2.5$. It can be seen from Fig.1 that the swarm members aggregate towards a bounded region. Then, they continuously move to group and follow the desired path.

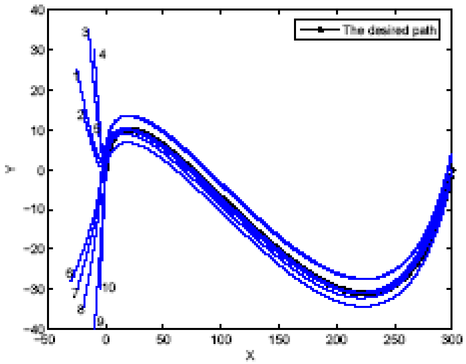

Fig. 1. The trajectories of the agents of the swarm model and the desired path Number 1, 2, 3, etc denote agent of the swarm model.

Fig. 2 below shows trajectory of the swarm center and the desired path. From the Figure 2, it can be seen that the swarm center exponentially traces the desired path.

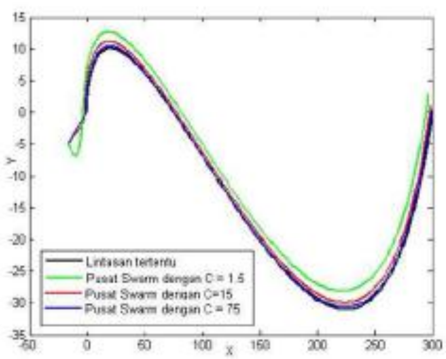

Fig. 2. The swarm center path and the desired path.

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