Research on Calibration Method for the Installation Error of three-axis Acceleration Sensor

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***Abstract*--** **When three-axis acceleration sensor is used to vehicle state measurement, the effect of the installation error cannot be ignored. In order to eliminate the error, a calibration method based on mathematical model is proposed. Without any other auxiliary instruments, the calibration process can be completed by modeling measurement data from the fixed sensor. Experiments result proves the validity of this method, and the calibration error is less than 3%.**

***Index Term*--** **three-axis acceleration sensor ; installation error ; calibration**

1. INTRODUCTION

In recent years, owing to the rapid development of Microelectromechanical Systems(MEMS) , acceleration sensors has been widely used in the area of automotive electronics. These applications usually need to measure acceleration data base on vehicle coordinate system, which requires every axis of acceleration sensor consistent with the one of the vehicle coordinate system[1, 2].However, due to the impact of the coarse installation process, the error between every axis of sensor and the ideal direction usually exists during measurement. The installation errors have crucial impact on measurement accuracy of the vehicle state data, so it is necessary to calibrate them.

In applications which require high-precision measurements such as aircraft, the traditional calibration methods for airborne equipment installation usually need precision measuring instruments like balance level, optical theodolite, laser radar and so on[3]. These instruments are not only high-price but also complex-operation, and to complete a calibration process is time-consuming, so it's not suitable for low cost applications.

In order to minimize the impact of the installation error in the case of ease installation, this paper analyzes and deduces the installation error model, and then establishes the error calibration method according to Euler's theorem. The calibration is completed only by calculating sensor self data, and the process is simple-operation and low cost without the aid of other measuring instruments. Finally this method is proved reliable by simulation test.

2. INSTALLATION ERROR ANALYSIS

When the vehicle state measurement device based on three-axis acceleration sensor is installed on the vehicle, the ideal installation state is to make axes of three-axis accelerometer sensor parallel to the ones of the vehicle coordinate system(Figure 1) [4], in this case, measured and recorded data can reflect the true state of vehicle running. Three-axis acceleration sensor coordinate system has been calibrated at the factory, and three axes are mutually orthogonal[5].

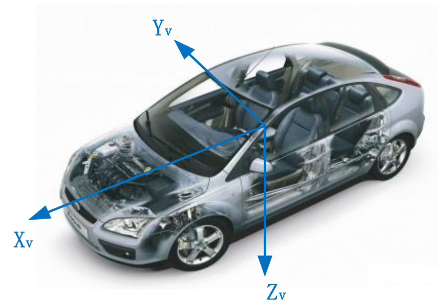

Fig. 1. Vehicle coordinate system

After installation, in fact it's impossible to ensure that three axes of sensor parallel to axes of the vehicle coordinate system, the deviation between the two coordinate systems is defined as the installation error. In order to describe the installation error more intuitively, the rotation matrix is used to express the relative relationship between the two coordinate systems, and Euler angles are used to define installation error angles. According to Euler's theorem, the two space Cartesian coordinate system can completely overlap after three times rotating around the axis[6]. The rotation relationship between sensor coordinate system($X_S Y_S Z_S$)and the vehicle coordinate system ($X_V Y_V Z_V$)is shown on Figure 2. So each rotation angle represents installation error angle, and the clockwise direction of rotation around the axis is defined as the positive direction.

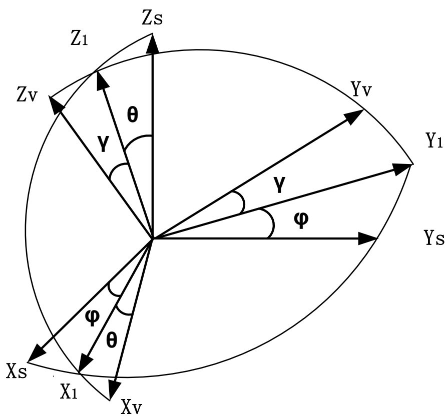

Fig. 2. rotating relation

Rotation process as follows,

$$C_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix}, C_\theta = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, C_\varphi = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The coordinate representation of any space vector A in the sensor coordinate system and that in the vehicle coordinate system satisfy the following equation

$$\overrightarrow{A_V} = C_\gamma C_\theta C_\varphi \cdot \overrightarrow{A_S} \quad [7] \tag{2.2}$$

$$XsYsZs \xrightarrow[\varphi]{\text{around } Zs} X_1Y_1Zs \xrightarrow[\theta]{\text{around } Y_1} XvY_1Z_1 \xrightarrow[\gamma]{\text{around } Xv} XvYvZv$$

Three rotation transformation matrixes are $C_\varphi, C_\theta, C_\gamma$, transformation between coordinate systems as follows

$$\begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} = C_\gamma C_\theta C_\varphi \begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix} \tag{2.1}$$

where

$\overrightarrow{A_V}$ is the vector in vehicle coordinate system, $\overrightarrow{A_V} = [A_{VX} \quad A_{VY} \quad A_{VZ}]^T$, $\overrightarrow{A_S}$ is the vector as the sensor output, $\overrightarrow{A_S} = [A_{SX} \quad A_{SY} \quad A_{SZ}]^T$, the transformation of the vector in coordinate systems is shown as matrix form,

$$\begin{bmatrix} A_{VX} \\ A_{VY} \\ A_{VZ} \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\varphi & -\cos\theta\sin\varphi & \sin\theta \\ \cos\gamma\sin\varphi + \sin\gamma\sin\theta\cos\varphi & \cos\gamma\cos\varphi - \sin\gamma\sin\theta\sin\varphi & -\sin\gamma\cos\theta \\ \sin\gamma\sin\varphi - \cos\gamma\sin\theta\cos\varphi & \sin\gamma\cos\varphi + \cos\gamma\sin\theta\sin\varphi & \cos\gamma\cos\theta \end{bmatrix} \cdot \begin{bmatrix} A_{SX} \\ A_{SY} \\ A_{SZ} \end{bmatrix} \tag{2.3}$$

Equation (2.3) indicate the transformation between the two coordinate systems, the three angles φ, θ, γ of Coefficient matrix is the installation error angle of the sensor. As long as installation error angle φ, θ, γ can be obtained, the value of acceleration in vehicle coordinate system can be calculated through equation (2.3) to overcome the impact of the installation error.

3. CALIBRATION METHOD OF INSTALLATION ERROR

The calibration of installation error is to calculate the three error angle φ、θ、γ. In the stationary state, every axis of the sensor has the output due to the gravity. As the Euler rotation relationship between the vehicle coordinate system and geographic coordinate system, When the vehicle is stationary on a plane, the Euler matrix equation can be established through transformation of acceleration of gravity between coordinate systems[8]. And this equation is used to establish the calibration model of the error together with equation (2.3).

3.1 Calibration model

In order to derive the calibration model of installation error, it's assumed that the vehicle parked on a plane, the angle between this plane and the horizontal plane is α (slope angle), the angle between the front direction of vehicle and the direction of max slope is β, then the relationship between the vehicle coordinate system and the geographic coordinate system can be expressed through twice rotating based on Euler's theorem. First assume that the vehicle parked on the horizontal plane, then the vehicle coordinate system

$X_V Y_V Z_V$ coincides exactly with the geographic coordinate system NEU (Figure 3), the twice rotation process is shown in Fig.4.

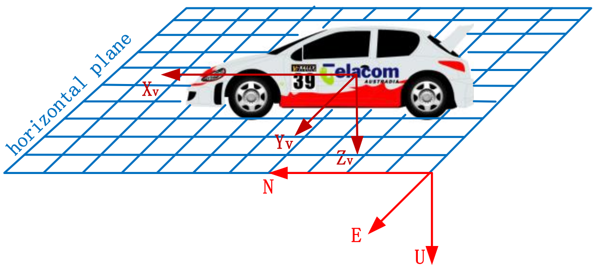

Fig. 3. coordinate systems coincide

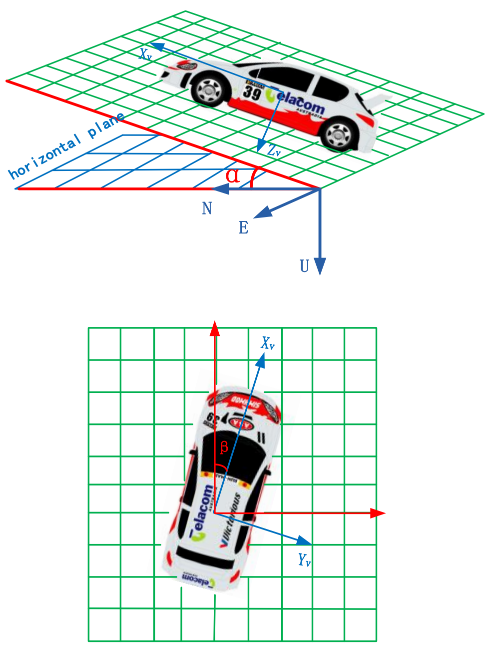

Fig. 4. rotating process

The rotation process is

$$\mathrm{NEU} \xrightarrow[\alpha]{\text{around E}} \mathrm{X_vEZ_v} \xrightarrow[\beta]{\text{around Z}_v} \mathrm{X_vY_vZ_v}$$

$$\begin{bmatrix} \cos\theta\cos\varphi & -\cos\theta\sin\varphi & \sin\theta \\ \cos\gamma\sin\varphi+\sin\gamma\sin\theta\cos\varphi & \cos\gamma\cos\varphi-\sin\gamma\sin\theta\sin\varphi & -\sin\gamma\cos\theta \\ \sin\gamma\sin\varphi-\cos\gamma\sin\theta\cos\varphi & \sin\gamma\cos\varphi+\cos\gamma\sin\theta\sin\varphi & \cos\gamma\cos\theta \end{bmatrix} \begin{bmatrix} A_{SX} \\ A_{SY} \\ A_{SZ} \end{bmatrix} = g \begin{bmatrix} -\cos\beta\sin\alpha \\ \sin\beta\sin\alpha \\ \cos\alpha \end{bmatrix} \tag{3.5}$$

where α is the slope angle, β is the angle between the front direction of vehicle and the direction of max slope, there are the two rotation matrixes, so transformation of any vector in two coordinate system can be expressed as

$$\overrightarrow{A_V} = C_\beta C_\alpha \cdot \overrightarrow{A_G} \tag{3.1}$$

where $\overrightarrow{A_G}$ is the vector in the geographic coordinate system, $\overrightarrow{A_V}$ is the vector after transformation in the vehicle coordinate system. When the vehicle is stationary, its acceleration is affected only by gravity, that is $\overrightarrow{A_G} = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T$, equation (3.1) can be written as

$$\begin{bmatrix} A_{VX} \\ A_{VY} \\ A_{VZ} \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{3.2}$$

(3.2) can become

$$\begin{bmatrix} A_{VX} \\ A_{VY} \\ A_{VZ} \end{bmatrix} = g \begin{bmatrix} -\cos\beta\sin\alpha \\ \sin\beta\sin\alpha \\ \cos\alpha \end{bmatrix} \tag{3.3}$$

From (2.2) and (3.1), we can get a equation as follows

$$\overrightarrow{A_V} = C_\gamma C_\theta C_\varphi \cdot \overrightarrow{A_S} = C_\beta C_\alpha \cdot \overrightarrow{A_G} \tag{3.4}$$

Taking (2.3) and (3.3) into (3.4), it can be written in matrix form as follows

As the angle α is fixed, the component of gravity in the Z_V -axis of vehicle coordinate system is a fixed value, that is

$g\cos\alpha$, which is not change with the angle β . From (3.5),we can obtain a equation as follows

$$(\sin\gamma\sin\varphi-\cos\gamma\sin\theta\cos\varphi)A_{SX}+(\sin\gamma\cos\varphi+\cos\gamma\sin\theta\sin\varphi)A_{SY}+(\cos\gamma\cos\theta)A_{SZ}=g\cos\alpha \tag{3.6}$$

As A_{SX}, A_{SY}, A_{SZ} are the measured value of components of gravity acceleration in every axis, can be calculated from equation group through several measurements. However, due to measurement error, directly solve may lead to no solution or large error. Therefore, numerical analysis is necessary to calculate.

3.2 Installation error solution

Equation (3.6) can be written as a space plane equation

$$A\cdot A_{SX}+B\cdot A_{SY}+C\cdot A_{SZ}=1 \tag{3.7}$$

where the parameters A, B, C can be expressed as

$$\begin{cases} A=\dfrac{\sin\gamma\sin\varphi-\cos\gamma\sin\theta\cos\varphi}{g\cos\alpha} \\ B=\dfrac{\sin\gamma\cos\varphi+\cos\gamma\sin\theta\sin\varphi}{g\cos\alpha} \\ C=\dfrac{\cos\gamma\cos\theta}{g\cos\alpha} \end{cases} \tag{3.8}$$

Assuming that the outputs of three-axis acceleration sensor are coordinates of a space point, then these points distribute on this space plane.

If the space plane equation (3.7) and the slope angle is conformed, equation group (3.8) can be solved to get the installation error angle φ，θ，γ[9].

The A, B, C in space plane equation (3.7) can be solved through the least-squares fitting method, assume that there are n groups measured data of sensor, $(x_i, y_i, z_i), i=1,2...,n$, and the equation of least-squares fitting for space plane is shown as follows

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix}=\begin{bmatrix} \sum^n x_i^2 & \sum^n x_i y_i & \sum^n x_i z_i \\ \sum^n x_i y_i & \sum^n y_i^2 & \sum^n y_i z_i \\ \sum^n x_i z_i & \sum^n y_i z_i & \sum^n z_i^2 \end{bmatrix}^{-1}\begin{bmatrix} \sum^n x_i \\ \sum^n y_i \\ \sum^n z_i \end{bmatrix} \tag{3.9}$$

From (3.9), the value of A, B, C can be calculated[10]. The value of also can be calculated from measured data of acceleration senor. The above-mentioned rotation relationship from sensor coordinate system to the vehicle coordinate system, (2.3) is established through the equation of Euler's theorem. Similarly make three times rotation from the vehicle coordinate system to the sensor coordinate system, three rotation angles are λ, δ, ω ,the equation is given by

$$\begin{bmatrix} A_{SX} \\ A_{SY} \\ A_{SZ} \end{bmatrix}=\begin{bmatrix} \cos\delta\cos\lambda & -\cos\delta\sin\lambda & \sin\delta \\ \cos\omega\sin\lambda+\sin\omega\sin\delta\cos\lambda & \cos\omega\cos\lambda-\sin\omega\sin\delta\sin\lambda & -\sin\omega\cos\delta \\ \sin\omega\sin\lambda-\cos\omega\sin\delta\cos\lambda & \sin\omega\cos\lambda+\cos\omega\sin\delta\sin\lambda & \cos\omega\cos\delta \end{bmatrix}\cdot\begin{bmatrix} A_{VX} \\ A_{VY} \\ A_{VZ} \end{bmatrix} \tag{3.10}$$

From (3.10) and (3.3) ,we can get a equation as

$$\begin{bmatrix} A_{SX} \\ A_{SY} \\ A_{SZ} \end{bmatrix}=g\begin{bmatrix} \cos\delta\cos\lambda & -\cos\delta\sin\lambda & \sin\delta \\ \cos\omega\sin\lambda+\sin\omega\sin\delta\cos\lambda & \cos\omega\cos\lambda-\sin\omega\sin\delta\sin\lambda & -\sin\omega\cos\delta \\ \sin\omega\sin\lambda-\cos\omega\sin\delta\cos\lambda & \sin\omega\cos\lambda+\cos\omega\sin\delta\sin\lambda & \cos\omega\cos\delta \end{bmatrix}\cdot\begin{bmatrix} -\cos\beta\sin\alpha \\ \sin\beta\sin\alpha \\ \cos\alpha \end{bmatrix} \tag{3.11}$$

If the two parking angles are different of 180° ,then the equation to sum the two groups data of sensor is shown as follows

$$\begin{bmatrix} A_{SX}(\beta) \\ A_{SY}(\beta) \\ A_{SZ}(\beta) \end{bmatrix} + \begin{bmatrix} A_{SX}(\beta+\pi) \\ A_{SY}(\beta+\pi) \\ A_{SZ}(\beta+\pi) \end{bmatrix} = g \begin{bmatrix} 2\sin\delta\cos\alpha \\ -2\cos\delta\sin\omega\cos\alpha \\ 2\cos\delta\cos\omega\cos\alpha \end{bmatrix} \quad (3.12)$$

From (3.12), we obtain

$$\cos\alpha = \frac{\sqrt{(A_{SX}(\beta)+A_{SX}(\beta+\pi))^2+(A_{SY}(\beta)+A_{SY}(\beta+\pi))^2+(A_{SZ}(\beta)+A_{SZ}(\beta+\pi))^2}}{2g} \quad (3.13)$$

Now, the value of α can be calculated.

Taking A, B, C and α into equation (3.8), we can get the installation error angles φ，θ，γ by solving the equation group(3.8), and the calibration of installation error is completed.

4 . VERIFICATION OF CALIBRATION METHOD

In order to verify the effectiveness of calibration method, we can test it through simulation experiment. Firstly it's assumed that error angles γ=5°,θ=6°,φ=7°,and the slope angle α =5° and that the parking angle β is 0°,30°,60°,⋯,330°, corresponding simulation data of sensor is shown in Table I, where X，Y，Z are data of every axis. Taking static random measurement error of sensor (±0.001g) into account, add random noise (0.001g) to the simulation data.

Table I
Simulation data

β (°)	0	30	60	90	120	150	180	210	240	270	300	330
X(×10^{-3}g)	-178	-162	-125	-082	-041	-013	-006	-023	-059	-104	-145	-174
Y(×10^{-3}g)	107	149	177	184	167	132	089	046	017	011	027	062
Z(×10^{-3}g)	977	976	975	979	984	992	995	999	999	996	989	983

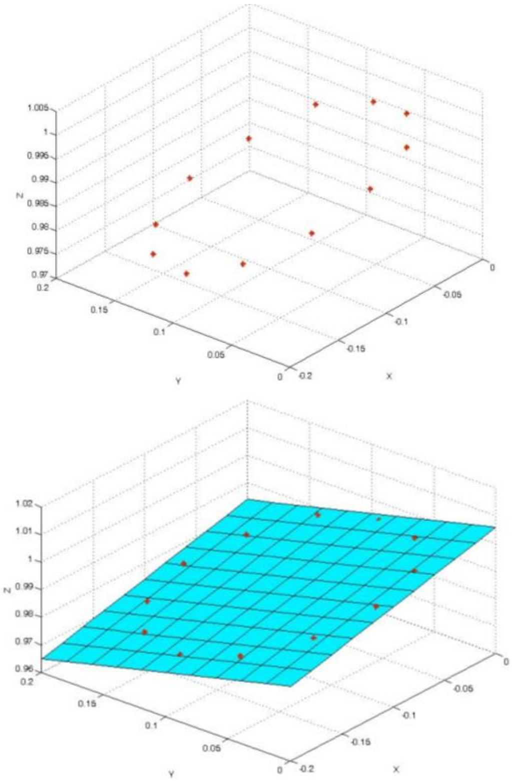

Fig. 5. (a) Distribution of data points fig.5(b) Fitting Plane

The distribution of 12 sets of data points in 3D space is shown in Fig.5(a), the space plane fitted by least-squares fitting is shown in Fig.5(b), the expression of the space plane is

$$-0.0944x+0.1094y+0.9941z=1 \tag{4.1}$$

Taking the data that β is different of 180° into equation (3.13), we can get 6 sets of α . Then the average value α =5.1192° , so the relative error $E_r(\alpha)$=2.38%。

Taking the parameters of (4.1) and into (3.8) we can get equation group as

$$\begin{cases}\sin\gamma\sin\varphi-\cos\gamma\sin\theta\cos\varphi=-0.0940\\ \sin\gamma\cos\varphi+\cos\gamma\sin\theta\sin\varphi=0.1045\\ \cos\gamma\cos\theta=0.9901\end{cases} \tag{4.2}$$

Solving (4.2), we can obtain the value of error angles γ =5.2999° , θ=6.1020° , φ=6.8984° , the relative error of installation error angle E_r=2.84%, so it can meet the need of engineering applications, and verify that the calibration method is effective.

5. CONCLUSION

In this paper, a calibration method for installation error of accelerometers is proposed which is based on mathematical model. The method only needs to park the vehicle on the plane several times without the aid of other precision instruments. And the simulation test verifies that deviation between results of the calibration method and the default value is less than 3% , and it's in an acceptable range, so the calibration method is effective.

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