On Problem of the Solution for the temperature and magnetic field

Nahed S.Hussien

Mathematics Department, Faculty of Science, Taif University, Saudi Arabia

Abstract:

In this paper, we studied the temperature and the magnetic field distribution in the infinitely long, eccentric tube carrying a steady, axial current and placed parallel to an infinite line current. Numerical example and graphs are presented for the temperature in the medium and for the magnetic field components in the region between the two cylinders is considered. The distance between the tube and the line current is let to vary and the results are discussed.. Numerical results are given and illustrated graphically.

1- Introduction

The dynamical problem of thermo elasticity has received much attention in the literature during the past decade. In recent years many thermo elastic which deals with the interactions among strain, temperature has drawn the attention of many researchers because of its extensive uses in divers field, such as geophysics for understanding the effect of Earth's magnetic field on seismic waves, damping of acoustic waves in magnetic field, emissions of electromagnetic radiations from nuclear devices, development of highly sensitive superconducting magnetometer, electrical power engineering, optics etc. Abd-Alla and Mahmoud[1] investigated magneto-thermo elastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model. Numerical solution of magneto-thermo elastic problem in non-homogeneous isotropic cylinder by the finite-difference method presented by Abd-El-Salam et al.[2]. Abd-Alla and Abo-Dahab[3] investigated time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. The thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder presented by El-Naggar et al.[4]. Abd-Alla et al.[5] investigated the thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. Abd-Alla et al.[6] studied the transient thermal stresses in rotating non-homogeneous cylindrically orthotropic composite tubes. Abd-Alla et al. [7] investigated thermal stresses in a rotating non-homogeneous cylindrically orthotropic composite tubes. Kumar and Mukhopadhyay[10], investigated effects of thermal relaxation time on plane wave propagation under two-temperature thermo elasticity, A unified generalized thermo elasticity; solution for cylinders and spheres were studied by A. Bagri, and Eslami[11]. Propagation of harmonic plane waves under thermo elasticity with dual-phase-lags investigated by Prasad et al.[12]. Bagri and Eslami[13] gave generalized coupled thermo elasticity of functionally graded annular disk considering the Lord–Shulman theory. Othman and Singh[14] investigated the effect of rotation on generalized micropolar thermo elasticity for a half-space under five theories. Generalized magneto-thermo elasticity in a perfectly conducting medium presented by Ezzat and Youssef[15]. Abd-Alla et al.[16] investigated Propagation of Rayleigh waves in generalized magneto-thermo elastic orthotropic material Under initial stress and gravity field.

The present problem is simple but physically realistic enough to lead to useful information and better understanding for the early time response of inhomogeneous materials to a dynamic input. In order to keep the formulation general of coupled thermo elasticity are used in the analysis.

The present paper deals with the problem of thermal stresses, we studied the temperature T and the magnetic field components Hi arising from the current in the tube and from the external line current. The analytical expressions for temperature and magnetic field are obtained. The results indicate that the effect of temperature and, magnetic field on tube are very pronounced. Numerical results are given and illustrated graphically.

2-Formulation of the problem

2.1 The temperature.

Equation of heat conduction

$$\nabla^2 T = -\frac{J^2}{k\sigma_e} \qquad \text{for } u_2 < u < u_1 . \tag{2.1}$$

The solution of this equation is looked for in the form

$$T(u,v) = T_H + T_P, \tag{2.2}$$

where T_H satisfies

$$\nabla^2 T_H = 0 \tag{2.3}$$

and T_P is any particular solution of Poisson's equation

$$\nabla^2 T_P = -\frac{J^2}{k\sigma_e}. \tag{2.4}$$

The solution for T_H which takes into account the symmetric properties of the configuration may be easily obtained using the method of separation of variables [5,p. 1212] as

$$T_H(u,v) = \frac{F_0(u-u_1)}{(u_2-u_1)} + L_0 + \sum_{n=1}^{\infty}\left(F_n \frac{\sinh n(u-u_1)}{\sinh n(u_2-u_1)} + L_n \frac{\cosh n(u-u_1)}{\cosh n(u_2-u_1)}\right)\cos nv. \tag{2.5}$$

To find the particular solution T_P, we write

$$\nabla^2 T_P = \frac{1}{h^2}\left(\frac{\partial^2 T_p}{\partial u^2} + \frac{\partial^2 T_p}{\partial v^2}\right) \tag{2.6}$$

$$\nabla^2 T_p = \left(\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2}\right), \tag{2.7}$$

let

$$z = x + iy, \qquad \bar{z} = x - iy \qquad (i = \sqrt{-1}). \tag{2.8}$$

Then

$$x = \frac{z + \bar{z}}{2}, \quad y = \frac{z - \bar{z}}{2i}, \tag{2.9}$$

from which we get

$$\nabla^2 T_p = 4 \frac{\partial^2 T_p}{\partial z \partial \bar{z}}.$$

Therefore,

$$4 \frac{\partial^2 T_p}{\partial z \partial \bar{z}} = -\frac{J^2}{k\sigma_e}.$$

Integration twice, yields

$$4T_p = -\frac{J^2}{k\sigma_e} z\bar{z},$$

i.e

$$T_P(u,v) = -\frac{J^2}{4k\sigma_e}(x+iy)(x-iy)$$

$$= -\frac{J^2}{4k\sigma_e}(x^2 + y^2)$$

$$= -\frac{J^2}{4k\sigma_e} r^2, \tag{2.10}$$

where r is the distance between the origin and any point between the two cylinders. Substituting x and y from (1-17), we obtain

$$r^2 = x^2 + y^2 = a^2 \frac{\sinh^2 u + \sin^2 v}{(\cosh u + \cos v)^2}$$

$$= a^2 \frac{\cosh^2 u - \cos^2 v}{(\cosh u + \cos v)^2} = a^2 \frac{\cosh u - \cos v}{(\cosh u + \cos v)} \tag{2.11}$$

Hence by (2.8)

$$T_P(u,v) = -\frac{J^2 a^2}{4k\sigma_e} \frac{\cosh u - \cos v}{(\cosh u + \cos v)}$$

$$= -\frac{J^2 a^2}{4k\sigma_e}\left(\frac{2\cosh u}{(\cosh u + \cos v)} - 1\right)$$

$$= -\frac{J^2 a^2}{4k\sigma_e}\left(2\coth u \frac{\sinh u}{(\cosh u + \cos v)} - 1\right). \tag{2.10}$$

On using (1.19),

$$T_P(u,v) = -\frac{J^2a^2}{4k\sigma_e}(-1+2\coth u + 4\coth u\sum_{n=1}^{\infty}(-1)^n e^{-nu}\cos nv). \qquad (2.13)$$

Then

$$T(u,v) = \frac{F_0(u-u_1)}{(u_2-u_1)} + L_0 + \sum_{n=1}^{\infty}(F_n\frac{\sinh n(u-u_1)}{\sinh(u_2-u_1)} + L_n\frac{\cosh n(u-u_1)}{\cosh n(u_2-u_1)})\cos nv)$$
$$-\frac{J^2a^2}{4k\sigma_e}(-1+2\coth u + 4\coth u\sum_{n=1}^{\infty}(-1)^n e^{-nu}\cos nv). \qquad (2.14)$$

For this solution to satisfy the boundary conditions

$$T|_{u=u_1} = T|_{u=u_2} = 0, \qquad (2.15)$$

the constants L_0, F_0, F_n, L_n appearing in (2.14) must be taken in the form

$$L_0 = \frac{J^2a^2}{4k\sigma_e}(2\coth u_1 - 1), \qquad F_0 = \frac{J^2a^2}{2k\sigma_e}(\coth u_2 - \coth u_1)$$

$$L_n = \frac{J^2a^2}{k\sigma_e}(-1)^n e^{-2nu_1}\coth u_1 \cosh n(u_2-u_1) \qquad (2.16)$$

$$F_n = \frac{J^2a^2}{k\sigma_e}(-1)^n(e^{-nu_2}\coth u_2 - \coth u_1\cosh n(u_2-u_1)e^{-nu_1}), \quad n = 1,2,...$$

2.2 The magnetic field.

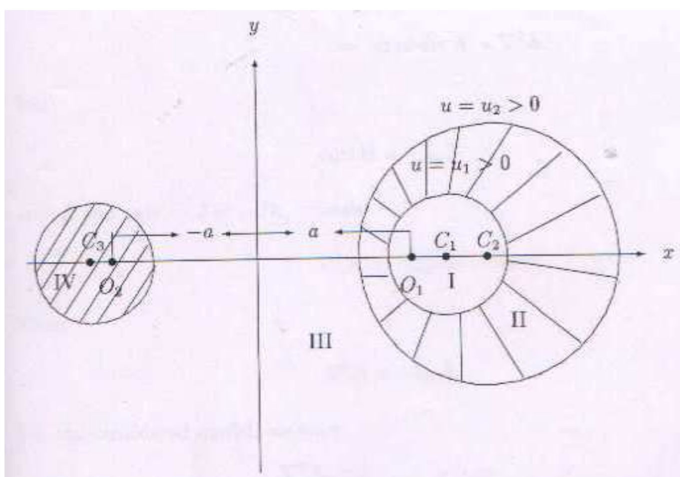

Figure 2.1. The model and the areas I, II, III, IV.

For the solution of the magnetic field in the model of section (1.3), we start by calculating the magnetic field due to an infinitely long tube, the normal cross-section of which is bounded by two non-concentric circles, carrying an axial, steady uniform electric current of density J_1 and placed parallel to a circular cylindrical conductor carrying an axial, steady uniform electric current of density J_2, then we pass to the limit when the external cylinder tends to a line current of intensity I.

We assume, as an approximation, that the magnetic permeability of the elastic body is independent of the deformation, i.e

$$\mathbf{B} \cong \mu_0 \ \mathbf{H}, \qquad (2.17)$$

and since

$$\mathbf{B} = \text{curl } \mathbf{A},$$

with

$$\text{div } \mathbf{A} = 0,$$

where $\mathbf{A} = A$ k is the vector potential with k a unit vector in the positive direction of the z-axis, then

$$\text{curl } \mathbf{B} = \text{curl curl } \mathbf{A}$$

$$= \text{grad div } \mathbf{A} - \nabla^2 \mathbf{A}.$$

But

$$\text{curl } \mathbf{B} = -\mu_0 \mathbf{J}$$

and, in our case, $\mathbf{J} = -J$ k, hence

$$\nabla^2 \mathbf{A} = -\mu_0 J \text{ k}. \qquad (2.18)$$

Then

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}. \qquad (2.19)$$

For the considered model, we have

$$\begin{aligned} &\nabla^2 A_1 = 0 && u > u_1 \\ &\nabla^2 A_2 = -\mu_0 J_1 && u_2 < u < u_1 \\ &\nabla^2 A_3 = 0 && u_3 < u < u_2 \qquad (2.20) \\ &\nabla^2 A_4 = -\mu_3 J_2 && u < u_3. \end{aligned}$$

The symbols A_1, A_2, A_3 and A_4 denote the restrictions of the function $A(u,v)$ to the areas I, II, III and IV of figure (2.1) respectively. Similar to the case of equation (2.1), the general solution of equation

$$\nabla^2 \mathbf{A} = b \qquad (2.21)$$

is looked for in the form

$$A = A_H + A_P, \qquad (2.22)$$

where A_H is the general solution of Laplace equation

$$\nabla^2 \mathbf{A}_H = 0 \qquad (2.23)$$

and A_P is any particular solution of Poisson's equation

$$\nabla^2 \mathbf{A}_P = b. \qquad (2.24)$$

The general solution of $\nabla^2 \mathbf{A}_H = 0$, which is even in the variable v, is obtained by the method of separation of variables [6] in the form

$$A_H = B_0 u + C_0 + \sum_{n=1}^{\infty} \frac{1}{n}(B_n e^{nu} + C_n e^{-nu}) \cos nv.$$

Following (2.10), a particular solution of equation (2.21) may be taken as:

$$A_P = \frac{b}{4} r^2 = \frac{a^2 b}{4} \frac{\cosh u - \cos v}{(\cosh u + \cos v)} = \frac{a^2 b}{4} \frac{2\cosh u - \cosh u - \cos v}{(\cosh u + \cos v)}$$

$$= \frac{a^2 b}{4}(-1 + \frac{2\cosh u}{(\cosh u + \cos v)}) = \frac{a^2 b}{4}(-1 + 2\coth u \frac{\sinh u}{(\cosh u + \cos v)})$$

$$= \frac{a^2 b}{4}(-1 + 2\coth u \frac{x}{a}) \qquad (2.25)$$

$$x = a\ (\text{sig}\ u)\ [1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-nu} \cos nv]. \qquad (2.26)$$

Hence, the general solution of (2.24) is given by

$$A(u,v) = B_0 u + C_0 + \sum_{n=1}^{\infty} \frac{1}{n}(B_n e^{nu} + C_n e^{-nu}) \cos nv$$

$$+ \frac{a^2 b}{4}(-1 + 2\coth u \frac{x}{a}). \qquad (2.27)$$

Using (2.27), the solution of equation $(2.20)_1$ which is finite for $u > u_1$ takes the form:

$$A_1 = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{n} e^{-n(u-u_1)} \cos nv, \qquad (2.28)$$

where A_1, A_n, $n = 1,2,...$ are arbitrary constants to be determined. Similarly, the solution of equation $(2.20)_2$ is obtained from (2.27) by setting $b = -\mu_0 J$ as $\mu_1 \to \mu_0$

$$A_2 = B_0 u + C_0 + \sum_{n=1}^{\infty} \frac{1}{n}(B_n e^{nu} + C_n e^{-nu}) \cos nv$$

$$- \frac{J_1 \mu_0 a^2}{4}(-1 + 2\coth u + 4\coth u \sum_{n=1}^{\infty} (-1)^n e^{-nu} \cos nv). \qquad (2.29)$$

Also, the solution of equation $(2.20)_3$ is written from. (2.24) by setting $b = 0$ as

$$A_3 = \alpha_0 u + r_0 + \sum_{n=1}^{\infty} \frac{1}{n}(\alpha_n e^{n(u-u_1)} + r_n e^{-n(u-u_1)}) \cos nv. \qquad (2.30)$$

The finite solution of equation $(2.20)_4$ is written from (2.27) by setting $b = -\mu_3 J_2$ as $\mu_2 \to \mu_3$

$$A_4 = D_0 + \sum_{n=1}^{\infty} \frac{D_n}{n} e^{nu} \cos nv - \frac{J_2 \mu_3}{4} r^2 \tag{2.31}$$

$$r^2 = a^2(-1 + 2\coth u \frac{x}{a}). \tag{2.32}$$

In this region, $u < 0$. Therefore,

$$x = -a(1 + 2\sum_{n=1}^{\infty} (-1)^n e^{-n|u|} \cos nv) \tag{2.33}$$

which leads, together with (2.31), to

$$A_4 = D_0 + \sum_{n=1}^{\infty} \frac{D_n}{n} e^{nu} \cos nv$$

$$+ \frac{a^2 J_2 \mu_3}{4}(1 + 2\coth u + 4\coth u \sum_{n=1}^{\infty} (-1)^n e^{nu} \cos nv). \tag{2.34}$$

Now, since

B = curl **A** = μ **H**,

the two components of the magnetic field with respect to the bipolar system of coordinates are

$$H_u = \frac{1}{\mu h} \frac{\partial A}{\partial v}$$

$$H_v = -\frac{1}{\mu h} \frac{\partial A}{\partial u} \tag{2.35}$$

From (2.28) and (2.35), one has

$$H_u^{(1)} = -\frac{1}{h} \sum_{n=1}^{\infty} A_n e^{-n(u-u_1)} \sin nv \tag{2.36}$$

$$H_v^{(1)} = -\frac{1}{h} \sum_{n=1}^{\infty} A_n e^{-n(u-u_1)} \cos nv \tag{2.37}$$

From (2.29) and (2.35), we get

$$H_u^{(2)} = -\frac{1}{\mu_0 h}[\sum_{n=1}^{\infty} (B_n e^{nu} + C_n e^{-nu}) \sin nv$$

$$- a^2 J_1 \mu_0 \coth u \sum_{n=1}^{\infty} (-1)^n e^{-nu} n \sin nv] \tag{2.38}$$

$$H_v^{(2)} = -\frac{1}{\mu_0 h}[B_0 + \sum_{n=1}^{\infty} (B_n e^{nu} - C_n e^{-nu}) \cos nv + \frac{a^2 J_1 \mu_0}{4}[2\cos\text{ech}^2 u$$

$$+4\,\mathrm{cosech}^2 u\sum_{n=1}^{\infty}(-1)^n e^{-nu}\cos nv + 4\coth u\sum_{n=1}^{\infty}(-1)^n e^{-nu}\cos nv]] \quad (2.39)$$

From (2.30), using (2.35) we get

$$H_u^{(3)} = -\frac{1}{h}\sum_{n=1}^{\infty}\alpha_n e^{n(u-u_2)} + r_2 e^{-n(u-u_2)})\sin nv, \qquad (2.40)$$

$$H_v^{(3)} = -\frac{1}{h}[\alpha_0 + \sum_{n=1}^{\infty}(\alpha_n e^{n(u-u_2)} - r_n e^{-n(u-u_2)})\cos nv]. \qquad (2.41)$$

From (2.34), one obtains with the help of (2.35)

$$H_u^{(4)} = -\frac{1}{\mu_3 h}[\sum_{n=1}^{\infty}D_n e^{nu}\sin nv + a^2 J_2\mu_3\coth u\sum_{n=1}^{\infty}(-1)^n e^{nu}\sin nv] \qquad (2.42)$$

$$H_v^{(4)} = -\frac{1}{\mu_3 h}[\sum_{n=1}^{\infty}D_n e^{nu}\cos nv - \frac{a^2 J_2\mu_3}{4}(2\,\mathrm{cosech}^2 u$$

$$+4\,\mathrm{cosech}^2 u\sum_{n=1}^{\infty}(-1)^n e^{-nu}\cos nv - 4\coth u\sum_{n=1}^{\infty}(-1)^n e^{nu} n\cos nv)]. \quad (2.43)$$

Applying the boundary conditions

$$\begin{aligned} H_v^{(1)} &= H_v^{(2)}, B_u^{(1)} = B_u^{(1)} \quad \text{at } u = u_1 \\ H_v^{(2)} &= H_v^{(3)}, B_u^{(2)} = B_u^{(3)} \quad \text{at } u = u_2 \\ H_v^{(3)} &= H_v^{(4)}, B_u^{(3)} = B_u^{(4)} \quad \text{at } u = u_3, \end{aligned} \qquad (2.44)$$

we get. using (2.36)-(2.43)

$$B_0 = -\frac{a^2 J_1\mu_0}{2}\,\mathrm{cosech}^2 u_1,$$

$$\alpha_0 = \frac{1}{\mu_0}(B_0 + \frac{a^2 J_1\mu_0}{2}\,\mathrm{cosech}^2 u_2) \qquad (2.45)$$

Then

$$\alpha_0 = \frac{a^2 J_1}{2}(\mathrm{cosech}^2 u_2 - \mathrm{cosech}^2 u_1) \qquad (2.46)$$

and also

$$\alpha_0 = -\frac{a^2 J_2}{2}\,\mathrm{cosech}^2 u_3. \qquad (2.47)$$

For the uniqueness of α_0 it is required that

$$J_1(\operatorname{cosech}^2 u_2 - \operatorname{cosech}^2 u_1) = -J_2 \operatorname{cosech}^2 u_3 = -I, \quad (2.48)$$

where I is the total current in both regions $u_2 < u < u_1$, $u < u_3$. We get from the other boundary conditions system of (6) equations which have to be solved to get B_n, C_n, A_n, α_n, r_n and D_n in the form

$$A_n = B_n e^{nu_1} + C_n e^{-nu_1} - a^2 J_1 \mu_0 n \coth u_1 (-1)^n e^{-nu_1} \quad (2.49)$$

$$\mu_0 A_n = -a^2 J_1 \mu_0 (\operatorname{cosech}^2 u_1 + n \coth u_1)(-1)^n e^{-nu_1} - B_n e^{nu_1} + C_n e^{-nu_1} \quad (2.50)$$

$$\mu_0(\alpha_n - r_n) = B_n e^{nu_2} - C_n e^{-nu_2} + J_1 \mu_0 a^2 (\operatorname{cosech}^2 u_2 + n \coth u_2)(-1)^n e^{-nu_2} \quad (2.51)$$

$$\alpha_n + r_n = B_n e^{nu_2} + C_n e^{-nu_2} - a^2 J_1 \mu_0 n \coth u_2 (-1)^n e^{-nu_1} \quad (2.52)$$

$$\mu_3(\alpha_n e^{n(u_3-u_2)} - r_n e^{-n(u_3-u_2)} = D_n e^{nu_3} - a^2 J_2 \mu_3 (\operatorname{cosech}^2 u_3 - n \coth u_3)(-1)^n e^{-nu_3} \quad (2.53)$$

$$\alpha_n e^{n(u_3-u_2)} + r_n e^{-n(u_3-u_2)} = D_n e^{nu_3} + a^2 J_2 \mu_3 n \coth u_3 (-1)^n e^{-nu_3}. \quad (2.54)$$

The solution of this system leads to

$$2\mu_0 \alpha_n = (\mu_0 + 1) B_n e^{nu_2} + (\mu_0 - 1) C_n e^{-nu_2} + a^2 J_1 \mu_0 (\operatorname{cosech}^2 u_2 + n(1-\mu_0) \coth u_2)(-1)^n e^{-nu_2} \quad (2.55)$$

$$2\mu_0 r_n = (\mu_0 - 1) B_n e^{nu_2} + (\mu_0 + 1) C_n e^{-nu_2} - a^2 J_1 \mu_0 (\operatorname{cosech}^2 u_2 + n(1+\mu_0) \coth u_2)(-1)^n e^{-nu_2} \quad (2.56)$$

$$A_n(1-\mu_0) = 2B_n e^{nu_1} + a^2 J_1 \mu_0 (-1)^n e^{-nu_1} \operatorname{cosech}^2 u_1. \quad (2.57)$$

The constants B_n and C_n are obtained as

$$\left(\frac{(1+\mu_0)(1-\mu_3)e^{nu_3} + (1+\mu_3)(\mu_0 - 1)e^{-n(u_3-2u_2)}}{(\mu_0 - 1)(1-\mu_3)e^{n(u_3-2u_2)} + (1+\mu_3)(1+\mu_0)e^{-nu_3}} - \frac{\mu_0 + 1}{\mu_0 - 1} e^{2nu_1} \right) B_n$$

$$= \frac{2a^2 J_1 \mu_0 \mu_3 \operatorname{cosech}^2 u_3}{((\mu_0 - 1)(1-\mu_3)e^{n(u_3-2u_2)} + (1+\mu_0)(1+\mu_3)e^{-nu_3})}$$

$$-(-1)^n \operatorname{cosech}^2 u_2 a^2 J_1 \mu_0 \frac{((1-\mu_3)e^{n(u_3-2u_2)} - (1+\mu_3)e^{-nu_3})}{((1+\mu_0)(1+\mu_3)e^{-nu_3} + (\mu_0-1)(1-\mu_3)e^{n(u_3-2u_2)})}$$

$$+a^2 J_1 \mu_0 n(-1)^n \coth u_2 + (-1)^2 \frac{a^2 J_1 \mu_0}{(\mu_0-1)}(\operatorname{cosech}^2 u_1 + n(1-\mu_0)\coth u_1) \quad (2.58)$$

$$\left(\frac{(\mu_0-1)(1-\mu_3)e^{n(u_3-2u_2)} + (1+\mu_0)(1+\mu_3)e^{-nu_3}}{(\mu_0-1)(1-\mu_3)e^{n(u_3-2u_2)} + (1+\mu_3)(1+\mu_0)e^{-nu_3}} - \frac{\mu_0-1}{\mu_0+1}e^{-2nu_1}\right)C_n$$

$$= \frac{2a^2 J_2 \mu_0 \mu_3 \operatorname{cosech}^2 u_3}{((1+\mu_0)(1-\mu_3)e^{nu_3} + (1+\mu_3)(\mu_0-1)e^{-n(u_3-2u_2)})}$$

$$-(-1)^n \operatorname{cosech}^2 u_2 a^2 J_1 \mu_0 \frac{((1-\mu_3)e^{n(u_3-2u_2)} - (1+\mu_3)e^{-nu_3})}{((1+\mu_0)(1-\mu_3)e^{nu_3} + (1+\mu_3)(\mu_0-1)e^{-n(u_3-2u_2)})}$$

$$-(-1)^n n \coth u_2 a^2 J_1 \mu_0 \frac{((1-\mu_0)(1-\mu_3)e^{n(u_3-2u_2)} - (1+\mu_0)(1+\mu_3)e^{-nu_3})}{((1+\mu_0)(1-\mu_3)e^{nu_3} + (1+\mu_3)(\mu_0-1)e^{-n(u_3-2u_2)})}$$

$$+(-1)^2 \frac{a^2 J_1 \mu_0}{(1+\mu_0)} e^{-2nu_1}(\operatorname{cosech}^2 u_1 + n(1-\mu_0)\coth u_1). \quad (2.59)$$

In what follows, we shall be interested In the case where the external cylinder carrying the current tends to be a line current. In this case, the above coefficients B_n and C_n take the form

$$B_n = (-1)^n a^2 J_1 \mu_0 \frac{n(\mu_0^2-1)(\coth u_2 - \coth u_1) + (\mu_0-1)\operatorname{cosech}^2 u_2}{(\mu_0-1)^2 e^{2nu_2} - (1+\mu_0)^2 e^{2nu_1}}$$

$$+(-1)^n a^2 J_1 \mu_0 \frac{(1+\mu_0)\operatorname{cosech}^2 u_1}{(\mu_0-1)^2 e^{2nu_2} - (1+\mu_0)^2 e^{2nu_1}} \quad (2.60)$$

$$C_n = (-1)^n a^2 J_1 \mu_0 \frac{(1+\mu_0)e^{2nu_1}\operatorname{cosech}^2 u_2 + (\mu_0-1)e^{2nu_2}\operatorname{cosech}^2 u_1}{(1+\mu_0)^2 e^{2nu_3} - (\mu_0-1)^2 e^{2nu_2}}$$

$$+(-1)^n a^2 J_1 \mu_0 \frac{n((1+\mu_0)^2 e^{2nu_1}\coth u_2 - (\mu_0-1)^2 e^{2nu_2}\coth u_1)}{(1+\mu_0)^2 e^{2nu_1} - (\mu_0-1)^2 e^{2nu_2}}, \quad (2.61)$$

A_n, α_n, r_n are obtained from (2.54), (2.55), (2.56) by substituting for B_n and C_n.

3 Numerical results and discussion.

We have considered a numerical example for which we have chosen

$$\mu_0 = 0.9999$$

and

$$r_1 = 0.5\, r_2.$$

In this case,

$$\frac{d}{r_2} = \frac{3}{8}\frac{r_2}{a}$$

We have allowed the ratio $\frac{a}{r_2}$ to vary in the limits from 1 to 10, in order to take the line current farther from the tube, thus weakening its effect on the latter. As noticed above, when the ratio $\frac{a}{r_2}$ increases indefinitely, the boundary of the internal circle moves away from the boundary of the external one at $v = 0$ and the eccentricity of the tube becomes smaller.

The calculations for the temperature and for the magnetic field components were carried out using 20 terms of each series.

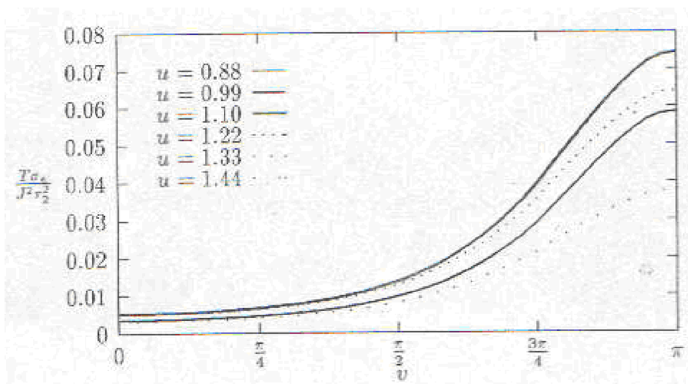

Figure 2.2. The temperature for $\frac{a}{r_2} = 1$.

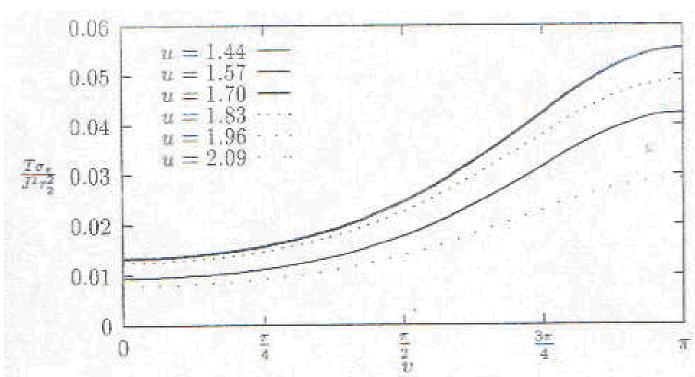

Figure 2.3. The temperature for $\frac{a}{r_2} = 2$.

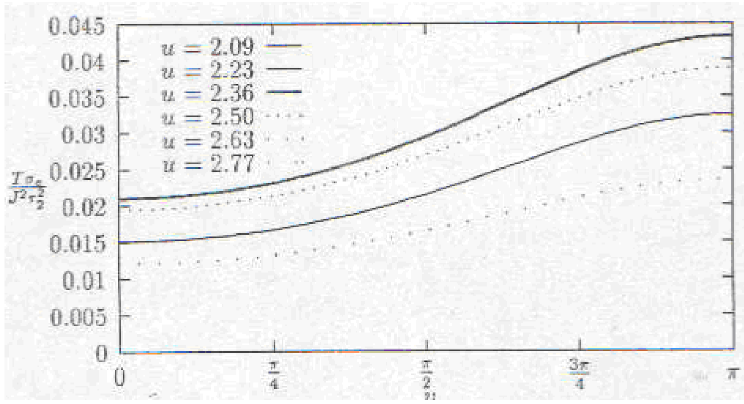

Figure 2.4. The temperature for $\frac{a}{r_2} = 4$.

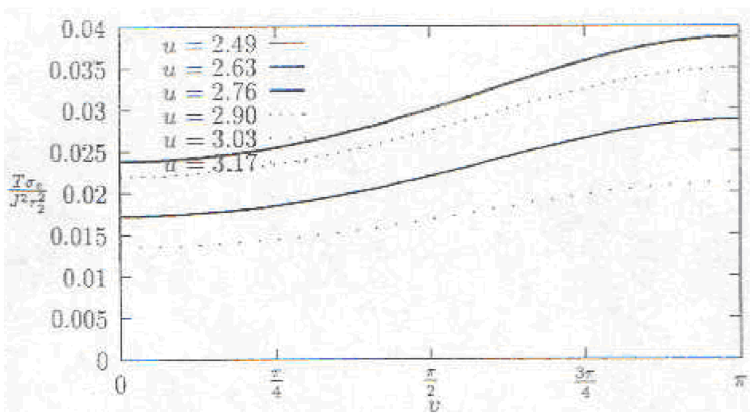

Figure 2.5. The temperature for $\frac{a}{r_2} = 6$.

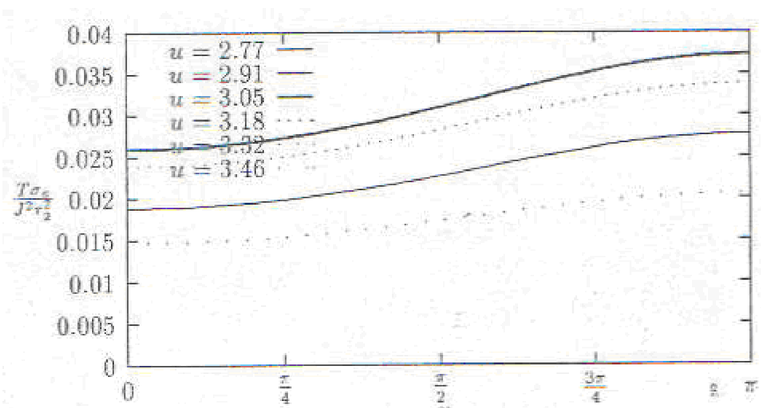

Figure 2.6. The temperature for $\frac{a}{r_2} = 8$.

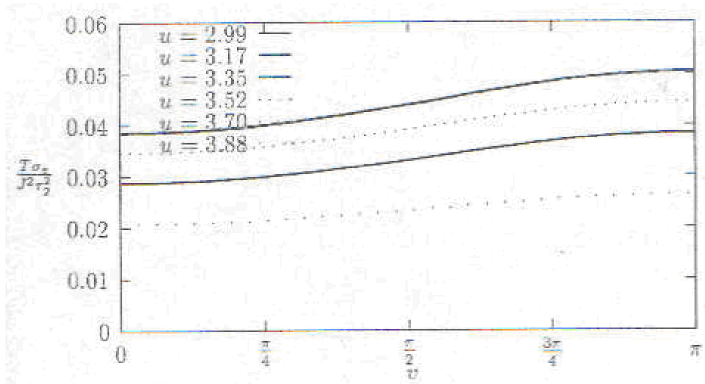

Figure 2.7. The temperature for $\frac{a}{r_2}$ = 10.

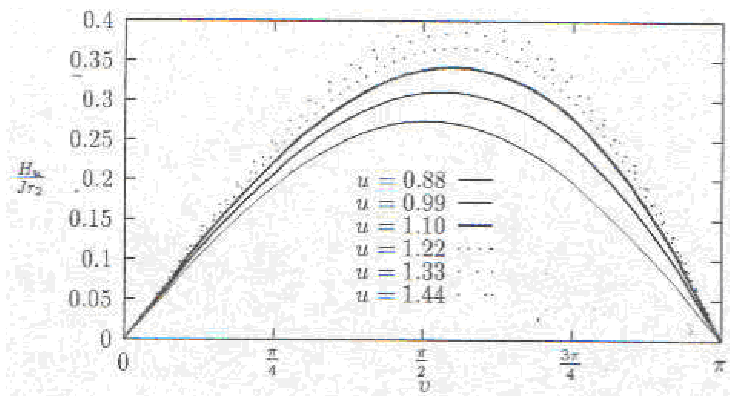

Figure 2.8. The magnetic field component H_u for $\frac{a}{r_2}$ = 1.

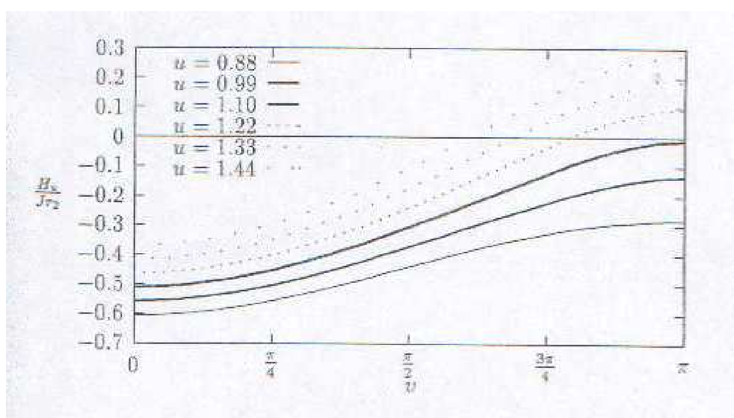

Figure 2.9. The magnetic field component H_v for $\frac{a}{r_2}$ = 1.

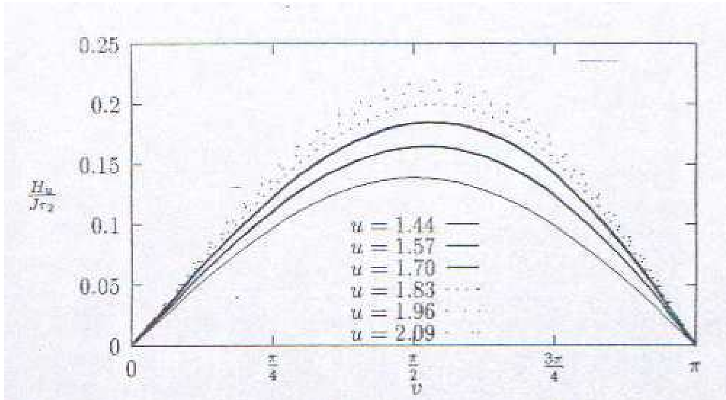

Figure 2.10. The magnetic field component H_u for $\frac{a}{r_2} = 2$.

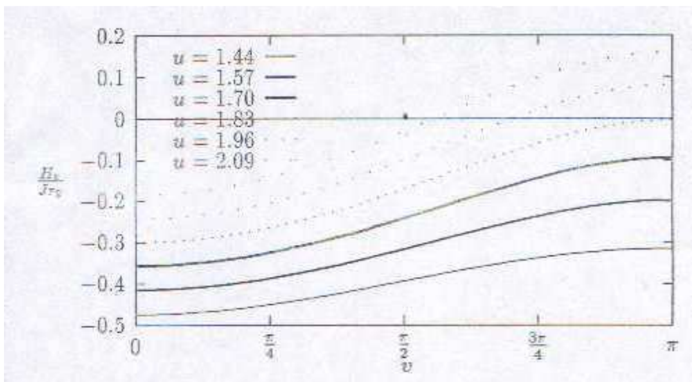

Figure 2.11. The magnetic field component H_v for $\frac{a}{r_2} = 2$.

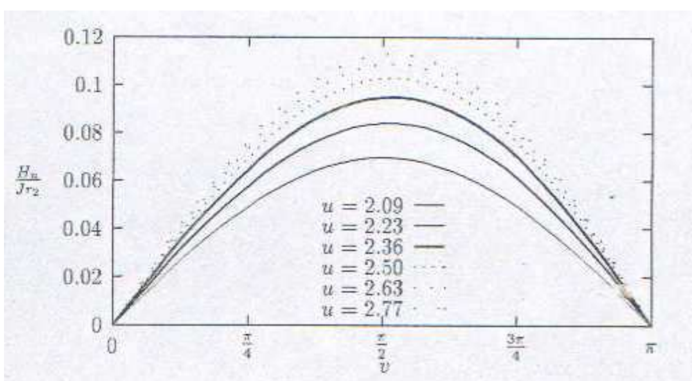

Figure 2.12. The magnetic field component H_u for $\frac{a}{r_2} = 4$.

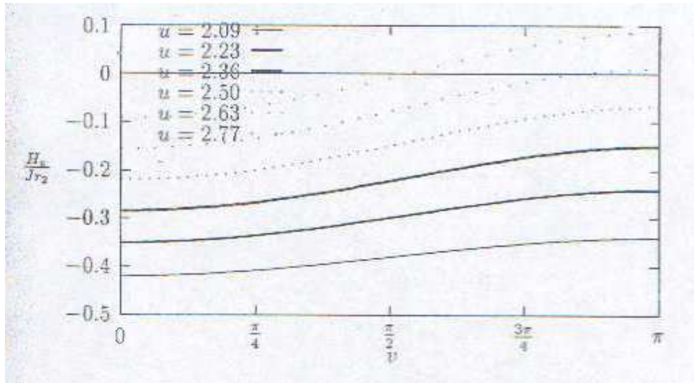

Figure 2.13. The magnetic field component H_v for $\frac{a}{r_2} = 4$.

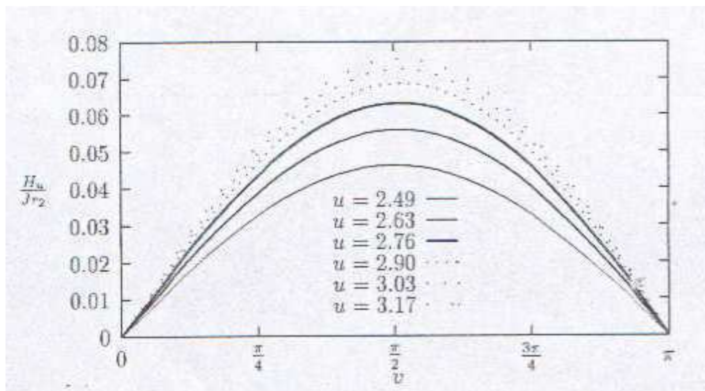

Figure 2.14. The magnetic field component H_u for $\frac{a}{r_2} = 6$.

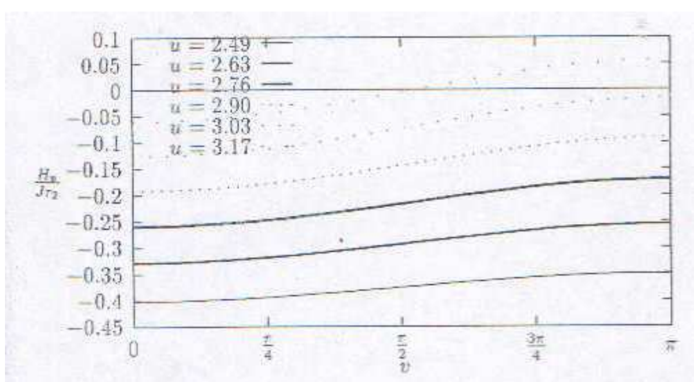

Figure 2.15. The magnetic field component H_v for $\frac{a}{r_2} = 6$.

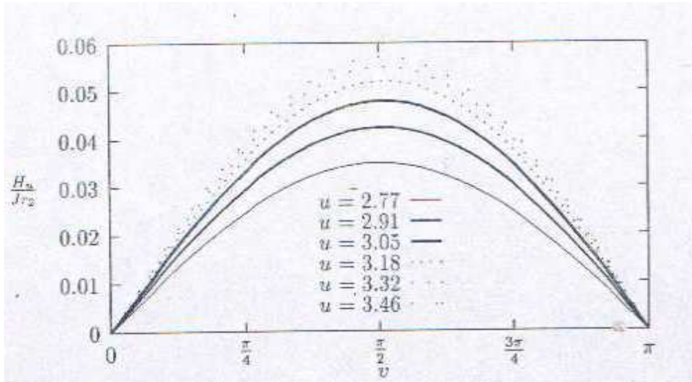

Figure 2.16. The magnetic field component H_u for $\frac{a}{r_2} = 8$.

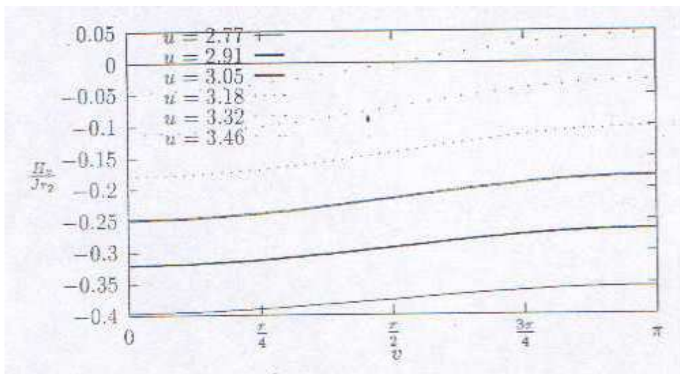

Figure 2.17. The magnetic field component H_v for $\frac{a}{r_2} = 8$.

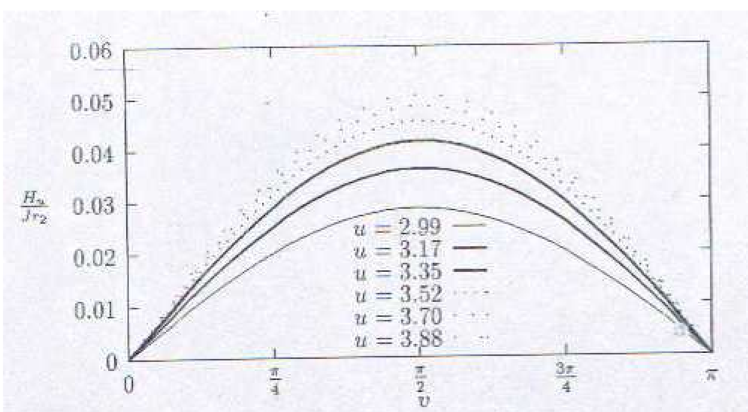

Figure 2.18. The magnetic field component H_u for $\frac{a}{r_2} = 10$.

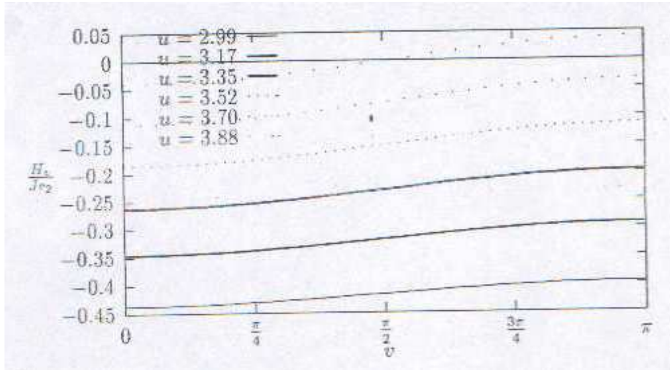

Figure 2.19. The magnetic field component H_v for $\frac{a}{r_2} = 10$.

Figures (2.2)-(2.7) show the distribution of temperature in the tube, calculated at different circles, as function of the angle v, for different values of the geometric parameter $\frac{a}{r_2}$.

4-Numerical results and discussion

1. The curves for the temperature at the two boundaries coincide with the v-axis, thus continuing the fulfillment of the thermal boundary conditions.
2. The temperature always attains its maximum value at $v = \pi$ (in the region of larger thickness of the tube).
3. As the parameter $\frac{a}{r_2}$ increases, the variations of the temperature become less significant, i.e. the temperature is more uniform, as a result of the decrease in eccentricity of the tube.
4. The maximal temperatures are naturally attained around the middle sections of the tube.
5. It may be noticed that the variations of the temperature due to the changes in values of the parameter $\frac{a}{r_2}$ are explained by the variations in the geometry of the tube, and not by the distance of the external line current from the tube, since the thermal effect of this line current is not taken into account from the outset.

Figures (2.8)-(2.19) exhibit the distribution of the two magnetic field components in the tube.

1. The curves for the component H_u as functions of the angle v are parabolic-shaped, give zero values at $v = 0$ and r $v = \pi$ and are skewed towards the region $v = \pi$. This skewness gradually vanishes with the increase if the parameter $\frac{a}{r_2}$ as a result of the decrease in eccentricity.

The maximum value of Hu decreases as $\frac{a}{r_2}$ increases, the component itself tends to vanish as the tube approaches the circular shape, in agreement with the results of [2].

The values of Hu globally increase with u.

As to the magnetic field component Hv, it exhibits the following Features:

It globally increases with u.

It is a monotonic increasing function of v.

As the parameter $\frac{a}{r_2}$ increases, the variations of Hv become smaller.

The values of Hv lend to zero at the inner surface as the parameter $\frac{a}{r_2}$ increases, in agreement with the results for the circular tube [2].

5- References

[1] Abd-Alla, A.M. and Mahmoud S.R. Magneto-thermoelastic problem in rotating non-homogeneous orthotropic hollow cylinder under the hyperbolic heat conduction model. Meccanica, 45, 451-461 (2010)

[2] Abd-El-Salam, M.R., Abd-Alla, A.M. and Hosham, H. A. Numerical solution of magneto-thermoelastic problem in non-homogeneous isotropic cylinder by the finite-difference method. Applied Mathematical Modeling, 31, 1662-1670 (2007).

[3] Abd-Alla, A.M. and Abo-Dahab, S.M. Time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation. Applied Mathematical Modeling, 33, 2388-2402 (2009)

[4] El-Naggar, A.M., Abd-Alla, A. M., Ahmed, S.M. and Fahmy, M.A.. Thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder. Heat and Mass Transfer, 39, 41-46 (2003)

[5] Abd-Alla, A. M., El-Naggar, A.M. and Fahmy,. M.A. Thermal stresses in a rotating non-homogeneous orthotropic hollow cylinder. Heat and Mass Transfer, 39, p 625-629 (2003)

[6] Abd-Alla, A.M., Abd-Alla, A.N. and Zeidan, N.A. Thermal stresses in a non-homogeneous orthotropic elastic multilayered cylinder. J. of Thermal Stresses; 23: 413-428 (2003).

[7] Abd-Alla, A.M., Abd-Alla, A.N. and Zeidan, N.A. Transient thermal stresses in a rotating non-homogeneous cylindrically orthotropic composite tubes. J. Applied ,Mathematics and Computation; 105; 253-269 (1999)

[8] M.H. Sadd, Elasticity: theory, application, and numeric's, Elsevier, Amsterdam, (2005).

[9] Ding , D.J., Wang, H.M. and Chen, W.O. A solutions of a non-homogeneous orthotropic cylindrical shell for axisymmetric plane strain dynamic thermoelastic problems. J Sound Vibration, 263, 815–829 (2003)

[10] Kumar, R. and Mukhopadhyay, S. Effects of thermal relaxation time on plane wave propagation under two-temperature thermoelasticity. International Journal of Engineering Science, 48, 128-139(2010),

[11] Bagri, A. and Eslami, M.R. A unified generalized thermoelasticity; solution for cylinders and spheres. International Journal of Mechanical Sciences, 49, 1325-1335 (2007)

[12] Prasad R., Kumar R. and Mukhopadhyay S. Propagation of harmonic plane waves under thermoelasticity with dual-phase-lags. International Journal of Engineering Science, 48, 2028-2043 (2010)

[13] Bagri, A. and Eslami, M.R. Generalized coupled thermoelasticity of functionally graded annular disk considering the Lord–Shulman theory. Composite Structures, 83, 168-179 (2008).

[14] Othman, M. I.A. and Singh, B. The effect of rotation on generalized micropolar thermoelasticity for a half-space under five theories. International Journal of Solids and Structures, 44, 2748-2762 (2007)

[15] Ezzat, M. A. and Youssef , H. M. Generalized magneto-thermoelasticity in a perfectly conducting medium. International Journal of Solids and Structures, 42, 6319-633 (2005)

[16] Abd-Alla, A.M., Abo-Dahab, S.M. and Hammad, H.A.H. Propagation of Rayleigh waves in generalized magneto-thermoelastic orthotropic material Uunder initial stress and gravity field. Applied Mathematical Modelling, 35, 2981-3000 (2011)

[17] Haddow, J.B. and Mioduchowski, A. Dynamic expansion of a compressible hyperelastic spherical shell. Acta Mechanica, 66, 205-216 (1987)