Attaining Aplanatism in Two-Mirror Telescopes

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***Abstract*---- The aim of the present work is to derive equations that enable optical designers to determine the profiles of two reflectors that form the principal optical elements of conventional two-mirror telescopes, namely the Gregorian and Cassegrainian. The mirrors have to work together as correctors to attain aplanatism. To achieve this, Wassermann and Wolf method has been developed and used in both,the Gregorian and Cassegrainian configurations of particular geometrical parameters. The Cassegrainian is taken as an example and its performance is examined**

***Index Term* ---- aplanatic systems, geometrical optics, optical design ,optical telescopes.**

I. INTRODUCTION

Two –mirror telescopes, classical or modified, are usually incorporated in some modern optical systems, such as laser detection and ranging, and guidance systems, where mostly narrow field of view is required. Attaining aplanatism in such systems resulting in an adequate optical performance. The aim of this work is to derive equations to attain aplanatism for the conventional two- mirror telescopes, namely, Gregorian and Cassegrainian . Many works on methods and techniques to achieve aplanatism were reported. Examples are, design of aplanatic telescopes [1]-[4], design of three and four aspheric surface systems[5],[6], design of two mirror telescopes,[7]-[9], and designing and testing aspheric correctors surfaces [10]-[16]. However, the method introduced by Wassermann and Wolf [11] which, essentially, employs the sine-condition is found to be more suitable for the present investigation. For each telescope configuration, two simultaneous first-order differential equations that may be integrated numerically by standard Runge and Kutta method are derived to describe the profiles of the telescope's two mirrors.

Cassegrainian is often preferred to the Gregorian arrangement because it is inherently short. Thus, an investigation on a Cassegrainian configuration of particular dimensions is carried out and showed that the optical performance is likely to be diffraction limited.

II. DETERMINATION OF MIRRORS PROFILS

This section is devoted for the derivation of equations that describe the behavior of an incoming ray from a distant object upon entering a two mirror system. The work will end up with the derivation of two simultaneous first order differential equations that describe the required profiles of the two mirrors to ensure aplanatism. Gregorian and Cassegrainian are examples of two mirror configuration which will be considered here. Both are centered optical systems in each of which the primary and secondary mirrors are optical neighbours separated from each other by a chosen distance as depicted in Figs. 1 and 3. Two sets of Cartesian coordinates may be introduced:

$(Z(\eta),\eta)$ for the primary mirror with an origin at pole O,

$(Z'(\eta'),\eta')$ for the secondary mirror with an origin at pole O',

With their $Z(\eta)$, axes along the optical axis of the system. The following dimensions may be identified:

OO'=D is the separation between the primary and secondary poles,

F_{effe}.O' =G is the separation between the focal point F_{effe}..(sytem's effective focal point) and secondary mirror pole O'.

A . Attaining aplanatism for Gregorian configuration

Fig. 1. demonstrates a typical Gregorian configuration ,which consists of two confocal concave mirrors. An incoming ray from a distant object point, parallel to the optic axis, u=0, hits the primary M_P at point T $(Z(\eta),\eta)$ and bounced back to hit the secondary M_S at point T'$(Z'(\eta'),\eta')$ after crossing the optic axis at or near the primary focal point FP . At T' the ray reflected towards the point Feffe . Below the coordinates Z,η and Z',η' of the points T and T' will be calculated.

As it can be seen in Fig. 2(a), the incoming ray from a distant object point hits the primary M_P at T which ,referred to the (η,Z) frame, has the equation:

$$|\eta| = Y \qquad (1_G)$$

Where Y is the height at which the incoming ray meets Oη.

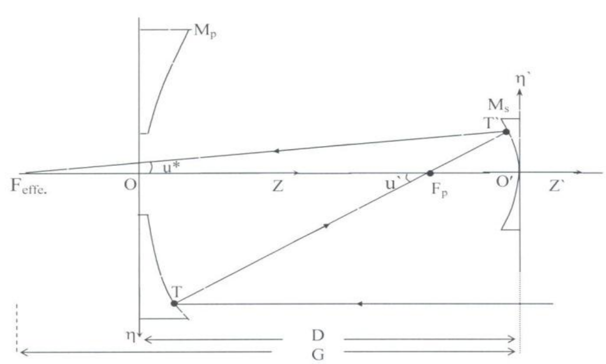
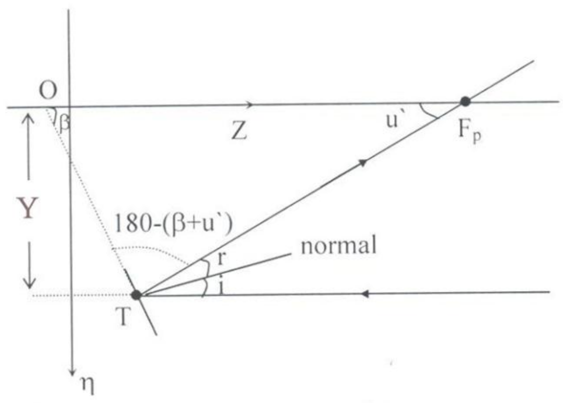
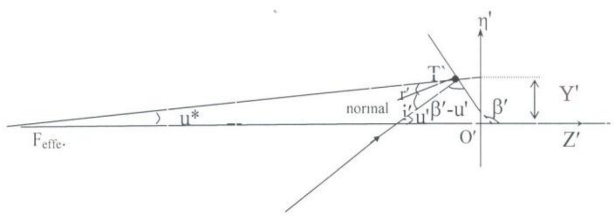


Fig. 1. A Gregorian configuration.



(a)



(b)

Fig. 2. Ray reflection at the Gregorian mirrors. (a) Ray behavior at the Gregorian primary Mp . (b) Reflection at the secondary Gregorian mirror Ms.

By differentiating (1_G) with respect to a chosen free parameter, which is in the present case Y (as the object is at infinity), the following equation is obtained:

$d\eta/dY = 1$ (2_G)

If the Gregorian is to be aplanatic, it must exactly satisfy the sine-condition. In the notation of Fig. 1., the condition may be indicated by :

$$\frac{Y}{sinu^*} = f_{effe} \quad (3_G)$$

Where f_{effe} is the effective focal length of the Gregorian, and u* is the angle which ray $T'F_{effe.}$ makes with the optic-axis.

Hence

$$tanu^* = Y(f^2_{effe.} - Y^2)^{-1/2} \quad (4_G)$$

is the optical constraint which must be satisfied to ensure aplanatim.

The equation of ray $T'F_{effe}$ referred to (Z',η') frame, Fig. 2(b), can be written as follows:

$$\eta' = Y' + Z'\, tanu^*, \quad (5_G)$$

where

$$Y' = G\, tanu^*, \quad (6_G)$$

is the height at which ray $T'F_{effe.}$ meets O'η'.

The first reflection takes place at the primary mirror, Fig. 2(a), where the reflected ray TT' cuts the optical axis at angle u'. The tangent to the primary surface at T makes an angle of inclination β to the axis.

Since $i = r$ (Snell's law) we have:

$$\beta = 90 - \frac{u'}{2}, \quad (7_G)$$

and

$$tan\,\beta = \frac{d\eta}{dZ}, \quad (8_G)$$

where η as a function of Z is the desired locus of T.

It is possible to write

$$\frac{dZ}{dY} = \frac{d\eta}{dY} \cdot \frac{dZ}{d\eta},$$

or ,

$$\frac{dz}{dY} = \frac{d\eta}{dY} .cot\beta,$$

and by using (2_G)

$$\frac{dz}{dY} = cot\,\beta, \quad (9_G)$$

Where $cot\,\beta = tan\,(\frac{u'}{2})$

and

$$tan\,u' = \frac{\eta + \eta'}{D - Z + Z'}. \quad (10_G)$$

The second reflection takes place at the secondary mirror, Fig. 2(b), where ray T'T' hits the mirror at point T' and reflects back to cross the axis at the focal point F_{effe}. The tangent to secondary surface at T' makes an angle of inclination β' to the axis and since i'=r' , we have,

$$\beta' = 90 + \frac{u' + u^*}{2}. \quad (11_G)$$

Now,

$$tan\beta' = \frac{d\eta'}{dZ'}, \quad (12_G)$$

Where η' as a function of Z' is the desired locus of T.

Also ,

$$\frac{d\eta'}{dZ'} = \frac{d\eta'}{dY} \cdot \frac{dY}{dZ'},$$

therefore ,

$$\frac{dZ'}{dY} = \frac{d\eta'}{dY} .cot\,\beta', \quad (13_G)$$

as T' always lies on the ray $T'F_{effe.}$ .Thus by differentiation (5_G) with respect to Y , and substituted in (13_G), with β' replaced by (11_G) gives,

$$\frac{dZ'}{dY} = \frac{\left[\frac{dY'}{dY} + Z\frac{d\,tanu^*}{dY}\right] tan\left(\frac{u'+u^*}{2}\right)}{\left[1 - tanu^*\, tan\left(\frac{u'+u^*}{2}\right)\right]}. \quad (14_G)$$

Clearly (9_G) and (14_G) permit , together with (10_G),(1_G) and (5_G), a complete computation of two mirrors. For ,by means of (10_G),(1_G)and(5_G) η and η' from (9_G)and (14_G) can be eliminated and thus two simultaneous first order differential equations of the form,

$\frac{dZ}{dY} = \zeta_G(Z, Z', Y)$, and $\frac{dZ'}{dY} = \xi_G(Z, Z', Y)$, are obtained.

These are subject to the boundary condition Z=Z'=0 for Y=0

B . Attaining aplanatism for a Cassegrain configuration

Fig. 3. shows a typical Cassegrain telescope that consists of a concave primary mirror followed by a convex secondary mirror so that its focal point located at or near the focal point of the primary F_P. Again an incoming ray from a distant object point, u=0, hits the primary at T and reflected

back to hit The same procedure that is adopted above will be followed here to calculate the coordinates of points T and T'.

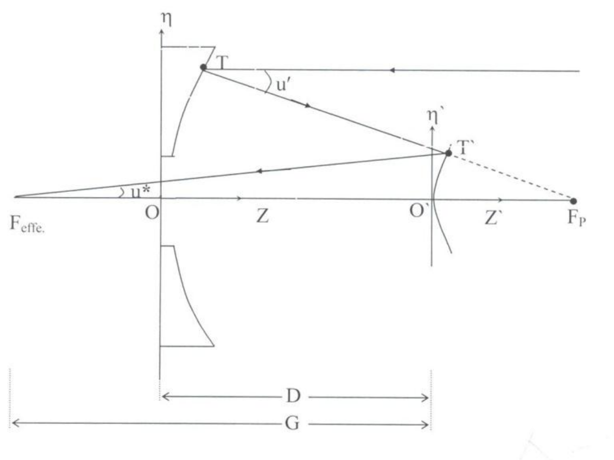


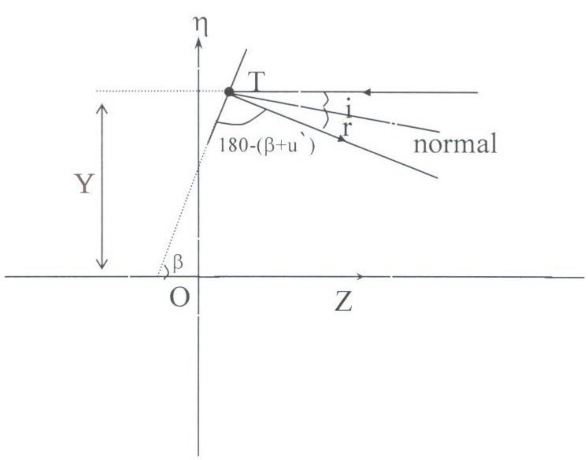
Fig. 3. A Cassegrain configuration.

Referring to the (η, Z) frame, Fig. 4(a) ,the incoming ray has the equation

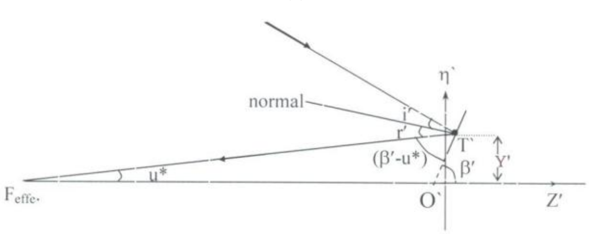
$\eta = Y$ (1$_C$)

where Y is the height at which the incident ray meets the axis Oη. So that,

$\frac{d\eta}{dY} = 1.$ (2$_C$)



(a)



(b)

Fig. 4. Ray reflection at the Cassegrin mirrors. (a) Reflection at the Cassegrainian primary mirror. (b) Reflection at the Cassegrainian secondary mirror.

In the notation of Fig. 3 ,the condition for attaining aplanatism may be ,

$\frac{Y}{sinu^*} = f_{effe.}$ (3$_C$)

Where $f_{effe.}$ is the effective focal length of the Cassegrain, and $u^*$is the angle at which ray $T'F_{effe.}$ makes with the optic-axis.

Thus,

$tanu^* = Y[f_{effe.}^2 - Y^2]^{-1/2},$ (4$_C$)

Is the optical constraint that must be met to achieve aplanatism.

Referring to the (Z',η') frame ,Fig. 4(b) , the equation of ray $T'F_{effe.}$ is

$\eta' = Y' + Z' \tan u^*,$ (5$_C$)

where the height at which ray $T'F_{effe.}$ meets the axis O'η' is

$Y' = G \tan u^*$ (6$_C$)

At the primary mirror, Figs. 3. and 4(a) , the reflected ray TT' makes an angle u' with the axis. The tangent to the primary surface at T makes an angle β to the axis .

Since $i = r$ (low of reflection) then ,

$\beta = 90 - \frac{u'}{2}$ (7$_C$)

and ,

$\tan\beta = \frac{d\eta}{dZ},$ (8$_C$)

again it is possible to write,

$\frac{dZ}{dY} = \frac{d\eta}{dY} \cdot \frac{dZ}{d\eta},$

or ,

$\frac{dZ}{dY} = \frac{d\eta}{dY} . \cot\beta ,$

and when using equation 8$_C$

$\frac{dZ}{dY} = \cot\beta ,$ (9$_C$)

where $\cot\beta = \tan(\frac{u'}{2}),$

and ,

$\tan u' = \frac{\eta - \eta'}{D - Z + Z'}$. (10$_C$)

At the secondary mirror, Fig. 4(b), the second reflection takes place so that the reflected ray cuts the axis at point $F_{effe.}$. The tangent to secondary surface at T' makes an angle of inclination β' to the axis and since i'=r' , then ,

$\beta' = 90 - \frac{(u' - u^*)}{2}.$ (11$_C$)

Now,

$\tan\beta' = \frac{d\eta'}{dZ'},$ (12$_C$)

also,

$\frac{d\eta'}{dZ'} = \frac{d\eta'}{dY} \cdot \frac{dY}{dZ'},$

therefore,

$\frac{dZ'}{dY} = \frac{d\eta'}{dY} .\cot\beta'$. (13$_C$)

By differentiating (5$_C$) with respect to Y, and substituted in (13$_C$), with β' replaced by (11$_C$) gives,

$$\frac{dZ'}{dY} = \frac{\left[\frac{dY'}{dY} + Z'\frac{d}{dY}tanu^*\right]tan\left(\frac{u'-u^*}{2}\right)}{\left[1 - tanu^* \, tan\left(\frac{u'-u^*}{2}\right)\right]} \quad . \qquad (14_C)$$

As with the Gregorian after eliminating η and η' ,two simultaneous first order differential equations of the form

$\frac{dZ'}{dY} = \zeta_C(z, z', Y)$, and $\frac{dZ'}{dY} = \xi_C(Z, Z', Y)$, are obtained which are subject to the boundary condition $Z=Z'=0$ for $Y=0$.

It is noticed that (1_G) to (14_G) and (1_C) to (14_C) are exactly the same. However, depending on the geometry of the system under consideration , the signs of the coordinates and the slopes of the tangents, must be taken into account.

III. Solving The Two Simultanous First Order Differential Equations;Determination of The Two Cassegrainian Mirrors Profiles.

Equations (9_C) and (14_C) may be integrated numerically by using Runge and Kutta method. As an example, a Cassegrainian configuration of primary focal length ratio =1.25 and system's focal ratio=6.25 and of initial parameters aspherics are imitated by sixteenth power polynomials. Preliminary investigation showed that the optical

A computer program has been written so that it prints out values of Z and Z' as well as η and η' for a range of parameter Y (step size ΔY =0.04 units) in tabulated form. The output (see Table I) shows the exact solutions of the two aspheric mirrors. For the purpose of ray tracing and evaluating the optical performance, the two calculated ,given in terms of units ,D=4,G=5 and $f_{effe.}$=25 is examined. performance of the system at the wavelength of 550 nm is likely to be diffraction limited for off-axis angles up to three arcmins.

TABLE I

The Exact Solutions of the Two Aspheric Mirrors (In Terms of Units) of the Cassegrainian.

Z	η (=Y)	Z′	η′
8.160793E-05	.04	1.958586E-05	8.079971E-03
3.264315E-04	.08	7.834294E-05	1.615982E-02
7.344714E-04	.12	1.762696E-04	2.423942E-02
1.305726E-03	.16	3.13363E-04	3.231862E-02
1.999995E-03	.2	4.799728E-04	3.999745E-02
2.93788E-03	.24	7.050348E-04	4.847542E-02
3.919986E-03	.28	9.406965E-04	5.599298E-02
5.222881E-03	.32	1.253314E-03	6.462914E-02
6.479959E-03	.36	1.554916E-03	7.198509E-02
7.999933E-03	.3999999	1.919566E-03	7.997954E-02
9.874462E-03	.4399999	2.369236E-03	8.885179E-02
.0117514	.4799999	2.819436E-03	9.692343E-02
1.351981E-02	.5199999	3.243564E-03	.103955
1.567976E-02	.5599999	3.76154E-03	.1119438
1.799968E-02	.6	4.317813E-03	.1199309
.0208912	.64	5.011045E-03	.1291935
2.358415E-02	.68	5.656572E-03	.1372564
2.591933E-02	.72	6.216265E-03	.1438807
2.887918E-02	.7600001	6.925571E-03	.1518598
3.199897E-02	.8000001	7.673085E-03	.1598364
3.527875E-02	.8400001	8.458798E-03	.1678107
3.871854E-02	.8800001	9.282688E-03	.1757825
4.316878E-02	.9200001	1.034837E-02	.1855834
.047004	.9600002	1.126658E-02	.1936286
5.100238E-02	1	1.222364E-02	.2016707
5.516396E-02	1.04	1.321955E-02	.2097098
5.831663E-02	1.08	1.397386E-02	.2155976
.0627161	1.12	1.502627E-02	.2235513
6.727549E-02	1.16	1.611667E-02	.2315017
7.199484E-02	1.2	1.724504E-02	.2394483
7.841908E-02	1.24	1.878059E-02	.2498528
8.355956E-02	1.28	2.000889E-02	.2578701
8.711237E-02	1.32	2.085763E-02	.263266
.0943299	1.36	2.258134E-02	.2738929
9.995975E-02	1.4	.0239254	.2818977
.1036693	1.44	2.481079E-02	.2870475
.1117088	1.48	2.672905E-02	.2978945
.1155067	1.52	2.763494E-02	.3028803
.1241102	1.56	2.968643E-02	.3138725
.1279835	1.6	3.060971E-02	.3186944
.1371641	1.64	3.279727E-02	.3298314
.1411	1.679999	.0337348	.3344893
.147898	1.719999	3.535362E-02	.3423789
.157968	1.759999	3.775046E-02	.3537306
.1619737	1.799999	3.870357E-02	.3581429
.1726525	1.839999	4.124338E-02	.3696364
.1766888	1.879999	4.220294E-02	.3738852
.1879892	1.919999	4.488844E-02	.3855191
.1920431	1.959999	.0458514	.3896046
.2039782	1.999999	4.868532E-02	.4013783

IV. CONCLUSION

The work under consideration is aimed at attaining aplanatism in any centered two mirror optical system. However, it has specifically been focused on deriving formulae for the design of the two aspherics of the Gregorian and Cassegrainian. These may be utilized by optical designers as a useful and simple tool for generating two-mirror aplanatic configurations of different dimensions. As an example, an aplanatic Cassegrainian is investigated. Ray trace calculations showed that it yields diffraction limited quality up to three arcmins field of view at 550nm wavelength. Optimizing the optical performance of the system may be achieved simply by varying the geometric configuration.

ACKNOWLEDGMENT

The author would like to thank Ms. S. Mahdi for her assistance in computer work.

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