Synthesis of Pencil Beam Linear Antenna Arrays using Simple FFT/CF/GA Based Technique

B. Eldosouky¹, A. H. Hussein¹, H. H. Abdullah², and S. Khamis¹
¹ Faculty of Engineering, Tanta University, Tanta, Egypt,
² Electronics Research Institute, Cairo, Giza, Dokki, Egypt.

Abstract— Many applications such as satellite communications and radar systems where the antenna array weight and size are limiting factors favor compact antenna arrays. In this paper, a new approach for the synthesis of linear arrays featuring a minimum number of antenna elements is presented. The method is based on the combination between Fourier transform technique, curve fitting technique, and the genetic algorithm to derive the optimum element spacing and elements excitations required to synthesize a prescribed array factor. The effectiveness and simplicity of the proposed algorithm will be demonstrated by comparison with other analytical and optimization techniques.

Index Term- Antenna arrays, fast Fourier transform (FFT), curve fitting (CF), and genetic algorithm (GA).

I. INTRODUCTION
Linear antenna array synthesis using optimum number of antenna elements has received a great attention in the electromagnetics community. Recently the matrix pencil method (MPM), the forward-backward matrix pencil method (FBMPM), the hybrid technique between the method of moments and the genetic algorithm (MoM/GA), genetic algorithm (GA) and particle swarm optimization (PSO) have been successfully applied in synthesizing linear antenna arrays. The number of antenna elements reduction has a significant importance in many applications where the cost and weight are critical such as, satellite and radar systems and mobile communications. Many research efforts attempted to reduce the number of elements by introducing non-uniform spacing between the antenna array elements [1-7]. A non-iterative algorithms based on the matrix pencil method (MPM) were introduced in [1-3]. However, the MPM introduces an ill conditioned matrix that needs special treatments such as the use of the singular value decomposition method (SVD).

On the other hand, new evolutionary algorithms based on the optimization techniques are used successfully to solve typically complicated radiation pattern synthesis problems. These algorithms include vector tabu search [7], simulated annealing [8], genetic [9], particle swarm optimization [10], and differential evolution algorithms [11]. The common step of these optimization techniques is based on finding the solution of many unknowns such as the excitation amplitudes, the phases and the location of each element.

Recently, a new algorithm based on a combination between the method of moments and the genetic algorithm (MoM/GA) is introduced [12]. The algorithm provides number of antenna elements reduction using either uniform or non-uniform element spacing. The MoM provides a deterministic solution for the excitation coefficients. On the other hand, the GA is used to estimate the optimum element locations to obtain the required radiation pattern within a minimum tolerance.

In this paper, a very simple new algorithm based on a combination between the Fourier transform technique [13,14], curve fitting technique, and the genetic algorithm [15] is introduced. The proposed algorithm provides a number of elements reduction using either uniform or non-uniform element spacing. The FFT/CF provides a deterministic solution for the excitation coefficients. In this scence, the FFT is used to determine the excitation coefficients for the desired pattern. The curve fitting technique is used to generate a fitting polynomial that relates the predetermined excitation coefficients using FFT to the corresponding elements positions. Thus, for a given number of antenna elements \( M \leq N \), (where \( M \) is the number of elements of the synthesized pattern and \( N \) is the number of elements of the desired pattern), the GA is used to estimate the optimum element positions those satisfy the fitting polynomial to obtain the required radiation pattern within a minimum tolerance. One of the main features of this algorithm is that it maintains a relatively fixed dynamic range ratio, DRR, (that is the ratio of the maximum excitation coefficient magnitude to the minimum excitation coefficient magnitude) as the estimated excitations lie within the fitting polynomial curve. The proposed algorithm is directly applied to the synthesis of pencil-beam patterns through a few tens or hundreds of iterations.

II. PROPOSED FFT/CF/GA ALGORITHM
In this paper, the usefulness of Fourier technique algorithm is established to generate the excitation coefficients of pencil beam uniformly spaced linear antenna arrays by talking Fourier transform of the array factor. Consider linear antenna consists of \( N \) elements, the array factor of the array is given by

\[
AF(\theta) = \sum_{n=0}^{N-1} a_n e^{j nk \cos(\theta)}
\]  

(1)

Let \( u = \cos(\theta) \), the array factor can be written in the form

\[
AF(\theta) = \sum_{n=0}^{N-1} a_n e^{j nk u}
\]  

(2)
where \( a_n \) is the complex excitation of the \( n^{th} \) element, \( k \) is the wave number \( (k = 2\pi/\lambda) \), \( \lambda \) is the wavelength, and \( \theta \) is the angular coordinate measured between the far-field direction and the array normal.

The FFT/CF/GA method uses the property that for an array antenna having a uniform inter-element spacing of the elements, a Fourier transform relationship exists between the array factor (AF) and the element excitations. This property is used to derive the array element excitations from the prescribed AF. It is required to synthesize a desired array factor using minimum number of antenna elements \( M < N \) such the synthesized pattern \( \text{AF}_s(\theta) \) is given by

\[
\text{AF}_s(\theta) = \sum_{m=1}^{M} e^{j k m d_o u} \approx \text{AF}(\theta) \quad (3)
\]

1. If the excitation coefficients of the original pattern are well known, we start at step 6. But if the excitation coefficients are not known we start at step 2.
2. Compute \( \text{AF}(\theta) = \sum_{n=0}^{N-1} a_n e^{j k n d u} \).
3. Compute the initial weights \( \{A_n\} \) from AF using a K-point FFT, with K+N.
4. Truncate \( \{A_n\} \) from K samples to N samples by making zero all samples outside the array.
5. Then, the final weights are obtained by windowing with a length-N window \( w \) where, \( a_n = w \ast A_n \).
6. By using polynomial curve fitting technique in MATLAB, a polynomial \( P(d_m) \) of order \( (Q) \) is generated to relate the predetermined excitation coefficients \( a_m \) using FFT to their corresponding elements positions

\[
P(d_m) = p_1(d_m)^Q + p_2(d_m)^{Q-1} + \cdots + p_Q(d_m) 
\]

\( , m = 1,2,3,...,M \quad (4) \)

where, \( d_m \) is \( m^{th} \) element position, \( M \) is the number of elements of the synthesized array, and \( \{p_1,p_2,p_3,...,p_Q, p_{(Q+1)}\} \) are the coefficients of the polynomial. By constructing the fitting polynomial, just the optimized element positions are applied to the polynomial to determine the synthesized excitation coefficients \( a_m \) required to synthesize the desired pattern.

7. Estimation of the minimum number of antenna elements is based on keeping the half power beamwidth (HPBW) constant with minimum variations in the side lobe level of the array pattern. In order to maintain the same HPBW, the array size of the synthesized array should be the same as the original array. The original array size is given by

\[
A_{\text{size}} = (N - 1)d 
\]

For array synthesis with minimum number of elements \( M_{\text{min}} \), it corresponds to maximum element spacing \( d_{\text{max}} \). In this case, the synthesized array size will be

\[
(M_{\text{min}} - 1)d_{\text{max}} 
\]

Equating both sides of equations (5) and (6),

\[
(M_{\text{min}} - 1)d_{\text{max}} = A_{\text{size}} 
\]

\[
M_{\text{min}} = \frac{A_{\text{size}}}{d_{\text{max}}} + 1 
\]

To avoid the appearance of the grating lobes, the maximum element spacing \( d_{\text{max}} \) should not exceed the wavelength \( (d_{\text{max}} \leq \lambda) \).

8. Estimation of the optimum element spacing \( d_0 \) is performed using the genetic algorithm (GA) optimization tool. The GA estimates the optimum element spacing \( d_0 \) that introduces minimum least mean square error between the absolute values original pattern and the synthesized pattern. The cost function \( F \) to be minimized under the constraint that the two patterns must have the same half power beamwidth is written as follows

\[
F = \frac{\sum_{k=0}^{R} |\text{AF}(k) - \text{AF}_s(k)|^2}{\text{HPBW}_o - \text{HPBW}_s} \quad (9)
\]

Where \( \text{AF}(k) \) and \( \text{AF}_s(k) \) are the original and the synthesized patterns respectively. \( \text{HPBW}_o \) and \( \text{HPBW}_s \) are the half power beamwidths of the original and the synthesized patterns respectively. \( R \) is the number of the angular samples taken to cover all the most important variations in the original pattern. The estimation of the optimum element spacing \( d_0 \) is performed applying the fitting polynomial \( P(d_m) \) Eq. (4).

For a given number of elements \( M \), assuming symmetrical array configuration, the synthesized array size will be

\[
SA_{\text{size}} = (M - 1)d_0 
\]

The \( m^{th} \) element position \( d_m \) can be determined as follows

\[
d_m = \frac{SA_{\text{size}}}{2} + [(m - 1) \times d_0] \quad , m = 1,2,3,...,M \quad (11)
\]

The GA algorithm searches for the optimum element spacing \( d_0 \) that exhibits optimum excitations \( a_m \) which satisfy the half power beamwidth constraints and introduce minimum LMSE between the original and the synthesized patterns. If this constraint is not satisfied, the number of elements is increased by one and the process is repeated until reaching the optimum \( d_0 \) and the minimum M. The GA optimization tool in MATLAB is used to estimate the optimum element spacing \( d_0 \) within a pre-assigned range \( 0.5 \lambda \leq d_0 \leq \lambda \). The corresponding excitation coefficients are determined directly using the fitting polynomial. The initial value of \( d_0 \) is set to 0.5 \( \lambda \) and the GA optimization process is performed until the minimum value of the LMSE is reached.
III. SIMULATION RESULTS

Consider a twenty elements broadside Tschebyscheff array with a half wavelength spacing between elements, and side lobe level, SLL = -30 dB [16]. In this case, the excitation coefficients of the Tschebyscheff array are well known, so there is no need for using FFT. Applying the polynomial fitting technique, a polynomial \( P(d_m) \) of order \( Q = 20 \) is obtained. The resultant polynomial perfectly fits the excitation coefficients as shown in Fig.1. The polynomial coefficients are listed in table (1). By applying Eq. (8), the minimum number of elements required for array synthesis is found to be \( M_{min} = 12 \). The GA optimization tool is applied to estimate the optimum element spacing required to synthesize the desired pattern. The options of the GA optimization tool are set as shown in table (2).

A satisfactory approximation of the desired pattern is synthesized with twelve elements of uniform element spacing \( d_0 = 0.844 \lambda \). A good agreement is obtained when it is compared to the analytical Tschebyscheff pattern as shown in Fig.2. The synthesized pattern has the same \( HPBW = 6.1879^\circ \)as the original Tschebyscheff pattern. The optimum spacing between the elements are obtained using only 51 GA iterations. The excitation coefficients, the element spacing, and the HPBW of the synthesized pattern are listed in table (3) compared to the the 20-elements \( \lambda/2 \) Tschebyscheff array and the reconstructed arrays using the MoM/GA algorithm. Fig.3 shows the synthesized pattern compared to the the 20-elements \( \lambda/2 \) Tschebyscheff pattern and the reconstructed patterns using the MPM, and the MoM/GA algorithms.

It worth noting that the proposed method provides much simpler solution than these algorithms, and provides more accurate results than the uniform spacing MoM/GA algorithm and the MPM where it provides side lobe level SLL = -29.5392 dB that is much close to the original side lobe level which equals -29.9845 dB.

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**Fig. 1.** The excitation coefficients and the fitting polynomial versus elements positions.

**Fig. 2.** The synthesized pattern using the FFT/CF/GA algorithm compared to the analytical Tschebyscheff pattern.

**Fig. 3.** The synthesized pattern using the FFT/CF/GA algorithm compared to the the 20-elements \( \lambda/2 \) Tschebyscheff pattern and the reconstructed patterns using the MPM, and the MoM/GA algorithms.
### Table I
Fitting polynomial coefficients

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### Table II
Options of the genetic algorithm optimization tool used in simulation

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options =
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    PopInitRange: [2x1 double]
    PopulationSize: 20
    EliteCount: 2
    CrossoverFraction: 0.8000
    ParetoFraction: []
    MigrationDirection: 'forward'
    MigrationInterval: 20
    MigrationFraction: 0.2000
    Generations: 100
    TimeLimit: Inf
    FitnessLimit: -Inf
    StallGenLimit: 50
    StallTimeLimit: Inf
    TolFun: 1.0000e-006
    TolCon: 1.0000e-006
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    PenaltyFactor: 100
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    FitnessScoringFcn: @fitscalingrank
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    CrossoverFcn: @crossoverscattered
    MutationFcn: @mutationadaptmixed
    DistanceMeasureFcn: []
    HybridFcn: []
```

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The analytical Chebyshev array

Reconstructed array

Uniform spacing MoM/GA

Reconstructed array

Non-uniform MoM/GA

Reconstructed array

Uniform spacing FFT/CF/GA

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HPBW = 6.1879° HPBW = 6.1879° HPBW = 6.1879° HPBW = 6.1879°
SLL = -29.9845 dB SLL = -28.3347 dB SLL = -29.0876 dB SLL = -29.5392 dB
DRR = 3.5 DRR = 3.56 DRR = 3.74 DRR = 3.66

VI. CONCLUSION

In this paper, a very simple new algorithm based on a combination between the Fourier transform technique, curve fitting technique, and the genetic algorithm is introduced. The proposed algorithm provides a number of elements reduction using either uniform or non-uniform element spacing. It provides more accurate results than the uniform spacing MoM/GA algorithm and the MPM. The proposed algorithm can be used for the synthesis of the shaped power patterns of complex excitation coefficients. In this case, it requires accurate curve fitting for both excitation coefficient magnitude and phase separately.

REFERENCES


Table III

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