

Multiple Linear Regression Model for Suspended Load Transport Rate Prediction and Its Evaluation Using Selected Transport Formulas

¹Assist. Prof. Dr. Thair Sh. Khayyun, ²Assist. Prof. Dr. Ayad S. Mustafa
¹[University of Technology, Building & Construction Eng. Dept., Baghdad-Iraq
 Email: THSHKHMA@YAHOO.COM]

²[Al-Anbar University, Civil Eng. Dept. , Al-Anbar-Iraq
 Email:AYAD_ENG2001@YAHOO.COM]

Abstract-- Water flow in rivers causes' sediment transport and the relationship between flow factors (velocity, depth) and amount of material transported is a very interesting but complex phenomenon and it has several important engineering aspects like erosion around structures, backfilling of dredged channels or reservoirs, erosion below a dam, morphological changes in rivers. Governing parameters were carefully selected based on a dimensional analysis and grouped into four parameters (transport, particle, shields and mobility). In this study, a new suspended load transport rate equation that is applicable for the Euphrates River at Al-Anbar Electrical Thermal Station in Iraq was developed using multiple linear regression analysis. The capability of suspended load transport rate formulas of (Leo c. Van Rijn (1984), Karim Kennedy (1990) and van Rijn (2007)) had been tested. A total of 25 series of field data had been applied which their suspended load concentration, average depth of flow, velocity, particle size of sediment and water surface width had been measured. A comparison of suspended load transport rate measured in the Euphrates River with computed transport rates of the modified formula showed a good agreement. The results of evaluations showed that, the new model performs better than the three formulas which were used. The accuracy of the new model is determined using the relative error, discrepancy ratio, and correlation coefficients. The computed suspended transport rates were found to be within a factor of (1.02) of measured values (discrepancy ratio) and 13.4% relative error. The formula of (Van. Rijn, (2007)) performed better with a factor of (1.26) of measured values and 39% relative error than the other two formulas (Van Rijn, 1984) and (Karim Kennedy, 1990). The percent of data that are between (0.5-2.0) of discrepancy ration vary between 72-100% for the formulas and new model.

Index Term-- Euphrates Rivers, suspended load, new model, evaluation.

1. INTRODUCTION

Engineers, geologists, and river morphologists have studied the subject of sediment transport (bed & suspended load) for centuries. Different approaches have been used for the development of sediment transport functions or formulas.

These formulas have been used for solving engineering and environmental problems. Results obtained from different approaches often differ largely from each other and from observations in the field. Some of the basic concepts, their limits of application, and the interrelationships among them have become clear to us only in recent years. Many of

the complex aspects of sediment transport are yet to be understood, and they remain among the challenging subjects for future studies.

A large number of comparison studies have been done to test the predictability of various sediment transport methods covering a wide range of flow conditions and sediment types (Van denBreg, T.H (1993); Winterwerp, J.C. (2001); Martin, Y. (2003); Pinto, L. (2006) and Baosheng, W.U. (2008)], but the accuracy of computational sediment transport models has remained a challenging question [ASCE (2004)].

Number of sediment transport models and formulas can be found in the literatures that are used to study sediment transport in alluvial channels. Most of the transport models are based on simplified assumptions that are valid in ideal laboratory conditions only and may not be true for much complicated natural river systems. Models based on more sophisticated theoretical solutions require a large number of parameters that are impossible or difficult to gather for a natural river system [Choudhury, P. (2010)].

Suspended transport is the dominant mode of transport in the lower reaches of rivers. The sediment beds are classified into subclasses, using the following class separation diameters: $d_{\text{gravel}} = 2000\mu\text{m}$, $d_{\text{sand}} = 62\mu\text{m}$, $d_{\text{silt}} = 32\mu\text{m}$ and $d_{\text{cs}} = 8\mu\text{m}$. A high-quality field data set of suspended transport rates in rivers is established and analyzed to reveal the basic influence of the flow regime (velocity, water depth) and the sediment particle size [Chih T.Y. (1996)]. Practically, fine particles (e.g. clay and silt) settle in a

laminar flow ($\frac{w_0 d_s}{\gamma} < 1$) while large particles (e.g. gravel and boulder) fall in a full turbulent flow motion ($\frac{w_0 d_s}{\gamma} > 1000$). The most common approach to model the

suspended sediment transport is based on the advection-diffusion theory representing the downward transport by gravity (settling) and the upward transport by turbulent processes (mixing), resulting in a Rouse-type sediment concentration profile over the water depth [Van Rijn, L.C. (2007)].

Suspended sediment transport refers to the particles or grain of sediment moved along a river within and wholly supported by the flow. In order for sediment grain to remain in suspension the upward directed forces associated with

turbulence in the flow must be strong enough to overcome the downward force of gravity acting on the grains. As physical reasoning implies, the suspended-sediment load consists largely of the finer fraction, the fine sand, silt and clay of the sediment available to the river. Because turbulence is generated of the channel boundary and is most intense there, suspended sediment tends to have higher concentrations and involve coarser material near the boundary and both sediment size and concentration decline as we move up through the water column towards the surface of the flow [Edward J. (2004)].

The objective of the study is to determine a new model for estimating the suspended load transport rate as a function of flow and sediment parameters for the Euphrates river in Ramadi-Iraq. In the present study suspended transport rate formulas of (Van Rijn (1984), Karim-Kennedy (1990) and

Van Rijn (2007)) were selected to test their efficiency with the new model.

2. STUDY AREA AND MEASURED DATA

2.1 Study area

The reach of the Euphrates River, considered in this study, locates near the intake of Electrical Thermal Station in Al-Ramadi city and lies upstream of Al-Ramadi gage station, Fig. (1). The length of the reach is 2.5 km and it is divided into 25 cross-sections. The average of discharge which is listed in this gage station is $640 \text{ m}^3/\text{s}$. The maximum and minimum discharge were 1279 and $247.5 \text{ m}^3/\text{s}$ respectively during the period from 1970-2010 (Ministry of Water Resources- Iraq (2012)).



Fig. 1. Study Area (reach of Euphrates River).

2.2 Measured data

For the collection of sediment data, guidelines were issued by the American Society for Testing and Materials (ASTM-DS387-1997) describing the parameters that should be measured or collected to obtain a complete sediment and hydraulic data set. A complete data set should include the following parameters:

- Sediment parameters: sediment discharge or sediment concentration of suspended load, size distributions of suspended load, bed material and their specific gravity.
- Hydraulic parameters: water discharge, velocity, width, depth and elevation of water surface.
- Other parameters: temperature.

- d. Description of field conditions such as bed forms present at time of data collecting methodology and instrumentation.

Suspended sediment discharge over an entire cross-section is usually measured by dividing the cross-section into a number of sections (5 sections). Sediment discharge passing through each section is obtained by taking measurements along the vertical within the portion of the section it represents. The sediment concentration in a vertical is sampled by depth integration. In multi-point methods, it is common to sample at 3 points at relative depths 0.2, 0.6 and 0.8 (ratio of the depth of the sampler to the stream depth). The velocity and the depth were measured by current meter and Echo-sounder respectively. The sediment discharge per unit width in each vertical is determined by using the equation (1):

$$q_s = \frac{d}{m} \sum_{i=1}^n K_i c_i v_i \quad \dots (1)$$

Where q_s is the sediment discharge per unit width in $\text{kg s}^{-1}\text{m}^{-1}$, m is the number of measuring points, c_i is the sediment concentration at the measuring point as determined in a field laboratory in mg/L or kg/m^3 , v_i is the velocity at the

measuring point in m/s , d is the depth in m , K_i is the fraction of depth each measurement represents (equal to 1.0) by the Chinese standards (1992). All samples are combined into a single representative weighted sample. In general sediment concentration is determined by using the evaporation method.

Sieve method was used for the determination of the size distribution of a sediment sample. The lower limit of sizes within which sieve analysis may be applied is 0.062 mm. The results of size analysis are expressed by a size gradation curve. Characteristic figures can always be interpolated from the curve such as d_{50} , d_{65} and d_{90} . Density or specific gravity is an important physical property of sediment particles, which was measured with a specific-gravity flask. In general, the specific gravity of sediment particles varies from 2.60 to 2.70.

The hydraulic data of the reach of this study used as the basis for calculations were measured for the monitoring period of 12 months in 2012. About 375 series of data regarding measured suspended sediments, flow velocity, particle size, water temperature and depth of water for the 25 cross-sections, which were measured at the same time, were selected and used in formulas, Table I.

Table I
Summary of Applied Data in Total (Suspended Load Calculation) of Euphrates River (AL-Ramadi Electrical Thermal Station)

Sediment concentration, mg/L	Flow rate, m^3/s	Velocity, m/s	Section width, m	d_{50} mm	d_{90} mm	Temperature, (degree)	Section area, m^2
80.93-376.42	271-835	0.50-0.77	160-420	0.09-0.15	0.19-0.24	9.5-26.9	460.7-979.2

3. METHODOLOGY

The hydraulic and sediment transport parameters needed in the computations were collected and processed on a personal computer. Governing parameters were carefully selected based on dimensional analyses and grouped into four parameters (transport, particle, shields and mobility). A new model for estimating the suspended load transport was developed by using the multiple linear regressions. This model was then compared with those of the measured values.

The suspended load transport rate was computed for the data set using the three selected formulas and then compared with those of the measured values. The correlation coefficient, discrepancy ratio and relative error were used for the comparison of performance. The accuracy order was prepared on the basis of data coverage between the discrepancy ratio of (0.5-2.0), the Min. relative error and the calculated values were plotted against the observed values so that the scatter about the perfect agreement line can also be considered.

4. SUSPENDED LOAD TRANSPORT RATE FORMULAS

The following three formulas for the prediction of suspended load transport rate in (m^3/sec) per unit width have been tested with the new model using the reach data of Euphrates River. These models were selected because they

are well known and they represent the main types of models (based on regression and stream power approaches).

4.1. Van Rijn (1984)

Van Rijn L.C. (1984) presented a method which enables the computation of the suspended load as the depth-integration of the product of the local concentration and flow velocity. The complete method to compute the suspended load (volume) per unit width should apply as:

1. Compute particle diameter, D_* :

$$D_* = D_{50} \left[\frac{(s-1)g}{v^2} \right]^{1/3} \quad \dots (2)$$

2. Compute critical bed-shear velocity according to Shields, U_{*cr} .
3. Compute transport stage parameter, T :

$$T = \frac{(U'_{*})^2 - (U'_{*,cr})^2}{(U'_{*,cr})^2}, U'_{*} = \left(g^{0.5} / c' \right) \bar{u}, c' = 18 \log(12R_b / 3D_{90}) \dots (3)$$

4. Compute reference level, a:
a= 0.01 d

... (4)

5. Compute reference concentration, C_a:

$$C_a = 0.015 \frac{D_{50} T^{1.5}}{a D_*^{0.3}}$$

... (5)

6. Compute particle size of suspended sediment, D_s:

$$\frac{D_s}{D_{50}} = 1 + 0.011(\sigma_s - 1)(T - 25), \sigma_s = 2.5$$

... (6)

7. Compute fall velocity of suspended sediment w_s (for suspended Particles in the range 100-1000 μm):

$$w_s = 10 \frac{v}{D_s} \left\{ \left[1 + \frac{0.01(s-1)g D_s^3}{v^2} \right]^{0.5} - 1 \right\}$$

... (7)

8. Compute β-factor:

$$\beta = 1 + 2 \left[\frac{w_s}{u_*} \right]^2, \text{ for } 0.1 < \frac{w_s}{u_*} < 1$$

... (8)

9. Compute overall bed-shear velocity, Eq. (9).

$$\text{Log} \frac{q_s}{(1.65gd_{50}^3)^{1/2}} = -2.279 + 2.972 \text{Log} \frac{v}{(1.65gd_{50})^{1/2}} + 1.060 \text{Log} \frac{v}{(1.65gd_{50})^{1/2}} \dots\dots(14)$$

$$* \text{Log} \frac{U_* - U_{*c}}{(1.65gd_{50})^{1/2}} + 0.299 \text{Log} \frac{D}{d_{50}} \text{Log} \frac{U_* - U_{*c}}{(1.65gd_{50})^{1/2}}$$

Where:

q_s= volumetric suspended discharge per unit width

d₅₀= median particle diameter

v= mean velocity

U_{*}= bed shear velocity= (gDs)^{1/2}

U_{*c}= shield's value of critical shear velocity at incipient motion.

D= water depth

$$U_* = (gds)^{0.5} \dots\dots (9)$$

10. Compute φ – factor:

$$\phi = 2.5 \left[\frac{w_s}{u_*} \right]^{0.8} \left[\frac{c_a}{c_o} \right]^{0.4} \text{ for } c_o = 0.65 \text{ and } 0.01 \leq \frac{w_s}{u_*} \leq 1$$

... (10)

11. Compute suspension parameter z and z':

$$z' = z + \phi, z = \frac{w_s}{\beta k U_*}, k = 0.4$$

... (11)

12. Compute F-factor:

$$F = \left[\left(\frac{a}{d} \right)^{z'} - \left(\frac{a}{d} \right)^{1.2} \right] / \left[\left(1 - \frac{a}{d} \right)^{z'} (1.2 - z') \right]$$

... (12)

13. Compute suspended load transport, q_s:

$$q_s = F_* \bar{U}_* d_* c_a \dots\dots (13)$$

4.2. Karim-Kennedy (1990)

Karim and Kennedy (1990) (Chih, T.Y. (1996)) used nonlinear, multiple- regression analyses (based on laboratory field data) to derive relations between flow velocity, sediment discharge, bed-form geometry, and friction factor of alluvial rivers. The sediment discharge and velocity relationships adopted by them have the following general forms Eq. (14):

4.3. Van Rijn (2007): Van Rijn L.C. (2007) found that the suspended transport in river was strongly dependent on particle size and on current velocity. The ranges of sediment diameters were (60-600 μm). The empirical trend line can be roughly approximated by the equation:

$$q_s = \left(d_{ref} / d_{50} \right)^2 (u - U_{cr})^3 \dots\dots\dots (15)$$

Where: q_s= suspended transport (kg/s/m), d₅₀= median grain size (m); d_{ref}= reference grain size= 0.0003m; u= depth –

averaged velocity (m/s); and U_{cr} = critical depth- averaged velocity = 0.30 (m/s).

5. DEVELOPMENT OF NEW SUSPENDED LOAD RATE EQUATION

The threshold of sediment motion describes the flow conditions and boundary conditions for which the transport of sediment starts to occur. The relevant parameters for the analysis of sediment transport threshold are: the bed shear stress τ_0 , the sediment density ρ_s , the fluid density ρ , the grain diameter d_s , the gravity accelerations g and the fluid viscosity μ , mean velocity of flow v , settling velocity w_0 and depth of flow d . The suspended load transport rate was investigated to obtain a new theoretically relationship supported by empirical or theoretical equations. The method of dimensional analysis was applied to the principal variables related to suspended load rate.

The suspended load rate q_s in (m³/sec) is a function of the following variables:

$$q_s = f(V, V^*, d, w_0, \rho, s^*, T^*) \quad \dots (16)$$

in which V is the mean velocity, V^* is the shear velocity, d is water depth, w_0 is the settling velocity, S^* is a dimensionless particle parameter

$$\left(S^* = \left(\frac{(s-1)g}{\nu^2} \right)^{1/3} d_{50} \right), T^* \text{ is a dimensionless}$$

shield's parameter $(T^* = \left(\frac{T}{(s-1)\rho g d_{50}} \right))$, s is the

specific gravity, d_{50} is sediment size at which 50% of material is finer and ν is kinematic viscosity. The following π - terms are obtained from dimensional analysis after q_s , d and ρ are selected as repeating variables:

$$f\left(\frac{q_s}{Vd^2}, \frac{q_s}{V^*d^2}, \frac{q_s}{w_0d^2}, S^*, T^*\right) = 0 \quad \dots (17)$$

The sediment transport term ($q_s/(vd^2)$) can be written as a function of the product of the other π - terms in the form:

$$\frac{q_s}{Vd^2} = F\left[\left(S^*\right), \left(T^*\right), \left(\frac{V^*}{w_0}\right)\right] \quad \dots (18)$$

The π - term $\left(\frac{q_s}{Vd^2}\right)$ in eq. (18) represents the transport

parameter, while the π - term $\left(\frac{V^*}{w_0}\right)$ represents the

mobility parameter for initiation of suspension. Suspended

sediment load occurs of $\frac{V^*}{w_0} > 2.5$ (Julien, P.Y. (1995)).

The particle motion in the direction normal to the bed is related to the balance between the particle fall velocity (w_0) and the turbulent velocity fluctuation in the direction normal to the bed. The turbulent velocity fluctuation is of the same order of magnitude as the shear velocity (V^*). The type of sediment motion is a function of the sediment properties and flow conditions (modified shields diagram). If the particle parameter S^* is smaller than 10, the shields parameter T^* must be larger than (0.06-0.4).

6. SUITABILITY ANALYSIS

The selection of appropriate suspended load rate formulas under different flow and sediment condition are important to sediment transport and river morphologic studies. Computed suspended load rate or concentration from different suspended transport formulas can give vastly different results from each other and from field measurement. So engineers must compare the accuracies and limits of application of different formulas before their final selection. The performance of predicting the suspended load rates were tested by:

6.1. Discrepancy ratio:

A discrepancy ratio was calculated for each measured suspended load by comparing the computed and measured suspended load rate and by using Eq. (19), (Yu, K. and Woo (1994)):

$$d.r. = \frac{q_s \text{ calculated}}{q_s \text{ measured}} \quad \dots (19)$$

where the acceptable range is 0.5-2.0. The value of discrepancy ratios was then averaged. The closer the value to unity; the better suited the equation is assumed for the data set.

6.2. Relative error

The following equation was used to calculate the percentage of the relative errors (R.E.) of the predicted values with respect to the measured values:

$$R.E.\% = \frac{q_s \text{ measured} - q_s \text{ predicted}}{q_s \text{ measured}} * 100 \quad \dots (20)$$

7. RESULTS AND DISCUSSION

7.1 New model

A new model is developed by using the computer program of multiple linear regressions of the 25 set of measured data and the four parameters (transport, particle, shield's and mobility parameters). The minimum and maximum values of these parameters were 0.00175-0.02340, 2.27-3.79, 3.21-6.66 and 4.88-11.49, respectively. For these flow conditions the shields diagram predict sediment in suspension for $T^* > T^*_c$ ($T^*_c=0.06-2.0$). The

particle Reynold number ($\frac{w_o d_s}{\nu}$) was 0.80-2.80. The multiple regression is used to obtain a set at coefficients for a linear model, also it is used to assess the relative

and how well does the linear model represent the observed data. The power law type regression Eq. (21) used in this study is obtained by using logarithm transformation. The following regression model is derived for the transport parameter $q_s / (vd^2)$:

importance of the predictor variables (S^* , T^* and $\frac{V^*}{w_o}$)

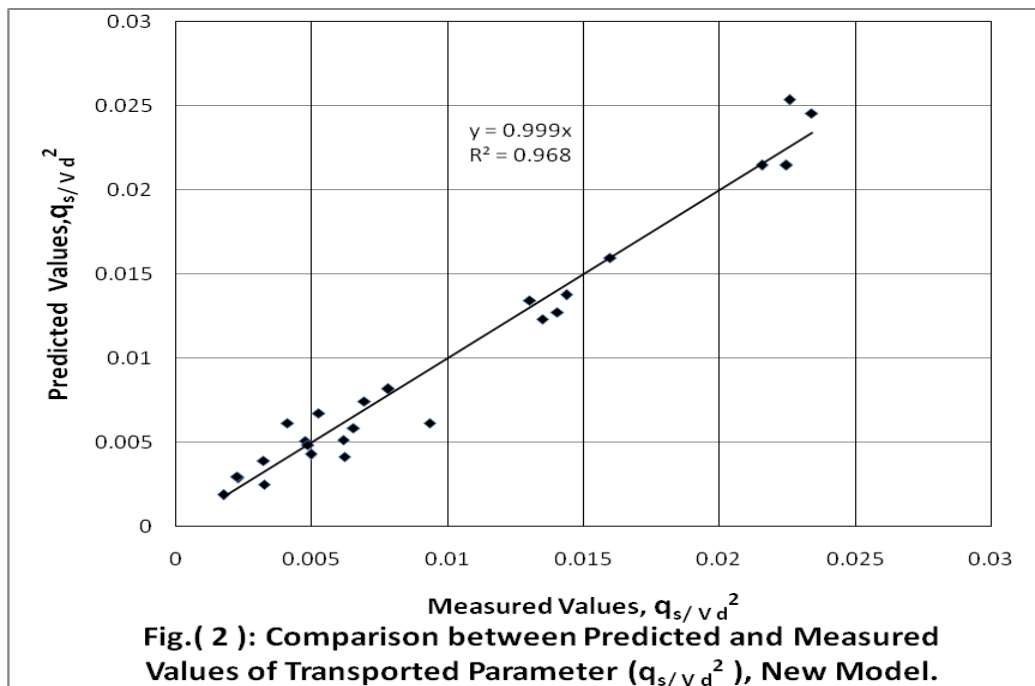
$$\frac{q_s}{Vd^2} = \left(\frac{V^*}{w_o}\right)^{-1.303} * \left(\left(\frac{(s-1)g}{\nu^2}\right)^{1/3} d_{50}\right)^{0.442} * \left(\frac{T}{(s-1)\rho g d_{50}}\right)^{-1.823} \dots (21)$$

Where, q_s is the suspended load rate in m^3/sec . The coefficients of determination; R^2 is (0.998). There are good agreements between the predicted and measured values of the transport permanent q_s/vd^2 , Fig.(2). The partial correlation

indicate that ($\frac{V^*}{w_o}$ and T^*) are the most important variables while (S^*) is less important than the others, Table (2).

Table II
Statistical Information for Prediction (q_s)

	$\frac{V^*}{w_o}$	S^*	T^*	q_s/vd^2
$\frac{V^*}{w_o}$	1.00	-0.94	0.88	-0.94
S^*	-0.94	1.00	-0.65	0.81
T^*	0.88	-0.65	1.00	-0.93
q_s/vd^2	-0.94	0.81	-0.93	1.00



7.2 Evaluation by the selected transport formulas

In this study and for evaluation purposes, the suspended load rate from the new model (from the measured data) was compared with the selected equations. Figs (3), (4), and (5) show these comparisons.

Many dimensionless parameters for comparison purposes were used: the discrepancy ratio and the relative error, Table (3). The results in Table (3) and Fig (6) indicate that the new

model and Van Rijn (2007) equation are the most accurate followed by Van Rijn (1984) and Karim- Kennedy (1990) equations. The percent of data that has discrepancy ratios between (0.5-2.0) is 100% for the new model and Van Rijn (2007) and with minimum relative mean error. The relative mean error for Karim-Kennedy (1990) equation is larger than the other equations because of using a laboratory

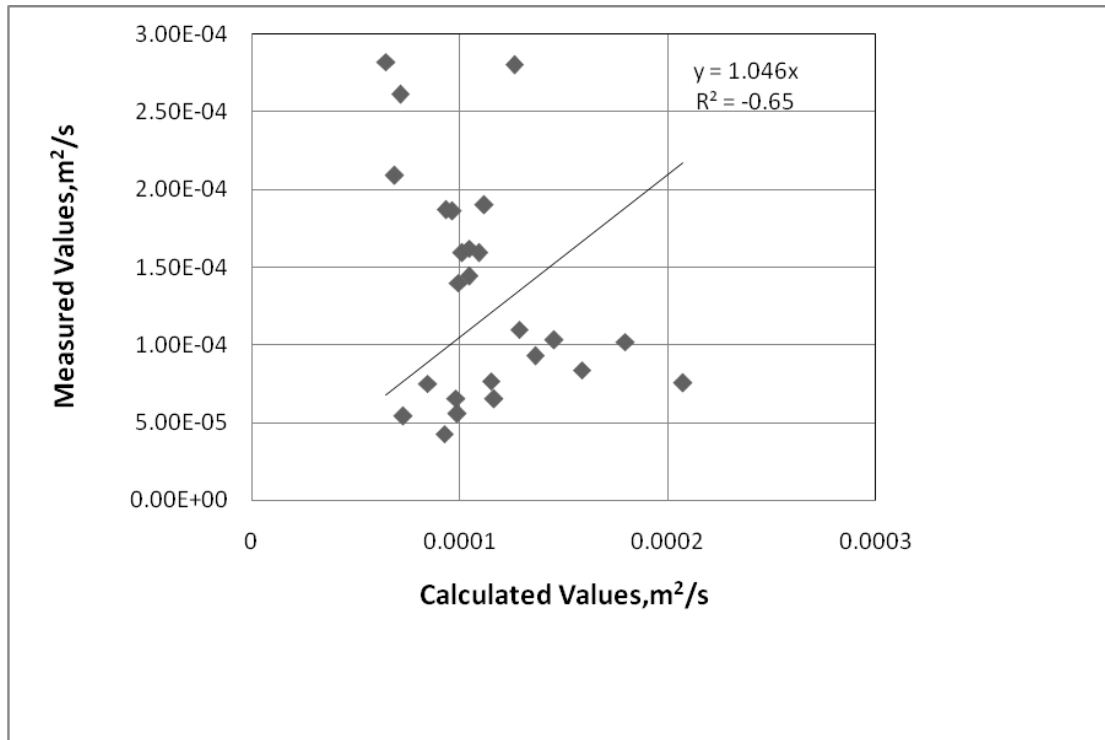


Fig. 3. Comparison Between Measured Suspended Load Rate and Computed Results of Leo C. Van Rijn(1984) Equation.

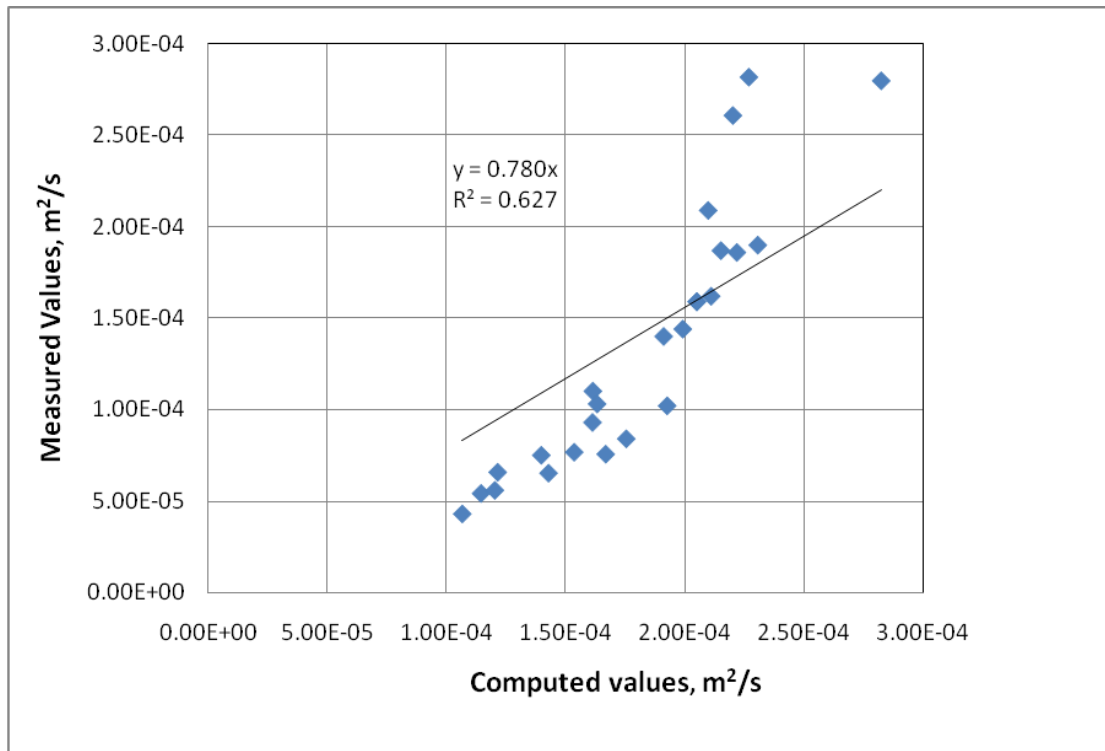


Fig. 4. Comparison Between Measured Suspended Load Rate and Computed Results of Karim-Kennedy(1990) Equation

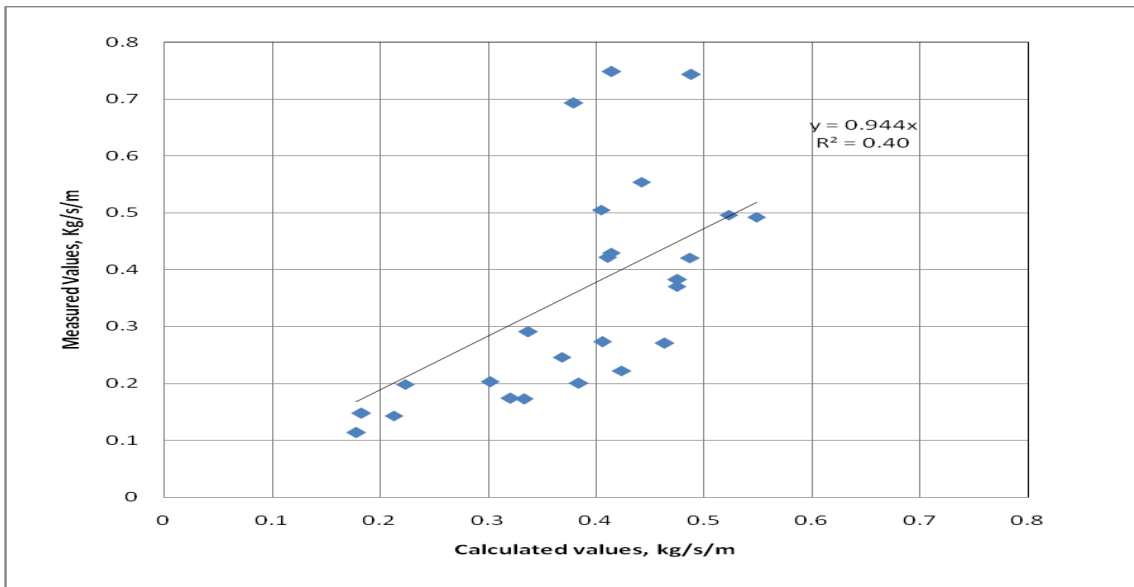


Fig. 5. Comparison between Measured Suspended Load Rates and Computed Results of Van Rijn(2007) Equation.

Table III
Summary of Comparison between Computed and Measured Suspended Load Rates (q_s).

Method	Equation and R^2	Discrepancy ratio (0.5-2.0)	R.M.E	Percent of data (0.5-2.0)
1. new model	$y=0.988X$ $R^2=0.94$	1.021	13.37	100%
2. Van Rijn (1984)	$y=1.046X$ $R^2=-0.65$	1.120	57.50	92%
3. Karim-Kennedy (1990)	$y=0.78X$ $R^2=0.63$	1.58	60.80	72%
4. Van Rijn (2007)	$y=0.944X$ $R^2=0.40$	1.261	39.73	100%

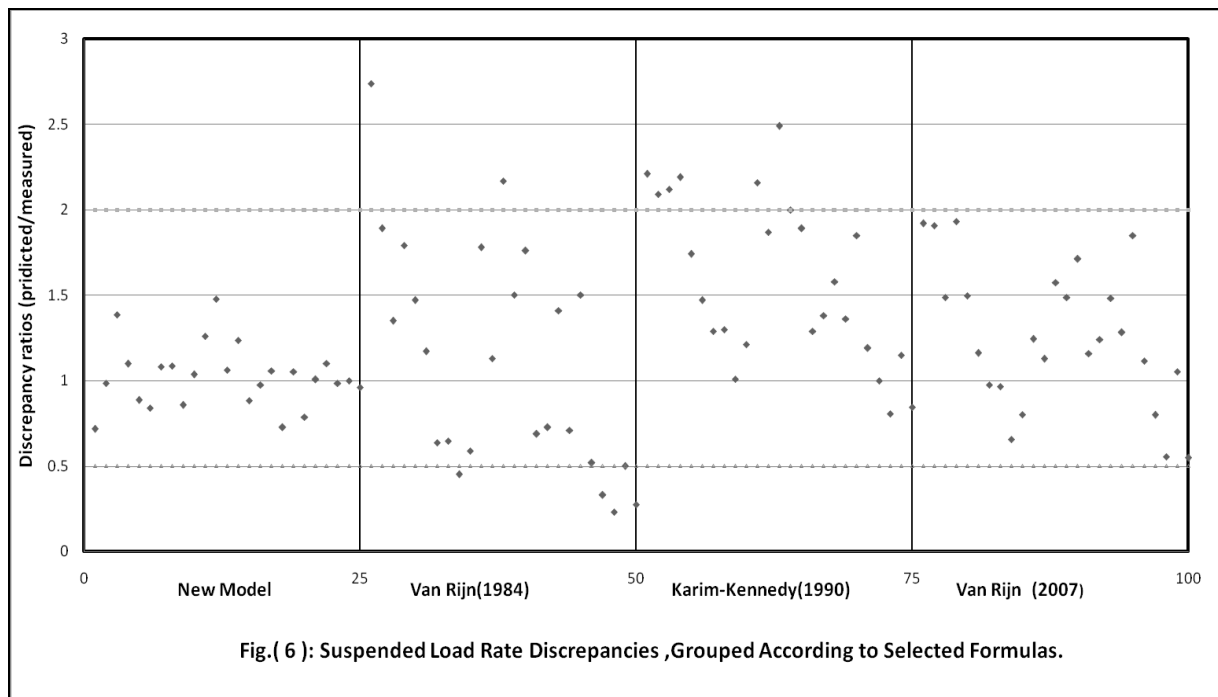


Fig.(6): Suspended Load Rate Discrepancies ,Grouped According to Selected Formulas.

flume data. Where sediment load was measured by special equipment (no reliable instrument). All the laboratory data has to be collected under steady equilibrium conditions. Natural river sediment and hydraulic data had to be collected with a day, and flow condition had to be fairly

steady to ensure a close relationship between sediment and flow conditions for a given set of river data.

8. CONCLUSIONS

In general, the high discrepancy ratio, coefficient of determination and small relative error indicate that the new

model can be predicted the transport parameter $\left(\frac{q_s}{Vd^2}\right)$ of

the reach of the Euphrates river, as a function of mobility, particle and shield parameters. The partial correlations indicate that the mobility and shield parameters are the most important variables while the particle parameter is less important than the others. The suspended sediment concentration measurements were very precise because the concentrations were determined separately for each subsection of the cross section considered. However, the new model gives detailed information about all parameters of importance to the suspended load rate process (sediment particle fall velocity and resistance of flow) and it can be used to obtain a quick estimate of suspended load rate. The new model is best suited for rivers with a d_{50} values within the range of (0.09-0.15mm).

The new model may give fairly accurate results for engineering purposes if the equation is applied to conditions similar to those from where the equation was derived.

The Van Rijn equation (2007), produced reasonable results for data sets which were used in this study, followed by the Van Rijn equation (1984) and Karim-Kennedy equation (1990) respectively. The computed suspended load rates were found to be within a factor of (0.5-2) of measured values.

Finally, remark is made with respect to the new model; it needs further verification by a flow and sediment data at many rivers. The differences in results for the selected formulas can be related to the rate of supply of fine material (wash load) which is not considered in these formulas. However, the percent of data within a factor of (0.5-2.0) of the measured values for the selected formulas and the new model vary between (72-100%).

NOTATION

The following symbols are used in this paper:

A	Cross—section area (m ²);
a	Reference level (m);
C _a	Reference concentration (%) (dimensionless);
d.r.	Discrepancy ratio (dimensionless);
D, d	Depth of flow (m);
D _s , d _s	Diameter of sediment particle (m);
D ₅₀ , d ₅₀	Median diameter of sediment material(m);
D ₉₀	Sediment size at which 90% by weight is finer (m);
D*	Particle parameter (dimensionless);
g	Acceleration due to gravity (m/s ²);
K	Von Karman factor ;
q _s	Suspended load rate (m ² /s or kg/s);

R.E.	Relative mean error (dimensionless);
R ²	Coefficient of determination (dimensionless);
S	Specific gravity (dimensionless);
S*	Sediment parameter (dimensionless);
T	Transport stage parameter (dimensionless);
T*	Shield's parameter (dimensionless);
\bar{U}	Average velocity of flow (m/s);
U*	Shear velocity (m/s);
U _{*c}	Critical shear velocity (m/s);
U _{cr}	Critical depth-averaged velocity (m/s);
V	Average velocity of flow (m/s);
V*	Shear velocity (m/s);
W _s , W ₀	Fall velocity of suspended sediment (m/s);
ν	Kinematic viscosity at water (m ² /s);
ρ	Water density (kg/m ³)

REFERENCES

- [1] ASCE Task Committee of Computational Modeling of Sediment Transport Processes (2004), "Computational Modeling of Sediment Transport Processes", J. Hydraulic engineering, 130(7): 597-598.
- [2] Baosheng, W.U., D.S. Van Maren and L. Linggan (2008), "Predictability of Sediment Transport in the Yellow River Using Selected Transport Formula", International J. Sediment Res, 23: 283-298.
- [3] Chih, T.Y. (1996), "Sediment Transport Theory and Practice", Chapter 3.
- [4] Chinese Standards (1992), "Standard for Measurement of Suspended Sediment in Rivers", G.B. 50159-92, Ministry of Construction.
- [5] Choudhury, P. and B. Sundar Sil (2010), "Integrated Water and Sediment Flow Simulation and Forecasting Models for River Reaches", Journal of Hydrology, 385: 313-322.
- [6] Edward J. Hickin (2004), "River Hydraulics and Channel Form", P.6.2.
- [7] Julein, P.Y. (1995), "Erosion and Sedimentation", Cambridge University Press.
- [8] Martin, Y., (2003), "Evaluation of Bed Load Transport Formulae Using Field Evidence from the Vedder River", British Columbia. Geomorphol, 53: 75-95.
- [9] Ministry of Water Resources (2012), "Flow Rates at Tigris and Euphrates Rivers", General Directorate of Water Resources Management, Baghdad-Iraq.
- [10] Pinto, L., A.B. Fortunato and P. Freire (2006), "Sensitivity Analysis of Non-Cohesive Sediment Transport Formula", Continental Shelf Res-, 26: 1826-1839.
- [11] Van denBerg, J.H., A. Van Gelder (1998), " Prediction of Suspended Bed Material Transport in Flows Over Silt and Very Fine Sand", Water Resources Res., 29 (5): 1393-1404.
- [12] Van Rijn, L.C. (1984), "Sediment Transport, Part II: Suspended Load Transport", Delft University of Technology.
- [13] Van Rijn, L.C. (2007), "Principles of Sediment Transport in Rivers", the Netherlands, (www.aquapublications.nl).
- [14] Van Rijn, L.C. (2007), "Unified View of Sediment Transport by Currents and Waves". Suspended Transport", ASCE, 0733-9429, 133: 6(668).
- [15] Winterwerp, J.C. (2001) "Stratification Effects by Cohesive and Non-Cohesive Sediment", J. Geophysical Res., 106 (c10): 22.559-22.574.
- [16] Yu.K., and Woo, H. (1994), "A Comparative Assessment of Some Selected Sediment Transport Formulas", "Proc., 9th Congress, App-IAHR, Singapore, 282-289.