Modeling of the impact on cylindrical composite shell as continuous patch loading

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***Abstract*— Due to great properties, Composite materials are utilized in many applications in advance and accurate industries such as aerospace and fuel vessel structures. In contrast with all of the composite advantages, susceptibility of these materials to impact damage became a major concern recently. The aim of this research is to study the dynamic response of the composite cylindrical shell subjected to impact by modeling the impact loading as a continuous patch loading on the contact surface of cylinder. Radial deflection of the cylinder as of the most important parameter in dynamic response has been studied in this research using analytical approach and FEM method. In analytical approach proper governing equations are derived and represented in Matlab. In the second approach we used finite element analysis and ABAQUS software to determine the accuracy of the results obtained analytically. The present study concludes that, amount of the radial deformation of cylinder is at its highest value in contact point and it will decrease gradually by getting far away from impact point on circumferential path on the cylinder surface.**

***Index Terms*— cylindrical Composite shell, Finite element analysis, Impact, Radial Deflection**

I. Introduction

LOOKING for cheaper, lighter and stronger materials, composites are attracting lots of interests [1]. Great performance of these materials introduced them as one of the most useful materials in many accurate and advance industries such as fuel vessel structures, aircraft, sporting goods and many other fields. Vulnerability of the composite structure to the impact loading became a major problem for the researchers and industries [1]. Since this type of loading (impact) creates a contact surface with high stress concentration which leads to serious undistinguishable defects on the composite structure, behavior of composite structures under such a loading is an important case for studying [2]. Although experimental study consider as the best method for obtaining the comprehensive, first hand and detailed results, due to difficulties and expensiveness of the study a few researches [3],[4] have been conducted to investigate the behavior of composite materials subjected to impact experimentally. According to the literature, most of the researches in this field deal with the analytical approach and FEM method for their investigation on the impact motion on composite structures [5]-[14]. One of the earliest studies on impact response of composite materials was done in 1987 using a proper 3D finite element method for calculation the amount of the distributed stress and strain in FRP composite laminate during transverse impact [5]. Researchers considered a rectangular plate with continuous fibers and they employed Hertzian contact low, Newmark method and Newton – Raphson method for their analytical approach and 8 node brick elements for their FEM simulation [5]. To improve the FEM results researchers in 1993 used a partial mixed 3D finite element method for their study [6]. An advanced coordinate system has been used by experts in 1998 to investigate the dynamic response of the fully – clamped FRP laminate composite plate subjected to the low velocity [7]. They defined the general coordinate system using the displacement of the hitter and the total displacement of composite [7]. Recently another study has been conducted to investigate the failures happens in composite due to impact [8]. Researchers used two famous criteria, critical matrix cracking and delamination at the interface, for their failure analysis. They did the failure analysis for two different types of impact, (point – loading and line – loading) [8].

II. Methodology

The aim of this project is to investigate the dynamic response of the cylindrical composite shell subjected to impact. For this purpose, two famous method (analytical and FEM) have been employed. In the first step, the impact motion on the composite cylinder will be modeled through the ABAQUS software. Since the properties of striker and cylinder and condition of impact motion have been chosen and modeled exactly the same as [10], their results will be used as the benchmark to validate the obtained data in ABAQUS software.

In the second step, the contact force history which has been achieved by ABAQUS and validated by the published data

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will be applied as a patch loading on the composite cylinder surface analytically. For this part the proper Fourier series term for the achieved patch loading and other essential governing equations will be derived and represented in Matlab to determine the radial deflection of the cylinder.

By the end the obtained results for both methods will be compared with each other to determine the accuracy of the research results. Fig.1 shows a schematic of impact motion on cylinder and it corresponding patch loading modeling.

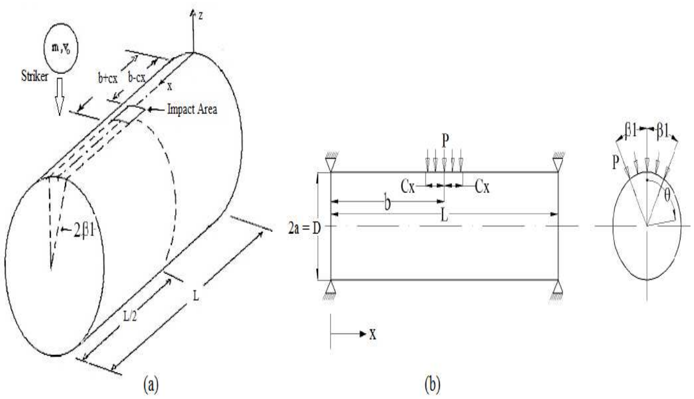

Fig. 1. (a) Schematic of impact motion on cylinder. (b) Patch loading model of impact on cylinder

III. FEM APPROACH

As mentioned earlier, impact motion on cylinder is modeled the same as [10] in ABAQUS software. Table 1 illustrates the composite cylinder and striker's properties. After modeling the cylinder and impactor the contact force history of impact motion is recorded for contact point on cylinder. Furthermore, final radial deflection of composite cylinder due to impact is investigated and plotted.

TABLE 1
MATERIAL AND STRUCTURAL PROPERTY DATA FOR THE COMPOSITE LAMINATES AND IMPACTOR [10]

Composite Cylinder	
Radius of Cylinder	*0.1 m*
Length of Cylinder	*0.42 m*
Thickness of Cylinder	*2.5 mm*
Fiber Orientation	*[0/90/0/90]* $_s$
E_{11}	*138.0 GPa*
E_{22}	*9.00 GPa*
G_{12}	*7.10 GPa*
G_{13}	*7.10 GPa*
G_{23}	*3.24 GPa*
υ_{12}	*0.3*
Density	*1540 Kg/m*^{3}
Impactor	
Diameter of Impactor	*12 mm*
Density	*7800 Kg/m*^{3}
Initial Velocity	*3 m/s*
Elasticity Modulus	*200 GPa*

IV. ANALYTICAL APPROACH

The contact force history which has been achieved by FEM method and ABAQUS software and validated with published data in previous section will be used as the patch loading which is acting on cylinder. To obtain the radial deflection of cylinder subjected to the parabolic contact force history (impact) analytically we will drive proper Fourier series term to model the impact motion as a patch loading.

According to the literature [15],[16] the applied force which is acting on the surface of cylinder (P_x, P_r, P_θ) and their corresponding displacement (U,V,W) can be represented in double Fourier terms as the patch loading. In our study we used the literature equations and manipulated them so that they can satisfy our problems boundary conditions. After manipulation, the double Fourier series of acting force can be rewritten as

$$P_x = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} p_{n,m}.\cos n\theta.\cos \lambda \, x/a \tag{1}$$

$$P_\theta = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} p_{n.m} \sin n\theta.\sin \lambda \, x/a \tag{2}$$

$$P_r = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} p_{n.m} \cos n\theta.\sin \lambda \, x/a \tag{3}$$

Using above double Fourier serious form, the deflection in all three direction (U,V,W) can express as follow

$$W = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} w_{n.m} \cos n\theta.\sin \lambda \, x/a \tag{4}$$

$$U = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} u_{n.m} \cos n\theta.\sin \lambda \, x/a \tag{5}$$

$$V = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} v_{n.m} \sin n\theta.\sin \lambda \, x/a \tag{6}$$

Where

$P = P_r$, $\lambda = m\pi \, a/L$, $-\beta \le \theta \le \beta$ and

$(b - Cx) \le x \le (b + Cx)$

$$p_{n,m} = \frac{8P}{m.n.\pi^2}.Sin\left(m.\pi.b/L\right)Cos\left(m.\pi.c/L\right)Sin(n.\beta) \tag{7}$$

Using the matrix equation and Cramer's rule value of $(u_{n,m}, v_{n,m}, w_{n,m})$ can be calculated as

$$w_{n,m} = a^2.p_{n,m}\left(C_{24}n^2\lambda^2 + C_{25}\lambda^4 + C_{26}n^4\right) / Den1_{n,m} \tag{8}$$

$$u_{n,m} = a^2.p_{n,m}\left(-C_7 n^2 \lambda - C_8 \lambda^3 n^2 - C_9 \lambda^3 + C_{10}\lambda^5 - C_{11} n^4 \lambda\right) \Big/ Den1_{n,m} \quad (9)$$

$$v_{n,m} = a^2.p_{n,m}\left(-C_{27} n\ \lambda^2 - C_{28}\lambda^4 n\ - C_{29}\lambda^2 n^3 - C_{30} n^3\right) \Big/ Den1_{n,m} \quad (10)$$

Where

$$Den1_{n,m} = C_{12} n^2 \lambda^2 - C_{13}\lambda^2 n^4 + C_{14}\lambda^2 n^6 + C_{15}\lambda^6 n^2 + C_{16}\lambda^4 n^4 - C_{17}\lambda^4 n^2 + C_{18}\lambda^4 + C_{19}\lambda^8 - C_{20}\lambda^6 + C_{21} n^4 - C_{22} n^6 + C_{23} n.m^8 \quad (11)$$

In the equation above C is a function of array stiffness matrix which is explained more in appendix the value of $p_{n,m}$ for the radial patch loading is given by the equation (7) that is consider of a length of cylindrical shell between two supports , of length (*L*), uniform thickness (*t*) of wall and main radius (*a*).

V. RESULTS AND DISCUSSION

A. *FEM Results*

After modeling the impact motion on the ABAQUS software the contact force history has been recorded and is plotted in fig.2. Furthermore, radial deflection of the cylinder shell along the circumferential direction is shown in fig.3.

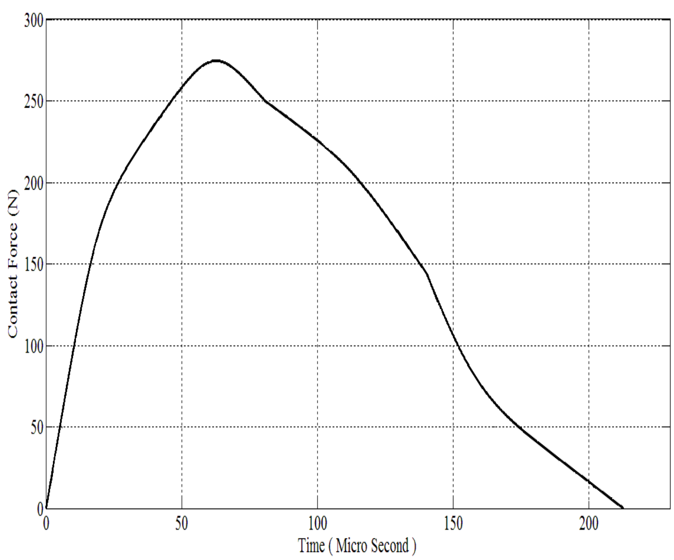

Fig. 2. Contact force of the composite cylindrical shell at the point of impact during impact period

As you can see in Fig.3, the maximum radial deflection occurs exactly at the contact point of the composite cylinder. By passing from contact point through the circumferential pass this deflection decreased due to deduction of the contact force effect. This value reached to almost zero when we passed the contact force by 60 degree and it remained in the same condition up to end.

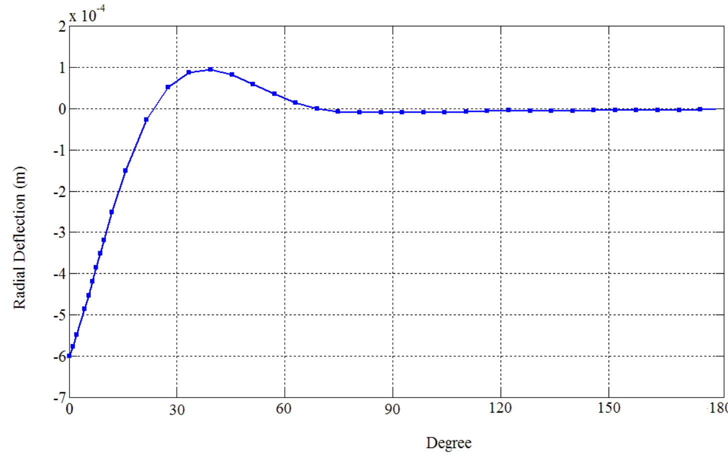

Fig. 3. Radial deflection of the composite cylindrical shell at the point of impact during impact period (200 μ s)

B. *FEM Results Validation*

Since our impact condition and properties was exactly the same as [10], as the benchmark and to determine the accuracy of the modeling and also the results obtained from FEM approach we compared the obtained contact force history with the one which is published in [10]. Fig.4 shows the comparison between current contact force and published one.

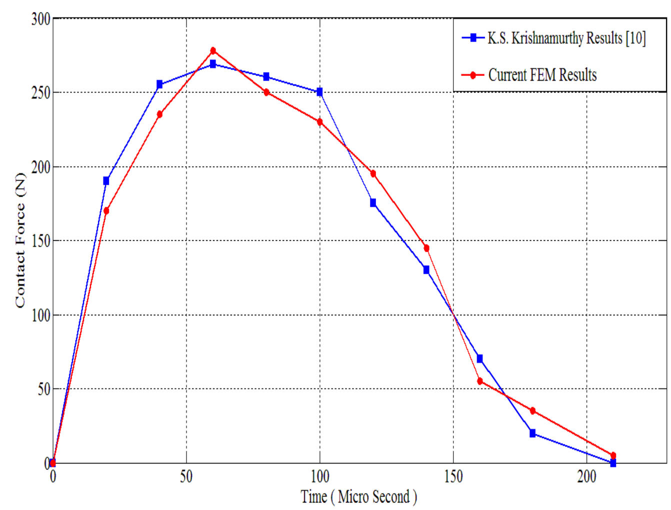

Fig. 4. Comparison of contact force history in cylinder's contact point

As you can see in above figures, the contact force history of the contact point on the cylinder which has been investigated in this study through the FEM modeling has a great similarity with the literature results. This resemblance of the data can show high accuracy of the modeling. Furthermore, as it was expected and mentioned in Literature [10] the impact motion produced a parabolic contact force which can be seen in both of the plotted data.

C. *Analytical Results*

Now the contact force history which was obtained in FEM results and validated with published data is used as continuous patch loading acting on the surface of the cylinder in analytical approach. In another word, the obtained contact force history as the radial pressure (P_r) is applied and used on the patch loading derived equations to calculate the Fourier series terms and corresponding deflection. For this purpose, the governing equations (1-11) are represented in Matlab code to determine the componential force ($P_{n,m}$) and its corresponding componential and finally total radial deflection. Fig.5 illustrates the radial deflection of composite cylinder subjected to a continuous patch loading for every (20 μs) passing from the loading time.

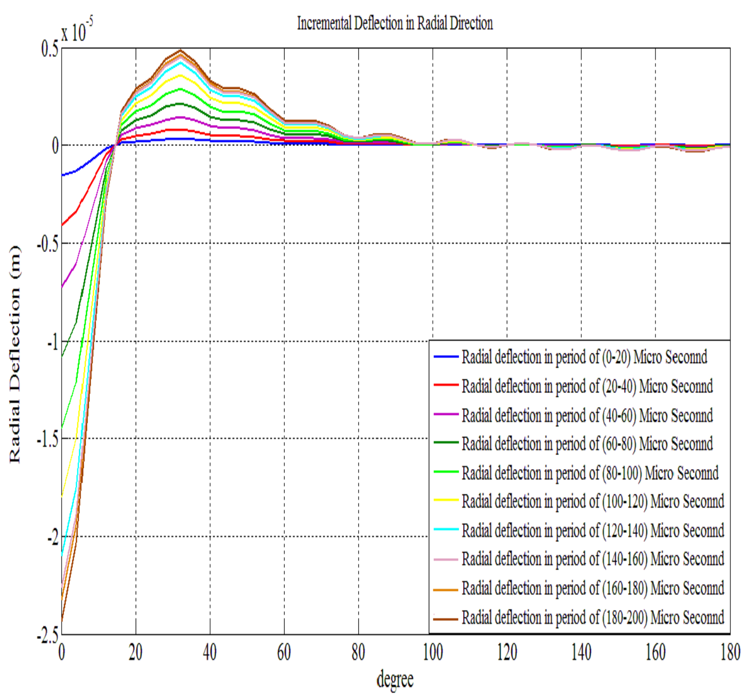

Fig. 5. Incremental radial deflection of composite cylinder subjected to continuous patch loading

By summation of all above incremental values of radial deflections the total amount of radial deflection will be achieved for the composite cylinder in whole impact duration. Fig.6 illustrates the total amount of radial deflection for the composite cylinder subjected to continuous parabolic patch loading in whole impact duration

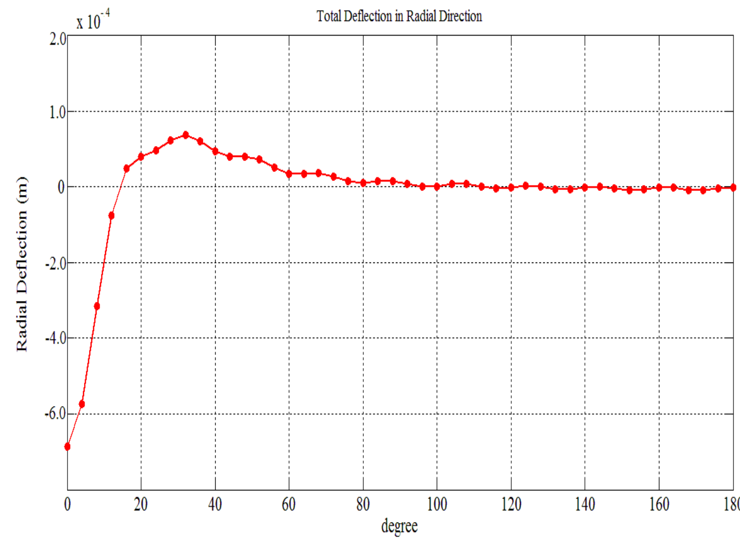

Fig. 6. Total amount of radial deflection on the composite cylinder subjected to continuous parabolic patch loading in whole impact duration

D. *Comparison of Radial Deflection of Impact Motion and Continuous Patch Loading*

Now to determine the applicability of modeling the impact loading as continuous patch loading, we compared the radial deflection of cylinder which has been achieved for exact impact modeling through FEM method with the one achieved through the a continuous patch loading. Fig.7 shows the radial deflection which was achieved through both modeling.

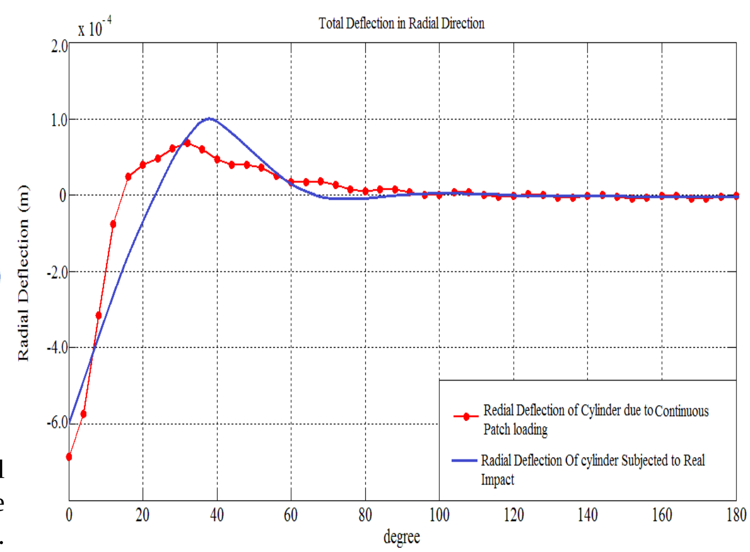

Fig. 7. Comparison of radial deflection in impacted composite cylinder and cylinder subjected to continuous patch loading

According to the comparison which is shown in above figure, radial deflection of the cylinder subjected to real impact loading has great similarities with the one which is subjected to the patch loading model of impact. As discussed earlier, in both results the maximum deflection occurred at the contact point of composite cylinder and by getting far away from this critical point the deflection decreased till it got to zero value.

VI. CONCLUSION

The developed simple method for modeling the impact on composite cylindrical shell as continuous patch loading can be used in different type of impact motions on composite cylinder with different operating parameters such as different initial velocity or mass of the striker. Furthermore, by manipulating the derived double Fourier series terms of deflection and applied forces dynamic response of the other composite structures subjected to impact can be investigated easily. Results of this research can be used for failure analysis of the impacted composite cylinder using the famous failure criterion such as Tsai-Wu, Tsai-Hill, strain energy and maximum stress.

APPENDIX

The relationship between the constant $(C_1, C_2, ...C_{30})$ which are used in equations (8-11) and the laminate stiffness is

$$C_1 = A_{66} + \frac{1}{a^2} D_{66} \tag{12}$$

$$C_2 = A_{12} \times A_{66} \tag{13}$$

$$C_3 = A_{66} + \frac{3}{a^2} D_{66} \tag{14}$$

$$C_4 = \frac{1}{a^2} D_{12} + \frac{3}{a^2} D_{66} \tag{15}$$

$$C_5 = A_{22} + \frac{1}{a^2} D_{22} \tag{16}$$

$$C_6 = \frac{2}{a^2}\left(D_{12} + 2D_{66}\right) \tag{17}$$

$$C_7 = A_{22}\left(C_2\right) - A_{12} A_{22} \tag{18}$$

$$C_8 = C_2 C_4 - \frac{1}{a^2} D_{11} A_{22} + \frac{1}{a^2} D_{66} C_3 \tag{19}$$

$$C_9 = -A_{12} C_3 \tag{20}$$

$$C_{10} = \frac{1}{a^2} D_{11} C_3 \tag{21}$$

$$C_{11} = \frac{1}{a^2} D_{66} A_{22} \tag{22}$$

$$C_{12} = A_{11} A_{22} C_5 - A_{11}\left(A_{22}\right)^2 + C_1 C_3 C_5 - \left(C_2\right)^2 C_5 - \left(A_{12}\right)^2 A_{22} + 2 C_2 A_{12} A_{22} \tag{23}$$

$$C_{13} = \frac{2}{a^2}\left[\begin{array}{l} A_{12} A_{22} D_{22} + D_{22} C_1 C_3 + A_{22} C_1 C_4 \\ - D_{22}\left(C_2\right)^2 - A_{12} A_{22} D_{66} + A_{22} D_{66} C_2 \end{array}\right] \tag{24}$$

$$C_{14} = \frac{1}{a^2} A_{11} A_{22} D_{22} + C_6 A_{22} C_1 + \frac{1}{a^2} C_1 C_3 D_{22} - \frac{1}{a^4} A_{22} (D_{66})^2 - \frac{1}{a^2} D_{22} (C_2)^2 \tag{25}$$

$$C_{15} = \frac{1}{a^2} A_{11} A_{22} D_{11} + C_6 A_{11} C_3 - A_{11} (C_4)^2 + \frac{1}{a^2} C_1 C_3 D_{11} - \frac{1}{a^2} D_{11} (C_2)^2 + \frac{2}{a^4} D_{11} (D_{66} C_3) \tag{26}$$

$$C_{16} = \frac{1}{a^2} A_{11} C_3 D_{22} + C_6 A_{11} A_{22} + \frac{1}{a^2} A_{22} (C_1) D_{11} + C_1 C_6 C_3 - C_1 (C_4)^2 - (C_2)^2 (C_6) - \frac{2}{a^2} C_4 C_2 D_{66} + \frac{2}{a^4} D_{11} A_{22} D_{66} - C_3 \frac{1}{a^4 (d_{66})^2} \frac{2}{a^4} D_{11} (D_{66} C_3) \tag{27}$$

$$C_{17} = 2\left(\begin{array}{l} \frac{1}{a^2} A_{11} D_{22} C_3 + A_{11} A_{22} C_4 - \frac{1}{a^2} A_{22} D_{11} C_2 \\ - A_{12} C_2 C_4 + \frac{1}{a^2} A_{12} A_{22} D_{11} - \frac{1}{a^2} A_{12} D_{66} C_3 \end{array}\right) \tag{28}$$

$$C_{18} = C_3\left[A_{11} C_5 - \left(A_{12}\right)^2\right] \tag{29}$$

$$C_{19} = \frac{1}{a^2} D_{11} C_3 \left(A_{11} - \frac{1}{a^2} D_{11} \right) \tag{30}$$

$$C_{20} = \frac{2}{a^2} D_{12} A_{11} C_3 \tag{31}$$

$$C_{21} = C_1\left[A_{22} C_5 - \left(A_{22}\right)^2\right] \tag{32}$$

$$C_{22} = \frac{2}{a^2} D_{22} A_{22} C_1 \tag{33}$$

$$C_{23} = \frac{1}{a^2} D_{22} A_{22} C_1 \quad (34)$$

$$C_{24} = C_1 C_3 + A_{11} A_{22} - (C_2)^2 \quad (35)$$

$$C_{25} = A_{11} C_3 \quad (36)$$

$$C_{26} = A_{22} C_3 \quad (37)$$

$$C_{27} = A_{11}(A_{22}) - A_{12} C_2 \quad (38)$$

$$C_{28} = A_{11}(C_4) - \frac{1}{a^2} D_{11} C_2 \quad (39)$$

$$C_{29} = \frac{1}{a^2} D_{66} C_2 + C_1 C_4 \quad (40)$$

$$C_{30} = A_{22} C_1 \quad (41)$$

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