Strength and Deformation Characteristics of Granular Materials under Extremely Low to High Confining Pressures in Triaxial Compression

Md. Abu Sayeed[1)], Kiichi Suzuki[2)], Md. Mizanur Rahman [3)]

***Abstract*— The shearing strength and volumetric relationship of granular materials change notably with a variation of confining pressure. In this study the effect confining pressure on the mechanical behavior of granular materials were observed using discrete element method (DEM). The triaxial tests were conducted on seven specimens covering a range from extremely low to high confining pressures. Each isotropically compacted specimen contains similar number spherical particles for same porosity. The strength and deformation characteristics show a good agreement with the experimental behavior of granular materials. At low to high confining pressures, there is a decrease in the peak angle of shearing resistance and dilation with the increase of confining pressures. However, the dependency of peak friction angle, dilatancy index and deformation characteristics on extremely low confining pressures are negligible. A unique relationship was also observed between normalized work and axial strain under different confining pressures. Microscopic parameter such as coordination number increases with the increase of confining pressure both for extremely low and low to high confining pressures. However, the tendency of evolution of the coordination number is quite different in extremely low and low to high confining pressures.**

***Index Term*-- Co-ordination number, Discrete element method Extremely low confining pressure, Normalized work.**

I. Introduction

Confining pressure is a vital factor which influences the shearing strength and deformation characteristics of granular materials like sand. Experimentally a number of researchers e.g., [1] have investigated the effect of medium to high confining pressures. Experimental studies [2], [3] have shown that with the increase of confining pressure load bearing capacity increases while the angle of shearing resistance (ϕ) and the dilation of sand decrease with the increase of confining pressure. By contrast, the value of ϕ and deformation characteristics are rather independent of extremely low confining pressure ranges from 5 to 20 kPa [4].

[1]Lecturer, Dept. of Civil Engineering, Rajshahi University of Engineering and Technology (RUET), Rajshahi-6204, Bangladesh. Phone : +8801711340441 (e-mail: **sayeed.ce00@yahoo.com**).
[2]Associate Professor, Graduate School of Science and Engineering, Saitama University, 255 Shimo-Okubo, Sakura-ku, Saitama City, Saitama 338-8570, JAPAN. (e-mail: kiichi95@yahoo.co.jp).
[3]Lecturer, Division of Information Technology, Engineering and the Environment , Mawson Lakes Campus, University of South Australia. (email: Mizanur.Rahman@unisa.edu.au).

However, membrane force is an important factor which adds some extra shear stress with the original confining stress on the specimen. Therefore, to obtain accurate values of ϕ, the effect of membrane force should be properly measured which is a difficult task in experimental studies. Such type of difficulties can be avoided by numerical approaches like discrete element method (DEM) [5] which offers a unique opportunity to obtain qualitative information of macro behavior and micro phenomena as well. Few researchers (e.g., [6]-[8]) investigated the effect of high confining pressures on the behavior of discrete materials using two dimensional DEM. However, extremely low confining pressure has not been considered in DEM based studies. Therefore, in this current study, three dimensional DEM is used to simulate the effect of confining pressures covering a range from extremely low to high on the mechanical behavior of granular materials and to understand the normalized work relationship as well as micro phenomena.

II. Numerical Experiment Details

A. Discrete Element Method and YADE

Reference [5] developed a computer programming code based on the basic elements of grains and their interactions to describe the behavior of granular media in two dimensional case. The DEM employs an explicit time finite difference scheme in which the calculation cycle includes the application of Newton's second law of motion to each particle, followed by the application of a simple force displacement law at all contacts. This tool has been authorized by different researcher, comparing the force vector diagram obtained from the DEM program with the corresponding plots achieved from the photo-elastic analysis of [9]. Currently, the DEM has been proven a sufficiently valid tool for fundamental research in granular materials [10]. This numerical technique also provides an excellent means to investigate the micro-mechanics of granular materials. An open source programming code YADE based on three dimensional DEM which was written by [11] and further updated by [12] is used in this investigation. The classes to extract micro-macro parameters have been programmed in YADE to computerize the extraction of the data. For the details of the YADE, readers are referred to [13].

B. Sample Preparation

To do this study numerical samples were prepared in two steps: initially similar number of particles generation and then isotropic compression. In the first step, sphere particles were randomly generated in a cubic shape frame such that their particle size distribution and initial particle orientation remains same in each sample. Each assembly contains 4000 spheres and their size distributions are shown in Figure 1. Mean radius of spheres was 2.27 mm. Although the discrete particles are irregular shape in nature, however, to keep a low calculation cost, spherical particles were used in this research work. In YADE, the inter-particle friction angle is an intrinsic parameter differs from the bulk friction angle and which relates to the friction generated between the particle surfaces when one particle slides slowly over another. During isotropic compression, different interparticle friction angles were chosen by trial and error method to obtain the target porosity of 39% for all the samples under different confining pressures (Table I). However, the inter-particle friction angles were reset to 30^{o} for all the isotropic samples prior to shear.

C. Material Properties and DEM Parameter

Discrete element analysis requires various parameters, e.g. normal stiffness, shear stiffness, damping etc. Selection of these parameters is challenging issue in discrete element modeling. Legality of results becomes questionable without any justification of input parameters [14]. Besides, different input values may affect simulation which may lead to different conclusions. Generally, these parameters are chosen by certain known behavior of granular materials. To investigate the qualitative behavior of granular materials, these parameters (Table II) were used in this present simulation which maintained the stability criteria.

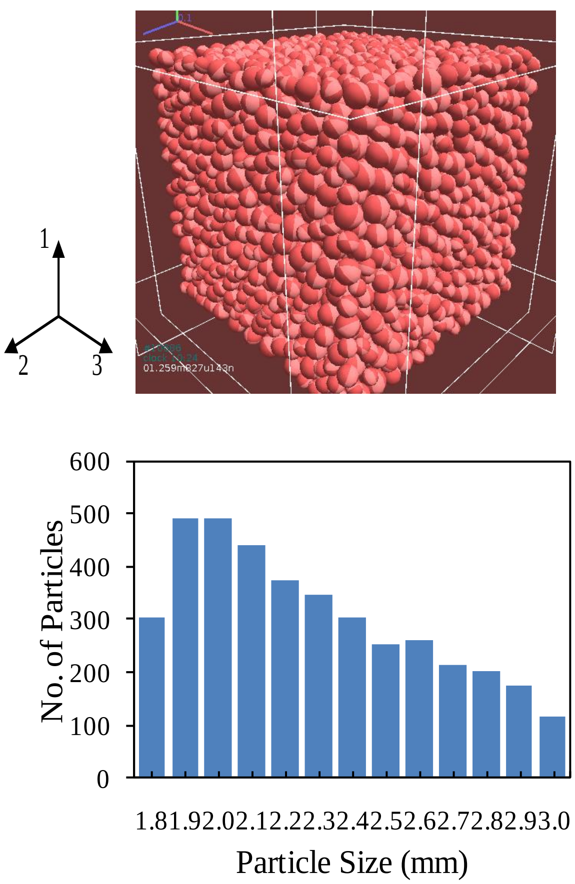

Fig. 1. (a) The numerical specimen and (b) Particle size distribution

TABLE I
The selected inter particle friction angles to obtain target porosity under different confining pressures during isotropic compression

Confining Pressure (kPa)	Inter-particle friction Angle (Degrees)	Porosity of Isotropically compressed sample
5	0.0	39.00%
10	0.045	
20	0.12	
50	0.63	
100	1.54	
200	3.44	
400	8.53	

TABLE II
Input parameters of DEM during triaxial compression

Input parameter	**Values**
Inter-particle friction angle (Degree)	30
Box friction angle (Degree)	0.0
Normal Stiffness, Kn (MN/m)	1.66
Stiffness Ratio (K_s/K_n)	0.5
Damping coefficient (α^{nv})	0.4
Strain rate (S^{-1})	0.001
Stability Criteria	0.01
Particle Density (Kg/m^3)	2650

D. Test procedure

Numerically drained triaxial compression tests were conducted on three isotropically compressed samples under extremely low confining pressure of 5 kPa, 10 kPa and 20 kPa using the updated DEM based programming code YADE (12). Another four samples were also tested under constant confining pressure of 50 kPa, 100 kPa, 200 kPa and 400 kPa. During triaxial compression, deformation of the granular assembly was controlled by six rigid boundary walls. The upper and lower walls moved vertically under strain-controlled condition, while the lateral confining pressure was controlled automatically by a servo mechanism. The numerical experiment was continued until an axial strain of 8%.

III. MACRO MECHANICAL RESPONSES

Figure 2(a) shows the relationship between the deviatoric stresses (q) verses axial strain (ε_1) under extremely low confining pressures, while Figure 2(b) depicts the relationship between deviatoric stresses verses axial strain under low to high confining pressures. Deviatoric stress is measured from the following relationship

$$q = \sigma_1 - \sigma_3 \tag{1}$$

It is observed that deviatoric stress increases with the increase of confining pressure. It is also noted that the deviatoric stress attains its peak state at a small strain level for extremely low confining pressures while the deviatoric stress peaked at a moderate strain level for low to high confining pressures.

Figures 3(a) and 3(b) show the relationship between volumetric strain (ε_v) verses axial strain (ε_1) for different confining pressures. However, the tendency of the evolution of the volumetric strain is quite different. Figure 3(a) clearly shows that the deformation characteristics are rather independent of confining pressures varying from 5 kPa to 20 kPa. When the confining pressure is extremely low, there is a very small restrain in the lateral direction, which allow more flexibility of the lateral walls movement in the horizontal directions and consequently more dilation as axial strain increases. By contrast, Figure 3(b) depicts that samples become more compressible as the confining pressure increases. These simulated results show good consistency at least qualitatively with the experimental results [2].

Figure 4(a) illustrates the relationship between the friction angle (ϕ) and confining pressure at peak stress state for extremely low confining pressures while Figure 4(b) shows the relationship between the angle of shearing resistance and confining pressure at peak stress state for low to high confining pressures. The angle of shearing resistance is obtained from the relationship

$$\sin\phi = (\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3) \tag{2}$$

It is noted that, when the confining pressure is extremely low, the peak angle of shearing resistance is rather independent of confining pressures while it decreases as the confining pressure becomes high. This tendency is also similar with experimental observation [4].

Figures 5(a) and 5(b) illustrate the relationship between dilatancy index (*DI*) verses axial strain for different confining pressures. Dilatancy index is calculated from the following equation

$$DI = \frac{d\varepsilon_v}{d\varepsilon_1} \tag{3}$$

Note that, the trend of dilatancy index is quite different

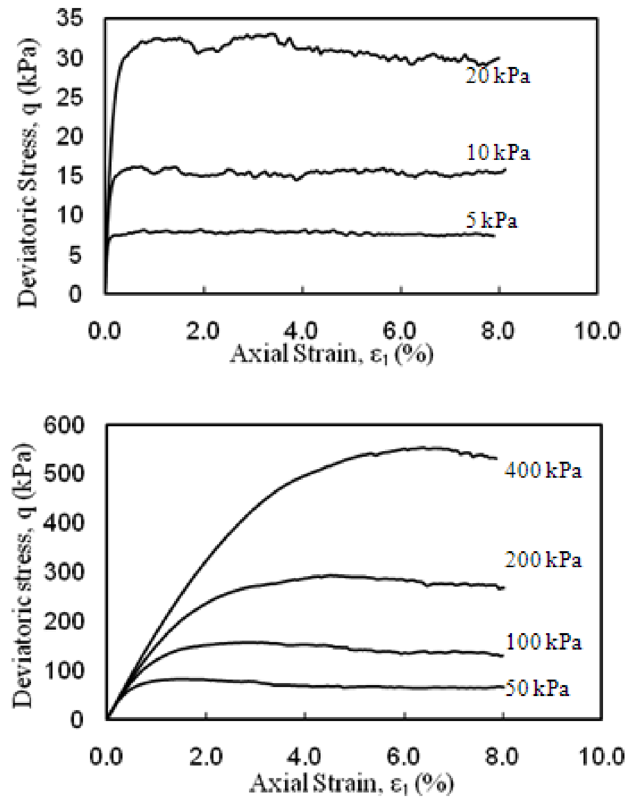

Fig. 2. Relationship between deviatoric stress and axial strain under (a) extremely low confining pressures, (b) low to high confining pressures.

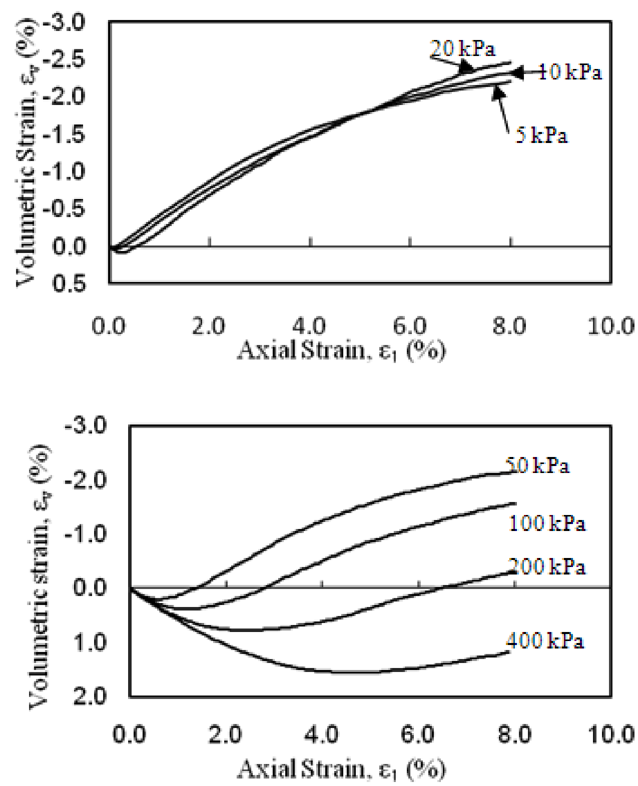

Fig. 3. Volumetric behaviors under (a) extremely low confining pressures, (b) low to high confining pressures.

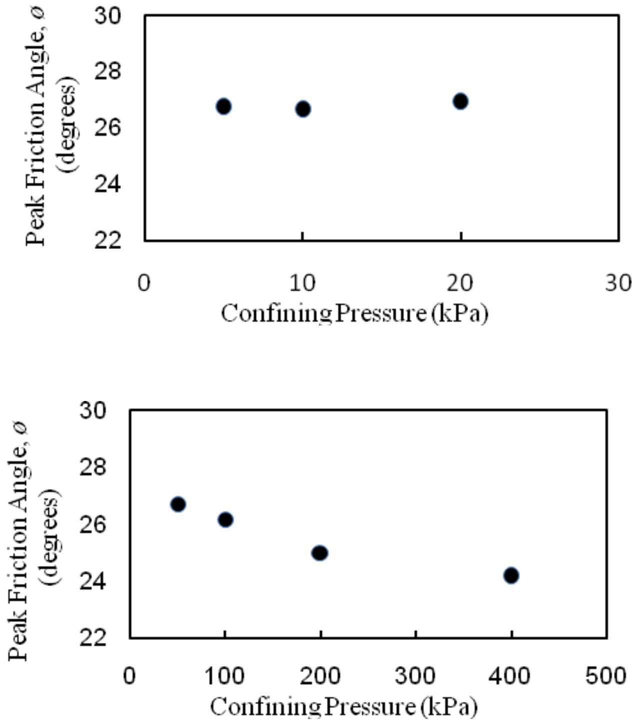

Fig. 4. Variation of peak friction angle for (a) extremely low confining pressures, (b) low to high confining pressures.

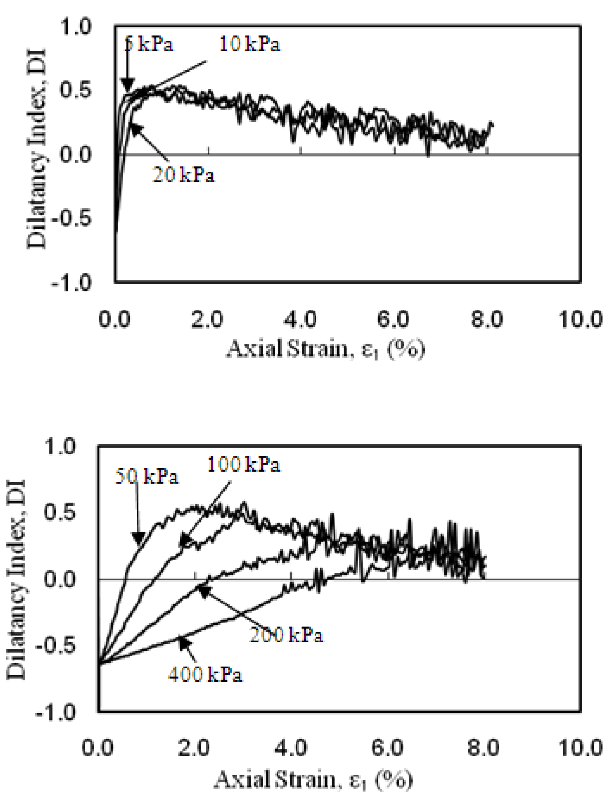

Fig. 5. Variation of dilatancy index with axial strain (a) for extremely low confining pressure, (b) for low to high confining pressure.

under different ranges of confining pressure. Figure 5(a) clearly shows that the dilatancy indexes are rather independent of extremely low confining pressures. On the contrary, Figure 5(b) shows that dilatancy indexes decreases with the increase of confining pressures. However, at residual strain (8%), it is independent of confining pressure.

The dependency of total shear work per unit volume on the confining pressure is also investigated. The work done per unit volume is obtained from the following expression

$$dW = \sigma_1 d\varepsilon_1 + \sigma_2 d\varepsilon_2 + \sigma_3 d\varepsilon_3 \quad (4)$$

where, $dW =$ incremental work, $d\varepsilon_1 =$ incremental axial strain, $d\varepsilon_2$ and $d\varepsilon_3$ is the incremental lateral strain. σ_1 is the vertical stress, σ_2 and σ_3 are lateral confining stresses. The total shear work (W) is than calculated by summation of incremental work. Then the normalized work, W_n is calculated from the following expression

$$W_n = \frac{W}{p} \quad (5)$$

where, p is the mean stress.

$$p = \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{3} \quad (6)$$

Figure 6(a) depicts the relationship between normalized work (W_n) and axial strain (ε_1) under extremely low confining pressures, while Figure 6(b) illustrates the relationship between normalized work (W_n) verses axial strain (ε_1) for low to high confining pressures. The relationships between normalized work with axial strain both for extremely low confining pressures and low to high confining pressures show that the normalized work-axial strain curves pattern are quite similar. It indicated that there is an almost unique relationship exists between normalized work and axial strain both for extremely low and low to high confining pressures.

IV Micro Mechanical Responses

A. *Co-ordination Number*

Coordination number is defined as twice the number of contacts between particles divided by the total number of particles in a specimen. Figures 7(a) and 7(b) show the

variation of coordination number with the axial strain at extremely low and low to high confining pressures respectively. With the increase in confining pressure particles come closer to each other and hence coordination number increases. During shear deformations, at extremely low confining pressures, initially there is a vast number of contact

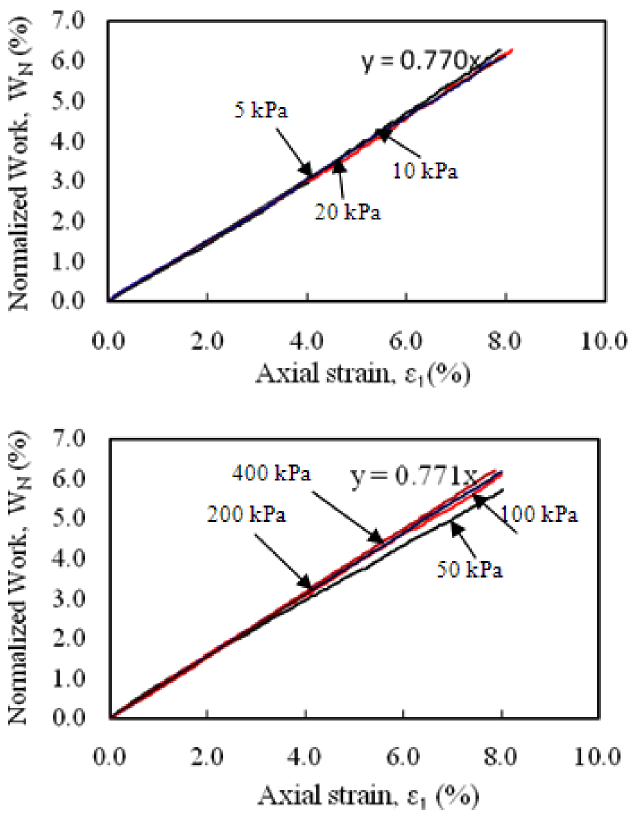

Fig. 6. Relationship between normalized work and axial strain under (a) extremely low confining pressures, (b) low to high confining pressures.

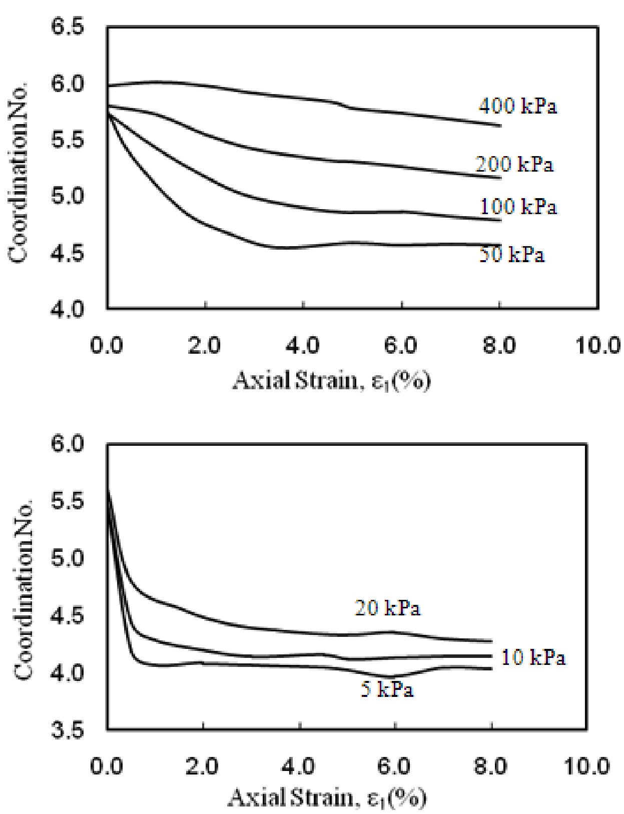

Fig. 7. Evolution of coordination number (a) at extremely low confining pressures, (b) at low to high confining pressures.

collapse related to sudden decrease in coordination number at very low strain and then it gradually reduces till the residual state. It means that the number of contacts formed and contacts collapsed are almost same at the residual state. However, Figure 7(b) depicts that coordination number decreases gradually with the axial strain. That indicates the number of contact collapsed is higher than the number of contact formation during triaxial simulation.

V CONCLUSION

The effect of confining pressure was studied using 3D DEM. The results explained that with the increase of confining pressure, deviatoric stress increases. At low to high confining pressure, there is a decrease in the peak angle of shearing resistance and dilation with the increases of confining pressure. At low strain, dilatancy index also decreases with the increase of confining pressure and at residual strain it is independent of confining pressure. However, the dependency of angle of shearing resistance, deformation characteristics and dilatancy index on confining pressure is very small when confining pressure is less than 20 kPa, similar to real granular materials. The result has verified that the DEM can be employed to simulate and understand the behavior of granular materials at extremely low confining pressure. An almost unique relationship exists between normalized work and axial strain both for extremely low and low to high confining pressures. Microscopic parameter such as coordination number increases with the increase of confining pressure both for extremely low and low to high confining pressure. However, the evolution pattern is quite different depending on the value of confining pressure. More micro results should be found out and finally, a constitutive law based on micromechanical information can be developed in the future

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About First Author:

Md. Abu Sayeed

Mr. Md. Abu Sayeed was born in Nek-Marad in the district of Thakurgaon, in Bangladesh on July 1982. He obtained the degree of Bachelor of Science in Civil Engineering from Rajshahi Univerisity of Engineering & Technology in the year 2005. Just after completing his B.Sc. Engg. Degree, he joined as a faculty member (Lecturer) in the Department of Civil Engineering, Rajshahi Univerisity of Engineering & Technology, Rajshahi, Bangladesh. He also successfully completed his Master of Engineering degree from Saitama University in Japan. At present he is doing teaching and consultancy in RUET.