Erection Analysis of a Large-Scale Radial Cable Net

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Abstract- **Characteristics of radial cable nets are studied and the concept of erection analysis and design are proposed to simulate the whole erection process. Erection schemes are discussed based on behaivor of cable nets. Mechanism motion and elastic deformation are classified in the erection process. The proposed method is verified using a large-scale radial cable net. The case study indicates the proposed concept could be used to provide aids for erection analysis and simulation during the erection process.**

Index Term-- **cable net, Construction simulation, tensile structure, prestress, erection analysis, and large deflection.**

I. INTRODUCTION

Cable net roofs as one type of tensile structures, due to their lightweight and high flexural rigidity, are widely accepted in large-span structures since Matthew Nowicki designed the State Fair Arena at Raleigh in North Carolina in 1950, as shown in Fig. 1.

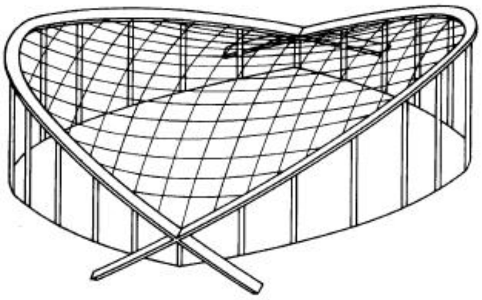


Fig. 1. State Fair Arena at Raleigh [1]

Natural spider webs seen in our daily life, illustrated in Fig. 2, provide analogical prototype structure to a radial cable net, even though spider nets have no any stable or rigid/valley cables.

Fig. 2. Spider web (picture from Bill Groce)

A typical radial cable net mainly consists of outer compression ring and tension members. Such tension members include inner hoop cables, upper ridge cables and bottom valley cables along radial direction, and bracing cables extending therebetween. Consequently, the inner hoop cable, the ridge cables, the valley cables and the bracing cables form a continuous tension network to offer a promising structural integrity. When the cables are prestressed, tensile forces will be supported by the outer compressive ring. Its topology, the load path and structural characteristics integrate organically with the whole building system. Therefore, it exhibits favorable configuration, reliability and cost-efficiency suitable for spanning delineated areas of various sizes for large-span stadiums. Such type of the structure has been used as the roof of the stadium in Frankfurt (German) and the century lotus large-scale stadium in Foshan city (China).

Much research has been conducted in refining the design and analysis of tensile structures. Various static/dynamic analyses [2-9] and designs [10,11] have been conducted to better understand the behavior of tensile structures under service-level loads. By contrast, the erection process of the tensile structure is often overlooked as a separated step from the design. Limited studies are on the erection analysis of cable-nets. Prestress distribution and magnitude level in the cables may be significantly affected by different construction procedures or methods, thereby causing the variation of structural stiffness. Hence, it is essential to properly select erection techniques, methods and sequences to ensure a reliable structural design. In this study, the concept of the erection analysis is proposed. The characteristics of the radial cable nets are described in the following sections and critical parameters on the behavior of the structure are discussed. A case study is presented to describe the concept of erection method.

II. CRITICAL PARAMETERS IN DESIGN OF CABLE-NETS

A radial cable-net, schematically presented in Fig. 3, has upper ridge cables and bottom valley cables along radial direction, and bracing cables extending therebetween. Ridge cables and valley cables are highly intercorrelated. The ridge cables serve as bearing members carrying applied loads and the valley cables serve as stable members when the cable net is subjected to vertical loads, while vice versa when the cable net is subjected to uplift loads (e.g., wind-induced uplifting). It is apparent that a proper layout of stable cables in the radial cable-net is of indispensability.

A. Dominant parameters in design

Ridge cables and valley cables are prerequisite to a radial cable net. A height-span ratio of the structure, h_1/l_1, is a critical variable affecting the structural characteristics, as shown in Fig. 3. Ridge cables carry further loads while valley cables are unloaded when subjected to vertical loads. The sag, f_1, of ridge cable segment AC is also one of the factors to influence the curvature of ridge cables. In general, ridge cables are assumed as a catenary or approximate parabola curve. A ratio h_2/l_2 and the sag, f_2, of valley cable segment BC will affect the structural behavior when the cable net is subjected to uplift loads.

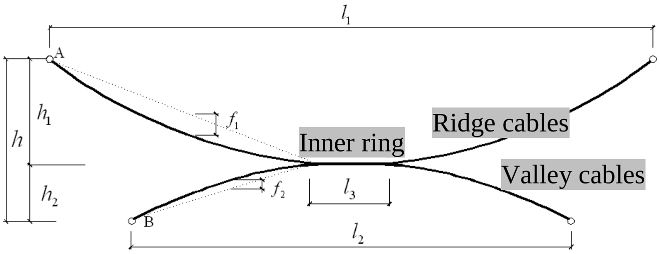


Fig.3. Parameters of the radial cable-net

The relationship between h_1 and h_2 is critical for the determination of the final configuration of the cable net if the span of the cable net (i.e., l_1 and l_2) is given. The variables f_1 and f_2 will affect the stiffness of ridge cables and valley cables under constant factors h_1 and h_2. Note that the internal forces of ridge and valley cables may decrease if values f_1 and f_2 are relatively high. Therefore, h_1, h_2, f_1 and f_2 are the main factors determining the structural behavior of cable nets.

The configuration of the radial cable net is uniquely determined by the variables l_1, l_2, h_1, h_2, f_1 and f_2. Hence, final topology and geometry of the radial cable net may rely on the prestress distribution and magnitude level. There is a one-to-one correspondence between structural configuration and the distribution of prestress under a definite topology.

For a definite topology, determination of prestress distribution in cable nets is an iterative procedure of form finding using either force density [11,12] or dynamic relaxation technologies [7,11]. It is indispensable that bearing cables, stable cables and bracing cables have enough strength to satisfy the reliability under various load cases, that is, any cable segment of the system should not be slack under any load case.

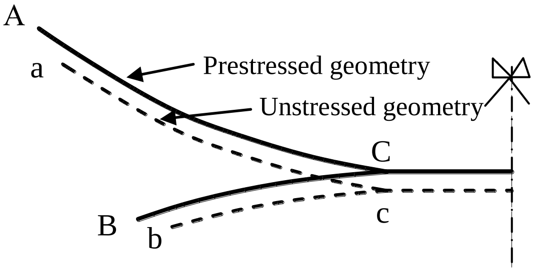


Fig. 4. Configuration of unstressed geometry and prestressed geometry

B. *Unstressed geometry*

Since cable-nets are a prestressed load-carrying system, initial unstressed geometry is critical for erection analysis in that unstressed geometry may provide fundamental information, including initial length of each cable segments in preliminary manufacture, and initial prototype for the onset of the erection analysis. The problem of finding a unstressed configuration of a cable-net is a reverse process of prestressing. For example, a half symmetric cable net is presented here for brevity, as shown in Fig. 4. The solid line denotes final prestressed geometry while the dashed line represents the initial unstressed geometry. With prestressed geometry in preliminary study, Unstressed geometry, shown in dashed lines in Fig. 4, is generally derived from unloading/releasing the pre-tension through finite element methods.

C. *Prestressing process*

With introduction of unstressed state, prestressing cable nets is the process of the erection, as shown in Fig.5.

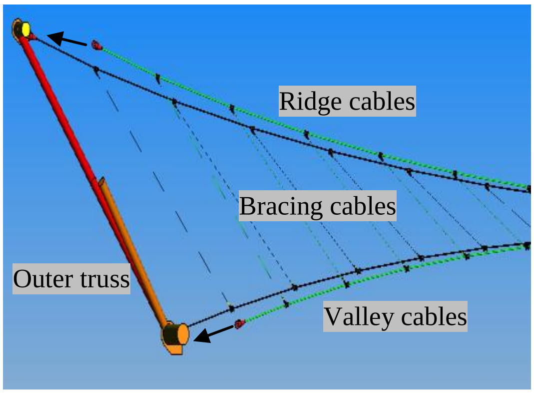
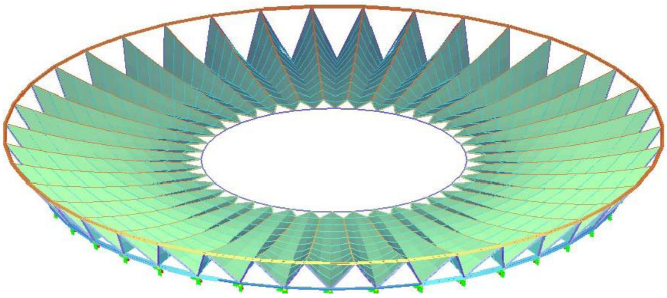


Fig.5. Placement of cable segments

Cables segments are fabricated according to unstressed length of cables derived from initial unstressed configuration and then assemblies on site. The ideal method of prestressing cable nets is to assembly the whole system and then apply loading on cables till all cables having predicted placement. A numerical analysis in preliminary study demonstrates that this erection scheme is cost-effective and effective. Final equilibrium shape could be achieved within two or three iterations while the prestress distribution is in high agreement with that predicted in preliminary design. Such erection, however, demands a great amount of labours and jack forces. Note that cable-nets under erection are quite different from their equilibrium configuration under in-service state. It is inevitable that parts of the cable net, due to incompleteness during construction, are a kinematic indeterminate system [13-19]. The observations in previous studies [10, 11, 13-17] in behavior of tensile structures reveal that cables may undergo infinitesimal or finite displacement before they are in state of equilibrium. This understanding leads to alternative step-by-step erection scheme more feasible for large scale cable-nets. The process of prestressing scheme consists of two steps: a) pulling cables; and b) tensioning cables. The former step intends to remove cable slack. Such placement of cables may experience large displacement without any extensive strain and thus no prestresses are applied to the cables. The latter step is to apply pre-tension to cables. Hence, the erection process of the cable net includes not only elastic deformation (tensioning/elongating cables), but also a large displacement with inextensive strain. Traditional nonlinear finite element method, a strong tool for nonlinear problems, is no longer valid due to the severe singularity of stiffness caused by a kinematic indeterminate system (e.g., finite motion during the construction of the cable nets).

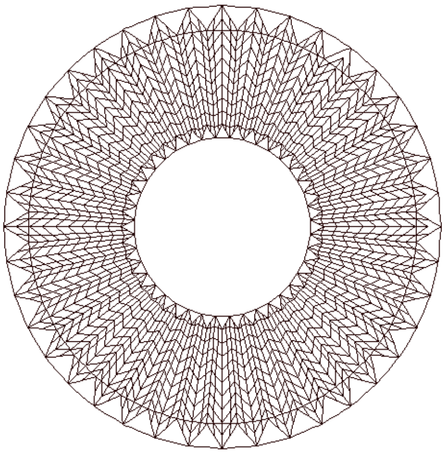
III. ERECTION ANALYSIS AND ERECTION CONTROL

Using final structural geometry as a reference is not valid for erection analysis of cable-net structures. It is because different stages may have different geometry while the distribution of prestress also changes during the construction process. Accurate prediction of their geometry and prestress may be highly associated with construction methods and technologies. Hence, the erection analysis is performed in accordance with the structural characteristics and updating configuration based on in-situ field-measured data.



(a) Radial cable net roof

(b) Elevation view



(c) Plan view

Fig. 6. Foshan century lotus radial cable net

The cable net is assembled by each component sequentially and approaches the predicted configuration through uplifting, pulling, and tensioning cables, by which the cable net acquired prestress and is stiffened as a load-carrying system. Specifically, the erection consists of assembling inner cable ring, uplifting and assembling ridge cables, assembling valley cables and bracing cables, respectively.

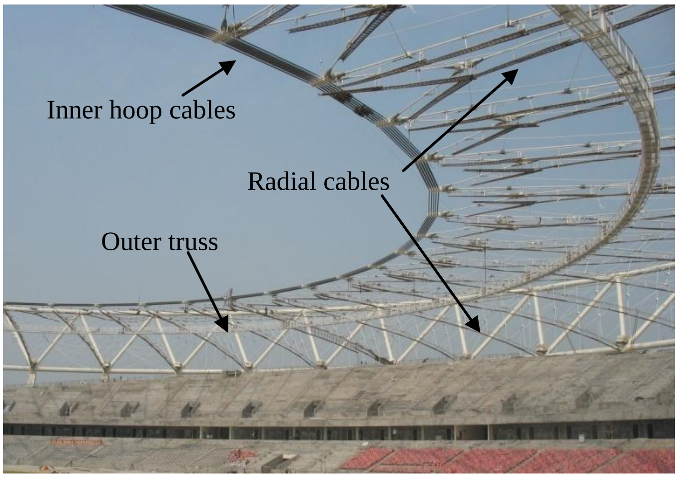
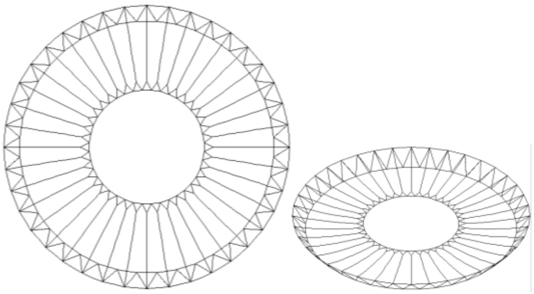
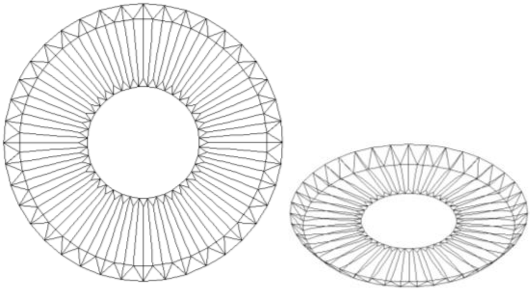


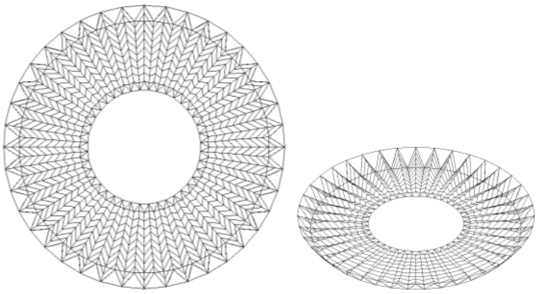
Fig. 7. Photo of the Foshan radial cable net during construction



(a) Assembling ridge cables



(b) Assembling valley cables



(c) Assembling bracing cables

Fig. 8. Design of erection procedures for the cable net

A. *Elastic deformation of cables*

Traditional displacement-based finite element method, [2, 6, 11] or force-based flexural method [15-19] may be used to solve the elastic deformation of cables, including cable internal forces or nodal displacements:

$$\boldsymbol{Af} = \boldsymbol{P}, \tag{1}$$

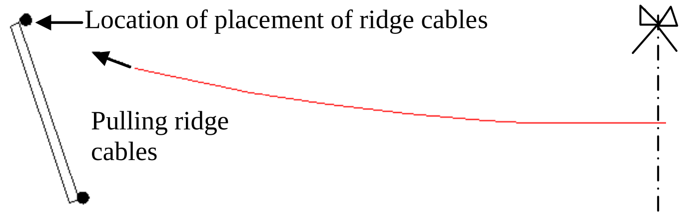
where, $\boldsymbol{A}$ is coefficient matrix, $\boldsymbol{f}$ is extension vector of lengths and $\boldsymbol{P}$ is load vector.

B. *Large displacement with inextensive strain*

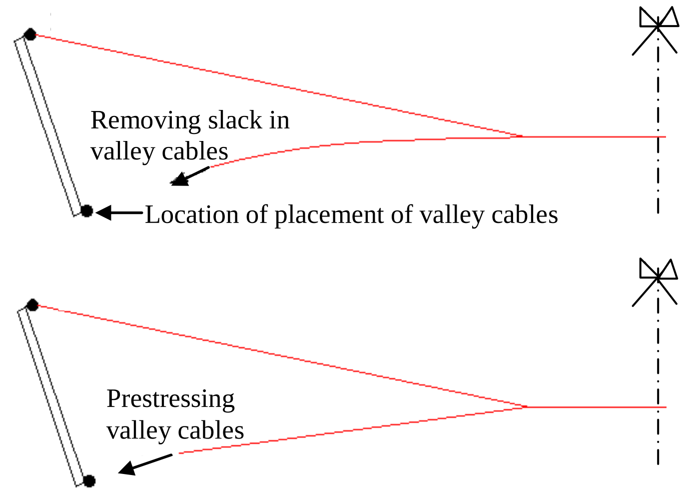
The finite element method is not valid for mechanism motion. Pulling cables acts as a constraint condition, $\boldsymbol{R}(x_1, x_2, \ldots x_n)$,

$$\boldsymbol{F}(\bar{\boldsymbol{u}}, \boldsymbol{R}(x_1, x_2, \ldots x_n)) \rightarrow \text{minimum}, \tag{2}$$

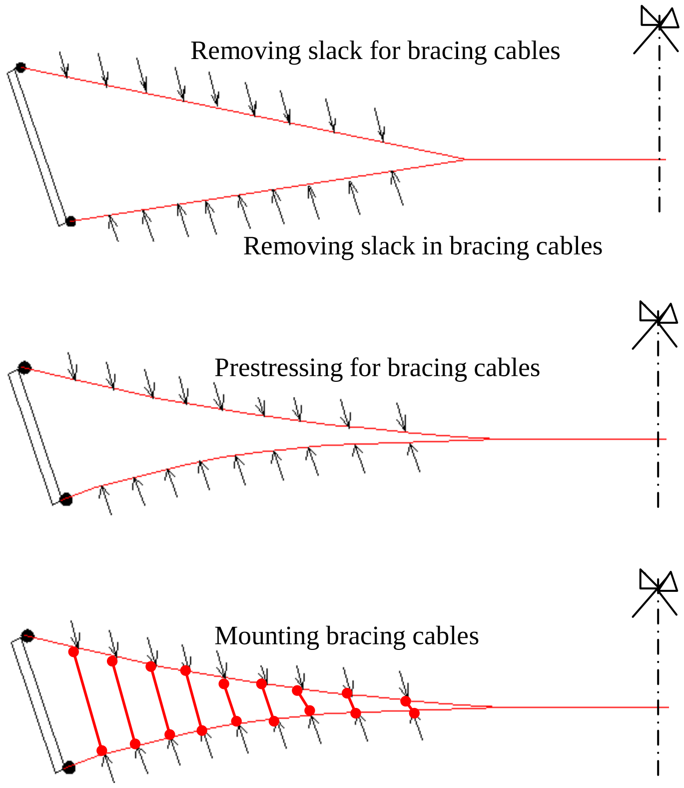
where, $\bar{\boldsymbol{u}}$ denotes mechanism in a system. The relationship of orthogonality [13] between the constrain condition and mechanism in the system is satisfied when the *minimum* in Eq. 2 is zero. Eq. 2 indicates that a system may undergo relatively large displacement, which ends up with stable configuration state of equilibrium under the constraint condition if and only if there is orthogonality between the constraint condition and mechanism.



(a) Placement of ridge cables



(b) Placement of valley cables



(c) Placement of bracing cables

Fig. 9. Erection process for the cable net (half span)

IV. A CASE STUDY OF A CABLE-NET

Consider Foshan century lotus cable net as an example to implement the erection analysis, as shown in Figs. 6a through 6c. Foshan century lotus stadium has a 310-meter in outer diameter and 20-meter in height radial cable-net roof, which comprises inner hoop cables, concave upward ridge cables and concave downward valley cables, and bracing cables, as illustrated in Fig. 7. Ends of ridge cables and valley cables were mounted to a compressive steel truss and the inner tensile hoop cables, respectively. The critical parameters of the radial cable net of l_1, l_2, h_1, and h_2 as mentioned in Fig. 3 are 310 m, 275 m, 125 m, 13 m and 7 m, respectively.

After the outer compressive steel truss is built, the inner hoop cables are assembled on the ground and also with other components in a hanging position without membrane. The main erection process consists of three stages: a) uplifting ridge cables (Fig. 8a); b) pulling valley cables (Fig. 8b) and c) assembling bracing cables (Fig. 8c), respectively.

A. Placement of ridge cables

Pulling ridge cables is the first step, as shown in Figs. 8a and 9a. The process is to pull ridge cables until the ends of cables are mounted on the outer steel truss. It is noted that the total length of cable segments in the radial direction is larger than the diameter of the stadium. No prestress is produced except internal forces due to own self-weight.

B. Placement of valley cables

Placement of valley cables consists of two steps as we discussed before: a) pulling valley cables without creating any prestress and b) prestressing valley cables. Pulling valley cables is the next step after the placement of ridge cables, as shown in Figs. 8b and 9b. Since the total length of valley cable segments along radial direction is less than the distance from the outer steel truss to the inner cable ring, there will exist a mechanism motion (pulling cables to remove slack) and elastic deformation (prestressing cables). Unlikely the process of placement of ridge cables which is a mechanism motion without any prestress, placement of valley cables is to prestress cables until the ends of the valley cables reach the mounted points on the outer steel truss, as indicated in Fig. 9b. Note that mechanism motion and elastic deformation could be intercorrelated during the whole construction.

C. Assembling bracing cables

Different to placement of ridge and valley cables, assembling bracing cables between ridge and valley cables demands much higher accuracy. It is because the distance at each mounted pair points on ridge and valley cables should equal to fabricated unstressed bracing cables length. It is schematically illustrated in Fig. 9c that a pair of forces is placed on corresponding nodes of ridge cables and valley cables by using temporary cables mounted to both ends. The processing of shortening temporary cables is to remove the slack in ridge and valley cables. Without cable slack, the ridge and valley cables are further prestressed until such cable length equals to the unstressed length of bracing cables. Replacement of temporaory cable by bracing cables, presented in Fig. 9c, allow bracing cables prestressing. Note that the process of placement of bracing cables experiences mechanism motion and elastic deformation.

Clearly, the erection process of cable nets accompanies large mechanism motion and elastic deformation. The mechanism motion during construction is frequently observed in placement of ridge, valley and bracing cables. Unlike traditional kinematics invariant structure, it is critical for tensile structures to assess potential mechanism motion during construction. It will overestimate the prestress, even leading to the wrong nodal coordinates and final geometry if using elastic/nonlinear finite element analysis for the whole erection control. Furthermore, since the actual in-situ geometry of tensile structure during construction is not consistent with the initial design geometry, the geometric model in the simulation should be updated in accordance with the field measured nodal coordinates and prestress on site.

V. CONCLUSIONS

Characteristics of radial cable nets are identified. The concept of erection analysis is proposed based on the behaviour of cable nets, with which mechanism motion and elastic deformation are classified. Without sufficient external constraints, such configuration may experience significant large deformation during the construction period. Such understanding leads to the proposed method. An erection process of a large-scale radial cable net is used to demonstrate the proposed concept. Appropriate erection analysis can effectively provide information for construction monitoring, and predict structural deformation and prestress distribution during the whole erection process.

In addition, the proposed erection analysis can be easily extended into other types of tensile structures, even in in-service state of structures such as load relieving roof system, in which the adjustment of structural configuration can offer releasing the energy by infinitesimal mechanism or finite motion. Although more work is required to explore the corresponding dynamic response due to finite motion during construction or even in-service states, it is expected that the construction of cable nets or other types of tensile structures can be more rationale with the proposed concept.

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