Experimental and Analytical Investigation of Flexural Behavior of Reinforced Concrete Beam

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***Abstract*-- Experimental based analysis has been widely used as a means to find out the response of individual elements of structure. To study these components finite element analyses are now widely used & become the choice of modern engineering tools for the researcher. In the present study, destructive test on simply supported beam was performed in the laboratory & load-deflection data of that under-reinforced concrete beams was recorded. After that finite element analysis was carried out by ANSYS, SAS 2005 by using the same material properties. Finally results from both the computer modeling and experimental data were compared. From this comparison it was found that computer based modeling is can be an excellent alternative of destructive laboratory test with an acceptable variation of results. In addition, an analytical investigation was carried out for a beam with ANSYS, SAS 2005 with different reinforcement ratio (under, balanced, over). The observation was mainly focused on reinforced concrete beam behavior at different points of interest which were then tabulated and compared. From these observation it shows that 1st cracking location is 0.43L ~ 0.45L from the support. Maximum load carrying capacity at 1st cracking was observed for over reinforced beam but on the other it was the balanced condition beam at ultimate load. Maximum deflection at failure was also observed for the beam that balanced reinforced.**

***Index Term*-- Nonlinear Behavior of Concrete and Steel, 1st Cracking, FEA, MacGregor Model**

I. INTRODUCTION

Concrete structural components exist in buildings and bridges in different forms. Understanding the response of these components during loading is crucial to the development of an overall efficient and safe structure. Different methods have been utilized to study the response of structural components. Experimental based testing has been widely used as a means to analyze individual elements and the effects of concrete strength under loading. While this is a method that produces real life response, it is extremely time consuming and the use of materials can be quite costly. The use of finite element analysis to study these components has also been used. In recent years, however, the use of finite element analysis has increased due to progressing knowledge and capabilities of computer software and hardware. It has now become the choice method to analyze concrete structural components. The use of computer software to model these elements is much faster, and extremely cost-effective.The use of FEA has been the preferred method to study the behavior of concrete (for economic reasons). Anthony J. Wolanski, B.S. (2004), studied "Flexural Behavior of Reinforced and Prestressed Concrete Beams Using Finite Element Analysis" [2]. This simulation work contains areas of study such as Behavior at First Cracking, Behavior at Initial Cracking, Behavior beyond First Cracking, Behavior of Reinforcement Yielding and Beyond, Strength Limit State, Load-Deformation Response of control beam and Application of Effective Prestress, Self-Weight, Zero Deflection, Decompression, Initial Cracking, Secondary Linear Region, Behavior of Steel Yielding and Beyond, Flexural Limit State of prestressed concrete beam[2].

Shing and Tanabe (2001) also put together a collection of papers dealing with inelastic behavior of reinforced concrete structures under seismic loads. The monograph contains contributions that outline applications of the finite element method for studying post-peak cyclic behavior and ductility of reinforced concrete columns, the analysis of reinforced concrete components in bridge seismic design, the analysis of reinforced concrete beam-column bridge connections, and the modeling of the shear behavior of reinforced concrete bridge structures [2].

Kachlakev, Miller, Yim, Chansawat, Potisuk (2001), studied "Finite Element Modeling of Reinforced Concrete Structures Strengthened with FRP Laminates" with ANSYS and the objectives of this simulation was examine the structural behavior of Horsetail Creek Bridge(This historic Bridge, built in 1914, is in use on the Historic Columbia River Highway east of Portland, Oregon, U.S.A), with and without FRP laminates; and establish a methodology for applying computer modeling to reinforced concrete beams and bridges strengthened with FRP laminates [4].

The objective of this paper was to investigate and evaluate the use of the finite element method for the analysis of reinforced concrete beams. Firstly, literature review was conducted to evaluate previous experimental and analytical procedures related to reinforced concrete components. Secondly, a calibration model using a commercial finite element analysis package (ANSYS, SAS 2005) was set up and evaluated using laboratory data. A mild-steel reinforced concrete beam with flexural reinforcement was analyzed to failure and compared to experimental results to calibrate the parameters in ANSYS, SAS 2005 for later analyses. The observation was focused on reinforced concrete beam behavior at first cracking, behavior beyond first cracking, behavior of reinforcement yielding and beyond, strength limit state, load-deformation response, and crack pattern. Discussion of the results obtained for the calibration model is also provided.

At last, an analytical investigation was carried out for a beam with ANSYS, SAS 2005 at different reinforcement ratio (under, balanced, over) and observation was focused on the same as before also comparison of first cracking load, ultimate load, work-done in linear and nonlinear region, and load-deflection nature between these different reinforcement ratio of the analytical beam.

II. METHODOLOGY

A. For experimental and analytical investigation

- Experimental
  - ❑ Mix design of concrete for desired strength
  - ❑ Casting of beams with same proportion as concrete cylinder
  - ❑ Test of concrete cylinder at 7 days and 28 days
  - ❑ Test of mild steel
  - ❑ Test of beam at 28-days
- Analytical
  - ❑ Graphical User Interface (GUI) method with ANSYS
  - ❑ Modeling, Meshing, Solution control, Loading, Solution, General post-processing, Time history post-processing
- Comparison between analytical and experimental results and finally with manual calculation

i) Experimental

TABLE I
COMPRESSIVE STRENGTH OF CONCRETE CYLINDER (28 DAYS)

Sl. No.	Dia, (in).	Load (lb)	Strength (psi)	Average Strength (psi)
1	6	115627	4421	4480
2	6	115627	4421	
3	6	120317	4598	

TABLE II
MILD STEEL TEST DATA

Sp. No.	Area, (in.^{2})	Average area, (in.^{2})	Yield Strength (psi)	Ultimate Strength (psi)
1	0.1196	0.12	40,441	66464
2	0.1183		40,253	66464
3	0.1187		43,806	66464

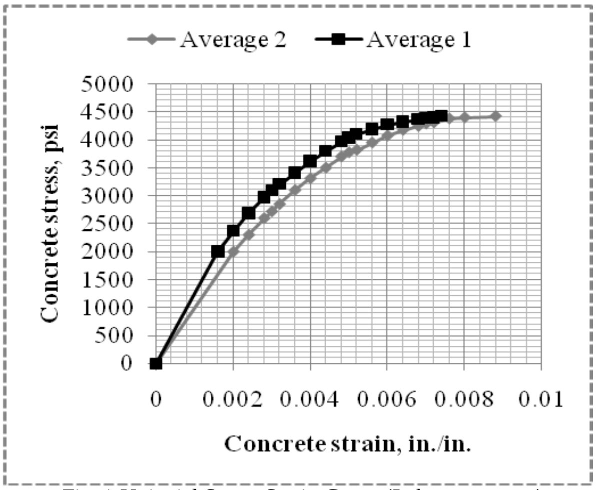


Fig. 1.Uniaxial Stress-Strain Curve (Laboratory test)

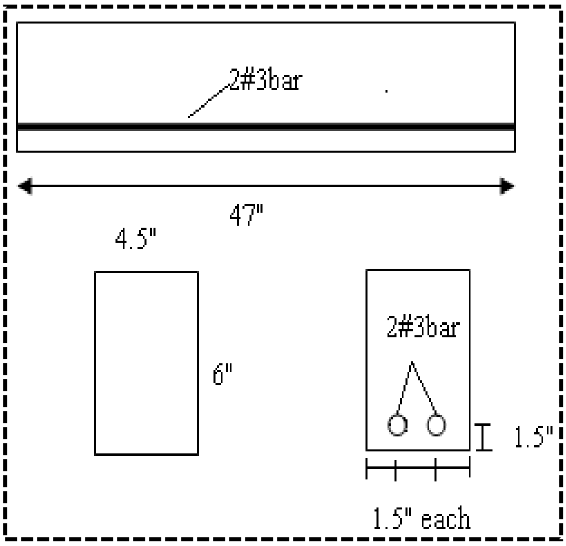


Fig. 2.Typical details for test beam.

Fig. 3. Loading and Supports for the Beam (Laboratory test)

Fig. 4.Different type of crack observed during test of the beam

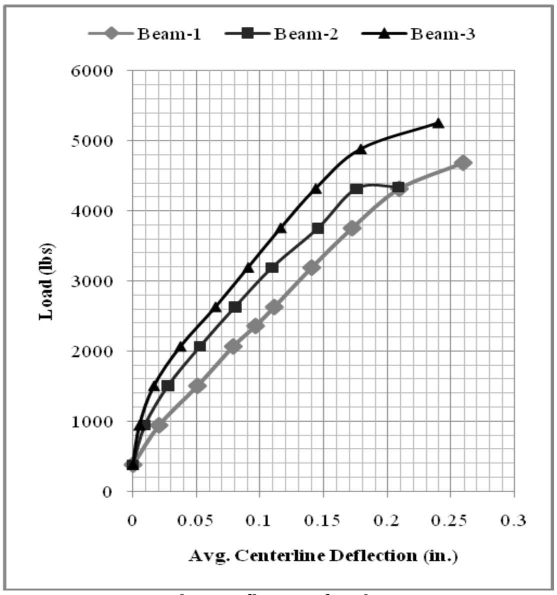


Fig. 5. Load Vs Deflection for the Test Beams

TABLE III
BEAM TEST DATA (28 DAYS)

Cross section of all test beams were 4.5 in.× 6 in. and length of 1st, 2nd, & 3rd beams were 46.75 in., 46.59 in, & 46.25 in. respectively.				
Sp. No.	load at 1st crack (lb)	Ultimate load (lb)	Avg. Deflection (1st crack) (in.)	Avg. Deflection (Ultimate) (in.)
1	2362	4687	0.0963	0.2596
2	3193	4340	0.1094	0.2087
3	2633	5250	0.065	0.2402

ii) Analytical

TABLE IV
ELEMENT TYPES FOR WORKING MODEL

Material Type	ANSYS Element
Concrete	Solid65
Steel Plates and Supports	Solid45
Steel Reinforcement	Link8

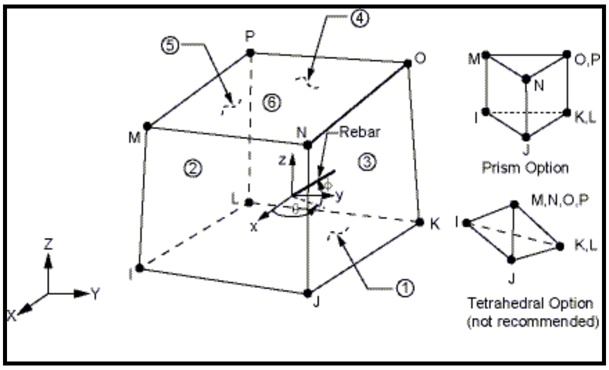


Fig. 6. Solid65 Element, (ANSYS, SAS 2005)[1]

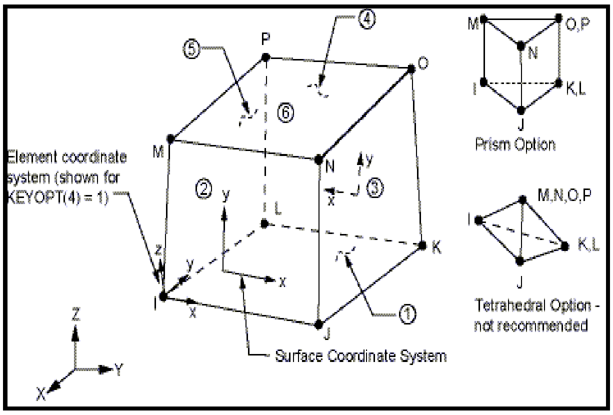


Fig. 7. Solid45 Element (ANSYS, SAS 2005)[1]

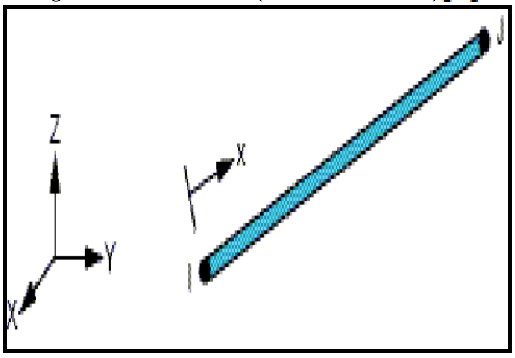


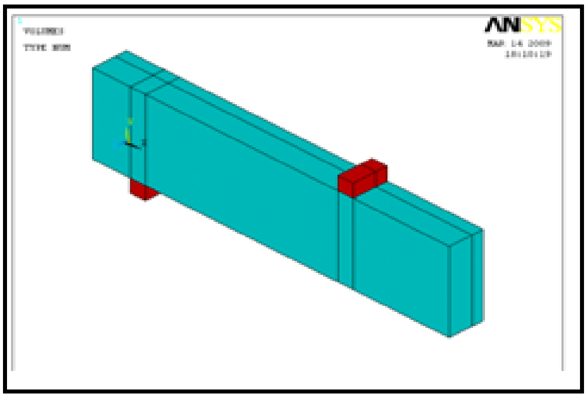
Fig. 8. Link8 Element (ANSYS, SAS 2005)[1]

TABLE V
REAL CONSTANTS FOR CALIBRATION MODEL

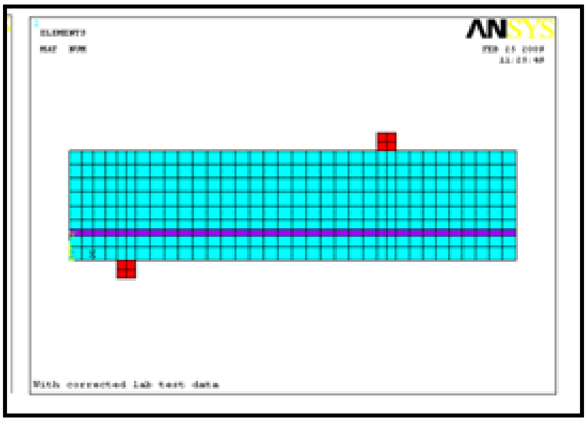
	Element Type	Cross-sectional Area (in^2)
Real Constant 1	Solid 65	0.12
Real Constant 2	Link 8	0.12

TABLE VI
MATERIAL PROPERTIES FOR CALIBRATION MODEL

Material Model Number	Element Type	Material Properties						
1	Solid65	Linear Isotropic						
			Average-1		Average-2		MacGregor Nonlinear model	
		EX	1250000 psi		1000000 psi		1000000 psi	
		PRXY	0.15		0.15		0.15	
		Multilinear Isotropic						
			Lab Test-1 (Average-1)		Lab Test-2 (Average-2)		MacGregor Nonlinear model	
			Strain (in./in.)	Stress (psi)	Strain (in./in.)	Stress (psi)	Strain (in./in.)	Stress (psi)
		Point 1	0.0016	2000	0.002	2000	0.0013266	1326.6
		Point 2	0.0024	2500	0.0028	2500	0.002	1901.768
		Point 3	0.0032	3000	0.0036	3000	0.003	2687.645
		Point 4	0.00374	3500	0.0044	3500	0.004	3315.068
		Point 5	0.004975	4000	0.0056	4000	0.005	3779.774
		Point 6	0.0056	4200	0.0066	4200	0.006	4095.91
		Point 7	0.00633	4300	0.0076	4300	0.007	4287.251
		Point 8	0.0074	4422	0.0088	4422	0.008	4380.1
							0.0088	4400
		Concrete						
		Shear transfer coefficient for open crack.	0.3					
		Shear transfer coefficient for open crack.	1.00					
		Uniaxial tensile cracking stress.	499					
		Uniaxial crushing stress (positive).	4422					
2	Solid45	Linear Isotropic						
		EX	29,000,000 psi					
		PRXY	0.30					
3	Link8	Linear Isotropic						
		EX	29,000,000 psi					
		PRXY	0.30					
		Bilinear Isotropic						
		Yield Stress	41,500 psi					
		Tangent modulus of elasticity	2,900 psi					



(a)



(b)

Fig. 9. (a) and (b). Volumes Created in ANSYS and Mesh of the Concrete, Steel Plate, Steel Support, and reinforcement.
N.B. Comparison of results was carried out with the beam-3 which is simulated in ANSYS with average data-2 since it was best among others.

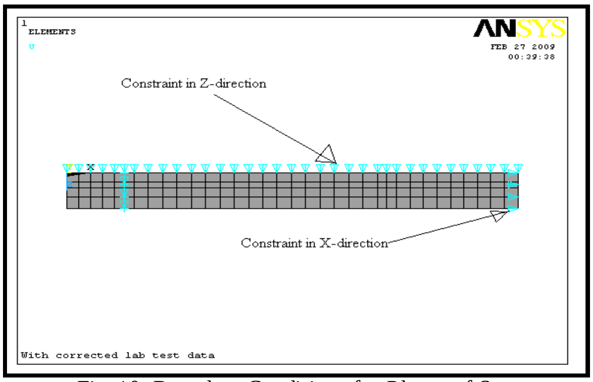


Fig. 10. Boundary Conditions for Planes of Symmetry

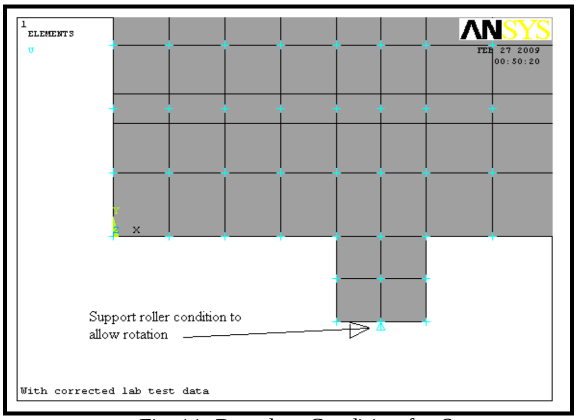


Fig. 11. Boundary Condition for Support

B. For Analytical Investigation

All these analytical beams were flexure control doubly reinforced concrete beam and support condition is simply supported.

Table VII
Specification for the analytical beams

Cross sections of all beams were 10in.✕ 15in. and length of all analytical beams was 15ft. Effective depth, d=11.25” and d’=2.5”			
Reinforcement ratio	f'_c (psi)	f_y (psi)	ρ
Under	4000	60000	0.016533
Balanced	4000	60000	0.028900
Over	4000	60000	0.042133

TABLE VIII
MATERIAL PROPERTIES FOR THE ANALYTICAL BEAMS

Material Model Number	Element Type	Material Properties		
1	Solid65	Linear Isotropic		
			MacGregor Nonlinear model	
		EX	3605000 psi	
		PRXY	0.2	
		Multilinear Isotropic MacGregor Nonlinear model*		
			Strain (in./in.)	Stress (psi)
		Point1	0.000333	1200.5
		Point2	0.0004	1396.7
		Point3	0.0008	2552.5
		Point4	0.0012	3347.8
		Point5	0.0016	3796.1
		Point6	0.002	3979.9
		Point7	0.00222	4000
		Point8	0.003	4000
		Concrete		
		Shear transfer coefficient for open crack.	0.35	
		Shear transfer coefficient for open crack.	1.00	
		Uniaxial tensile cracking stress.	474.34	
		Uniaxial crushing stress (positive).	4000	

Material properties of Solid45 and Link8 elements are same as before except yield stress of Solid65 is 60000 psi.

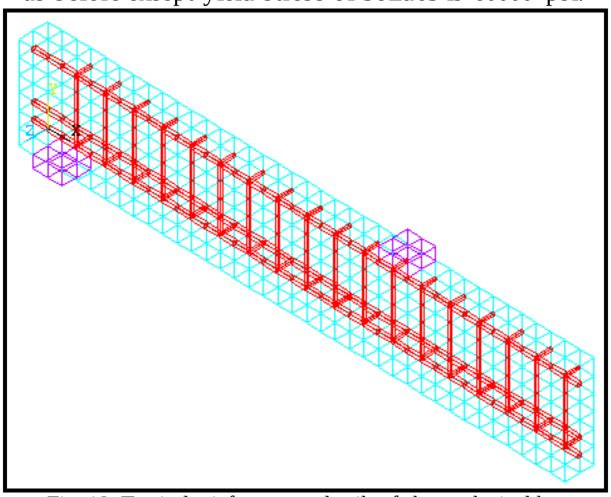


Fig. 12. Typical reinforcement details of the analytical beams (Quarter)

TABLE IX
REINFORCEMENT SPECIFICATION FOR THE ANALYTICAL BEAMS

Reinforcement ratio	Top bar	Bottom bar	Shear reinforcement
Under	2 #5bar	2 #5bar	#3 bar @ 5" C/C
Balanced	2 #7bar	4 #7bar	#4 bar @ 5" C/C
Over	2 #8bar	4 #8bar	#4 bar @ 5" C/C

III. RESULTS AND DISCUSSIONS

A. For experimental and analytical investigation

TABLE X
DEFLECTION AND STRESS COMPARISONS AT FIRST CRACKING

Model	Extreme tension fiber stress (psi)	Reinforcing steel stress (psi)	Centerline deflection (in.)	Load at first cracking (lb)
Manual calculation	513.912	6626.82	0.034126	2564.57
ANSYS	520.15	6908	0.03200	2701.48
Lab test	------*	------*	0.065	2633.00

* means these value couldn't possible to taken from test beam due lack of strain gauge during experiment. Here stress values were calculated and taken at first crack location. Table 11 shows the first crack location.

TABLE XI
LOCATION COMPARISONS AT FIRST CRACKING

First crack	Location from support (in.)	Load (lb)	Ratio of total length
Experimental	17.75	2633	0.4326L
ANSYS	16.99	2702	0.4144L

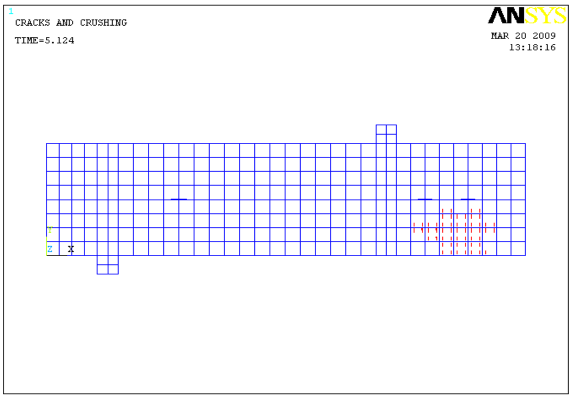


Fig. 13. 1st Crack of the Concrete Model at 2701.48 lb load

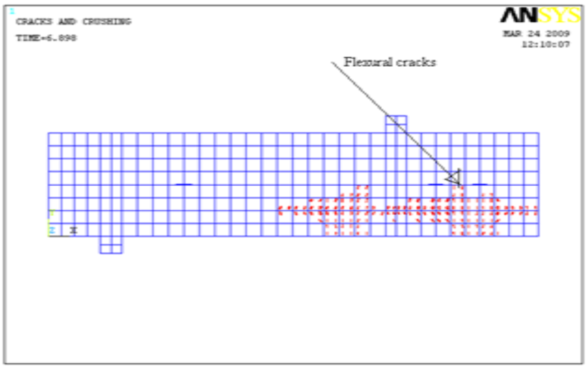


Fig.14. Cracking of the Concrete Model at 3698.4 lb

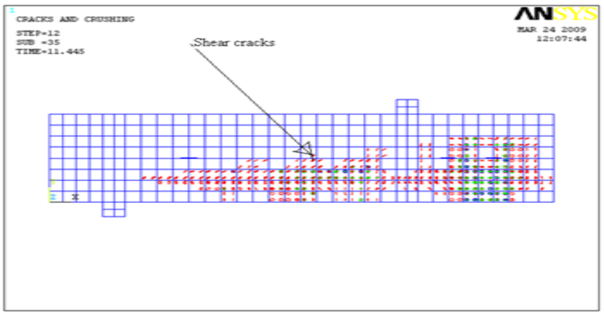
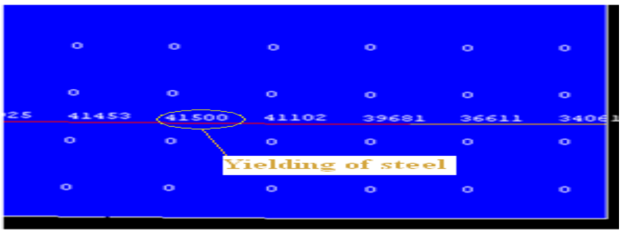
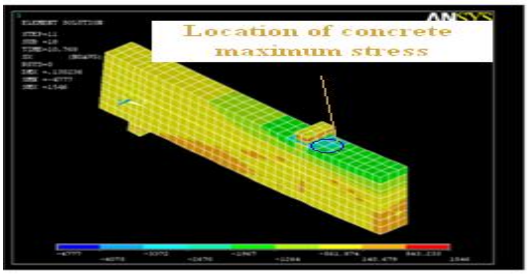


Fig. 15. Cracking of the Concrete Model at 6333.6 lb



(a)



(b)

Fig. 16. (a) and (b). Yielding (41500 psi) of steel at 5603.6 lb load and respective concrete stress in this section is 1986 psi<concrete cylinder strength 4422 psi.

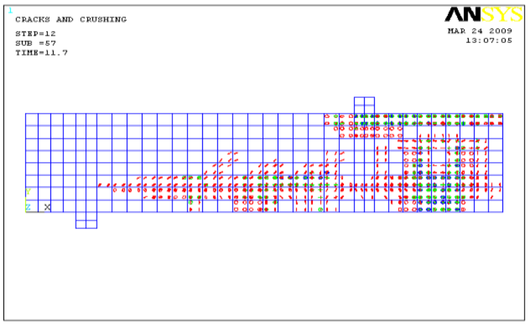


Fig. 17. Failure of the Concrete Beam

TABLE XII
DEFLECTIONS OF TEST VS. FINITE ELEMENT MODEL AT ULTIMATE LOAD

Beam	Load (lb)	Centerline deflection (in.)
Experiment (B-3)	5250	0.2402
ANSYS	6690.4	0.374839

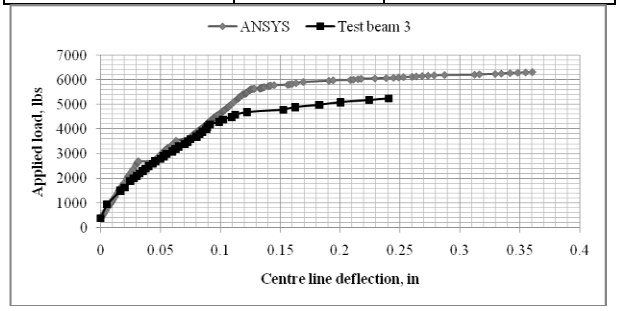


Fig. 18. Load vs. Deflection Curve Comparison of ANSYS and Laboratory test results

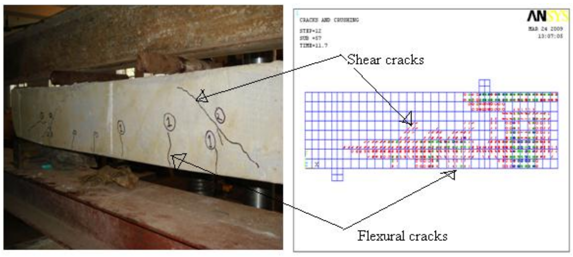


Fig. 19. Comparison of crack pattern in test beam and ANSYS

B. For analytical investigation

TABLE XIII
DEFLECTION AND STRESS COMPARISONS AT FIRST CRACKING AND ULTIMATE LOAD FOR THE ANALYTICAL BEAM

Analytical Model (ANSYS)	Extreme concrete fiber stress (psi) Tensile	Extreme concrete fiber stress (psi) Compressive	Reinforcing steel stress (psi)		Centerline deflection (in.)		Load at first cracking (lb)	Ultimate load (lb)
Under reinforced	496.76	3326.8	3277	60004	0.067581	0.93336	7854	24248
Balanced condition	483.42	4217.5	3021	60003	0.066749	1.007	8315	40128
Over reinforced	490.781	4053	2960	45759	0.065772	0.7440713	8833.6	39065

TABLE XIV
OBSERVATIONS AT FIRST CRACKING

First crack for steel ratio (ANSYS)	Location from support (in.)	Load (lb)	Location in fraction of total length
Under reinforced	77.5	7854	0.4306L
Balanced condition	82.5	8315	0.4583L
Over reinforced	80.0	8833.6	0.4444L

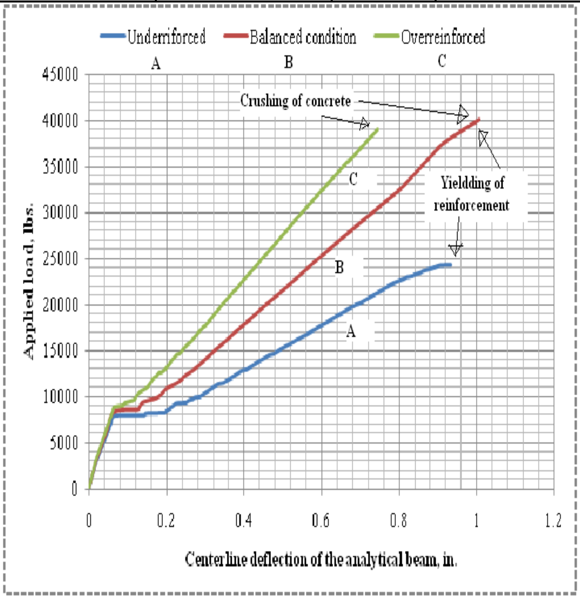


Fig. 20. Applied load vs. beam centerline deflection for the analytical beam at different reinforcement ratio

IV. CONCLUSIONS

A. For the experimental and analytical beams

The following conclusions can be stated based on the evaluation of the analyses of the calibration model.

(1) Deflections and stresses at the centerline along with initial and progressive cracking of the finite element model compare well to experimental data obtained from a reinforced concrete beam. Though some variation was observed in deflection causes due to the following constraints during test-
- Concrete stress-strain data in tests of cylinder was corrected before input in the ANSYS data table because the data was collected manually. And also Poisson's ratio was not possible to determine.
- Support condition was not truly hinge in one end for this reason during increasing load support sliding was observed.

(2) The failure mechanism of a reinforced concrete beam is modeled quite well using FEA and the failure load predicted is very close to the failure load measured during experimental testing.

(3) The analytical beam that was simulated with under reinforced test beam data is also under reinforced since the yielding of deformed bar initiated before the concrete reached its compressive strength capacity (Figure 10).

(4) The first flexural cracking of experimental beam was occurred 2.75” from the beam centerline, which was 3.51” in the ANSYS, SAS 2005. But theoretically this first crack should occur in the bottom face of the beam centerline. From experimental point of view, it was impossible to find out the causes of this variation in location of crack formation. Rather in the model generated by **ANSYS** it was well observed that principal tensile stress was developed earlier at that location mentioned bellow (Figure 18).

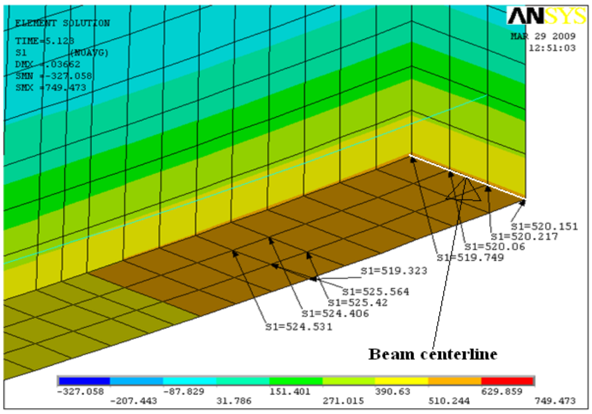


Fig. 21. Contour plot of principal stress

B. For the analytical beam

From the analytical investigation it was observed that under reinforced ratio is the best type of reinforcement ratio among the others since it shows greatest warning zone (Figure14) before failure. Where warning zone for balanced condition and over reinforcement ratios were 81.52% and 28.77% of under reinforcement condition respectively.

Maximum load carrying capacity at 1st cracking was observed for over reinforced beam but on the other it was the balanced condition beam at ultimate load. Maximum deflection at failure was also observed for the beam that balanced reinforced.

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