Degree Reduction of Interval WB Curves

O. Ismail, Senior Member, IEEE

***Abstract*— Wang-Ball (WB) curve is one of the generalized cubic Ball basis which was first put forward in CONSURF system by Ball. Wang extended it to arbitrary odd and even degrees. An algorithmic approach to degree reduction of interval Wang-Ball curve is presented in this paper. The four fixed Kharitonov's polynomials (four fixed WB curves) associated with the original interval WB curve are obtained. These four fixed WB curves are transformed into four fixed Bezier curves. The degree of the four fixed Bezier curves is reduced based on the matrix representations of the degree reduction process. The process of degree reductions k times are applied to the four fixed Bezier curves of degree n to obtain the four fixed Bezier curves of degree $n-k$ without changing their shapes. The four fixed reduced Bezier curves are converted into WB curves of the same degree. Finally the reduced interval WB control points are obtained from the four fixed reduced WB control points. An illustrative example is included in order to demonstrate the effectiveness of the proposed method.**

***Index Term*— Image processing, CAGD, degree reduction, interval Wang-Ball (WB) curve, interval Bezier curve.**

I. INTRODUCTION

In recent years there has been considerable interest in approximating the curves and surfaces that arise in computer-aided design applications by other curves and surfaces that are of lower degree, of simpler functional form, or require less data for their specification. The motivation for this activity arises from the practical need to communicate product data between diverse CAD/CAM systems that impose fundamentally incompatible constraints on their canonical representation schemes, e.g., restricting themselves to polynomial (rather than rational) forms, or limiting the polynomial degrees that they accommodate.

In image processing and visualization, comparing two bitmapped images needs to be compared from their pixels by matching pixel-by-pixel. Consequently, it takes a lot of computational time while the comparison of two vector-based images is significantly faster. Sometimes these raster graphics images can be approximately converted into the vector-based images by various techniques. After conversion, the problem of comparing two raster graphics images can be reduced to the problem of comparing vector graphics images. Hence, the problem of comparing pixel-by-pixel can be reduced to the problem of polynomial comparisons. In computer aided geometric design (CAGD), the vector graphics images are the composition of curves and surfaces. Curves are defined by a sequence of control points and their polynomials. In internet applications, there are many popular search engines such as Google, Yahoo and MSN.

The author is with Department of Computer Engineering, Faculty of Electrical and Electronic Engineering, University of Aleppo, Aleppo, (e-mail:oismail@ieee.org).

These search engines have provided efficient mechanisms in searching for the relevant media or documents from a given set of keywords. Nevertheless, comparisons can only be accomplished. Although some pictures can be matched to the given keywords, the comparisons are attained by matching from the information provided in those pictures. However, there are several multimedia depicted in the forms of pictures, figures or images. Thus, it is interesting to introduce the algorithms for seeking for these kinds of information.

Typically, images can be classified into two categories: raster graphics and vector graphics images. A raster graphics image is represented by a rectangular grid of pixels whereas vector graphics image is defined by a set of mathematical equations representing the geometric objects, e.g., points, lines, polygons, curves, and surfaces.

The information contained in the raster graphics image is a collection of pixel attributes: the coordinates and colors. Comparing two images, one needs to compare pixel-by-pixel, coordinate-by-coordinate or even color-by-color. Consequently, comparing two large images, e.g., photographic image in the Internet or in the image banks, it is inevitably needed to compare a plenty of images. Moreover, seeking for a simple geometric object composed in a bitmap image is concerned as a complicated task. In vector-based images, each element is represented in terms of the mathematic formula and its attributes. There are many properties of those geometric primitives that can make the image comparison easier. Using the relevant properties instead of computing the whole image can reduce the computational time. Fortunately in some particular applications, raster graphics images can be converted into the vector-based images. It is reasonable to transform raster graphics images into vector graphics images and compares those vector graphics images. In computer aided geometric design (CAGD), a vector graphics image is an aggregation of curves and surfaces. Curves and surfaces can be modeled in various techniques. One of those methods that has been commonly used is the polynomial curve and surface representation. There are several kinds of polynomial curves in CAGD, e.g., Bezier, Said-Ball, and Wang-Ball curves. These curves have some common and different properties. All of them are defined in terms of the sum of product of their blending functions and control points. They are just different in their own basis polynomials. In order to compare these curves, we need to consider these properties. The common properties of these curves are control points, weights, and their number of degrees. Control points are the points that affect to the shape of the curve. Weights can be treated as the indicators to control how much each control point influences to the curve. Number of degree determines the maximum degree of polynomials. In different curves, these properties are not computed by the same method. To compare different kinds of curves we need to convert these curves into an intermediate form.

The curve modeling plays an important role in geometric modeling because it can be generalized into the development of surfaces and solids. Typically, a curve construction is based on a sequence of the given control points that approximates the shape of this curve. In other words, the specified control points influence the appearance of the curve. Besides, this curve will pass through the first and the last endpoints but does not pass through every interior point. In addition, these polynomial curves can also be differently specified according to their blending functions (polynomials), e.g., Ball [1], [2], [3], Said-Ball [4], Bezier, and B-Spline curves [5]. The models of these curves are also dissimilar from their different polynomial formulations. Hence, these transformations will be taken into account for the curve comparisons.

A lot of research [6-18] effort has gone into curves and surfaces in the last 30 years because of these reasons. Many sophisticated curve methods are known today-some are specialized and others are general purpose.

In this work, the degree reducing matrix for the four fixed Bezier curves will be used to obtain the degree reduction of the four fixed WB curves associated with the original interval WB curve. First, the relationships between Bezier and WB fixed control points were used for converting WB fixed control points into Bezier fixed control points of degree n. The degree of the four fixed Bezier curves will then be reduced by one using the degree reduction algorithm. Finally, the $(n-k)^{th}$-degree four fixed Bezier curves can be readily transformed into the four fixed WB curves of degree $n-k$, and reduced interval WB control points can be obtained from the four fixed reduced WB control points.

This paper is organized in the following sections. Section II describes interval Bezier curves, and section III includes interval Wang-Ball curves whereas section IV provides interval Wang-Ball degree reduction, and section V presents a numerical example, while the final section offers conclusions.

II. THE BASIC RESULTS

In 1974, Ball [1][2][3] defined a set of basis functions for cubic curves. In 1989, Said [4] generalized the Ball model to higher degrees and developed the recursive algorithms for two generalized Ball curves. Let $\{[q_i^-, q_i^+]\}_{i=0}^n$ be a given set of interval control points which defines the interval Wang-Ball curve:

$$Q_n^I(u) = \sum_{i=0}^{n} [q_i^-, q_i^+]\, A_i^n(u), \qquad 0 \le u \le 1 \tag{1}$$

of degree n where $A_i^n(u)$ are the Wang-Ball basis functions defined as following:

$$A_i^n(u) = \begin{cases} (1-u)^{2+i}(2u)^i, & for\ \ 0 \le i \le \frac{n-3}{2} \\ (1-u)^{\frac{n+1}{2}}(2u)^{\frac{n-1}{2}}, & for\ \ i = \frac{n-1}{2} \\ \left(2(1-u)\right)^{\frac{n-1}{2}} u^{\frac{n+1}{2}}, & for\ \ i = \frac{n+1}{2} \\ \left(2u(1-u)\right)^{n-i} u^{n-i+2}, & for\ \ \frac{n+3}{2} \le i \le n \end{cases} \tag{2}$$

where n is odd.

$$A_i^n(u) = \begin{cases} (1-u)^{2+i}(2u)^i, & for\ \ 0 \le i \le \frac{n}{2}-1 \\ \left(2u(1-u)\right)^{\frac{n}{2}}, & for\ \ i = \frac{n}{2} \\ \left(2(1-u)\right)^{n-i} u^{n-i+2}, & for\ \ \frac{n}{2}+1 \le i \le n \end{cases} \tag{3}$$

where n is even.

The four fixed Kharitonov's polynomials (four fixed Wang-Ball curves) [19] associated with the original interval Wang -Ball curve are:

$$\begin{aligned} Q_n^1 &= q_0^- + q_1^- u + q_2^+ u^2 + q_3^+ u^3 + q_4^- u^4 + q_5^- u^5 + \cdots \\ &\equiv \beta_0^1 + \beta_1^1 u + \beta_2^1 u^2 + \cdots + \beta_n^1 u^n \\ Q_n^2 &= q_0^- + q_1^+ u + q_2^+ u^2 + q_3^- u^3 + q_4^- u^4 + q_5^+ u^5 + \cdots \\ &\equiv \beta_0^2 + \beta_1^2 u + \beta_2^2 u^2 + \cdots + \beta_n^2 u^n \\ Q_n^3 &= q_0^+ + q_1^+ u + q_2^- u^2 + q_3^- u^3 + q_4^+ u^4 + q_5^+ u^5 + \cdots \\ &\equiv \beta_0^3 + \beta_1^3 u + \beta_2^3 u^2 + \cdots + \beta_n^3 u^n \\ Q_n^4 &= q_0^+ + q_1^- u + q_2^- u^2 + q_3^+ u^3 + q_4^+ u^4 + q_5^- u^5 + \cdots \\ &\equiv \beta_0^4 + \beta_1^4 u + \beta_2^4 u^2 + \cdots + \beta_n^4 u^n \end{aligned} \tag{4}$$

The relationships between fixed Bezier and fixed WB curves can be defined and proved in similar way by polar form approach in [21]. The Bezier fixed control points, $\{\alpha_i^j\}_{i=0}^n$ for $(j = 1,2,3,4)$associated with the four fixed Wang-Ball curves provided in terms of the transformation matrix as shown in the following proposition.

Proposition 1. The Bezier control points associated with the four fixed Wang-Ball curves of degree n can be given in terms of the multiplication of the WB control points and an $(n+1)\times(n+1)$ conversion matrix M_n^n by:

$$\begin{bmatrix}\alpha_0^j & \alpha_1^j & \cdots & \alpha_n^j\end{bmatrix} = \begin{bmatrix}\beta_0^j & \beta_1^j & \cdots & \beta_n^j\end{bmatrix} \cdot M_n^n \tag{5}$$

$$(j = 1,2,3,4)$$

where M_n^n is the conversion matrix from Wang-Ball control points associated with the four fixed Wang-Ball curves into Bezier control points associated with the four fixed Bezier curves that can be defined by:

$$M_n^n = \begin{bmatrix} m_{0,0} & m_{0,1} & \cdots & m_{0,n} \\ m_{1,0} & m_{1,1} & \cdots & m_{1,n} \\ \vdots & \vdots & \vdots & \vdots \\ m_{n,0} & m_{n,1} & \cdots & m_{n,n} \end{bmatrix} \quad (6)$$

which can be created from a square matrix as shown in the equation (7).

$$m_{k,l} = \begin{cases} 2^i \dfrac{\binom{n-2-2k}{l-k}}{\binom{n}{l}}, & for\ i < \left\lfloor \frac{n}{2} \right\rfloor \\ 2^{n-i} \dfrac{\binom{2k-2-n}{k-l}}{\binom{n}{l}}, & for\ i > \left\lfloor \frac{n}{2} \right\rfloor \\ \dfrac{2^l}{\binom{n}{l}}, & for\ i = l = \left\lfloor \frac{n}{2} \right\rfloor \\ \dfrac{2^{n-l}}{\binom{n}{l}}, & for\ i = l = \left\lceil \frac{n}{2} \right\rceil \\ 0, & otherwise \end{cases} \quad (7)$$

The conversion from Bezier into WB control points of the same curve can be rewritten from the equation of the conversion from WB into Bezier control points. Thus, the relationship can be shown in the following proposition.

Proposition 2. The WB control points associated with the four fixed Bezier curves of degree n can be defined in terms of Bezier control points and an $(n+1) \times (n+1)$ conversion matrix M_n^n by:

$$\begin{bmatrix} \beta_0^j & \beta_1^j & \cdots & \beta_n^j \end{bmatrix} = \begin{bmatrix} \alpha_0^j & \alpha_1^j & \cdots & \alpha_n^j \end{bmatrix} \cdot [M_n^n]^{-1} \quad (8)$$

$$(j = 1,2,3,4)$$

III. INTERVAL WB DEGREE REDUCTION

In order to find the resulting matrix of the WB degree reduction, some of the intermediate steps of the transformations are needed to be computed. Interval WB degree reduction can be created by the following steps:

Algorithm for the interval WB Degree Reduction

1. Transform the WB control points associated with the four fixed Wang-Ball curves of degree n into the Bezier control points of the same curve equation (5).
2. Reduce degree of the four fixed Bezier curves as explained in [21].
3. Convert the four fixed Bezier control points of degree $n-k$ into the fixed WB control points of the same degree equation (8).
4. The reduced interval WB control points can be obtained from the four fixed reduced WB control points as follows:

$$[\beta_i^-, \beta_i^+] = \left[min\left(\beta_i^j\right), max\left(\beta_i^j\right) \right]$$

$$(i = 0, 1, \cdots\cdots, n-k) \text{ and } (j = 1, 2, 3, 4)$$

V. NUMERICAL EXAMPLE

Consider the interval Wang-Ball curve defined by five interval control points:

$$[p_0^-, p_0^+] = ([1.60,1.85], [1.40,1.75])$$

$$[p_1^-, p_1^+] = ([2.40,2.75], [3.25,3.65])$$

$$[p_2^-, p_2^+] = ([3.95,4.35], [3.40,3.85])$$

$$[p_3^-, p_3^+] = ([6.25,6.75], [1.80,2.25])$$

$$[p_4^-, p_4^+] = ([7.00,7.15], [2.00,2.50])$$

It is required to reduce the degree of the given interval Wang-Ball curve defined by them to 3 without changing its shape.

The reduced interval vertices $\{[\beta_i^-, \beta_i^+]\}_{i=0}^3$ of the given interval Wang-Ball curve are obtained after applying the algorithm explained in section III as follows:

$$[\beta_0^-, \beta_0^+] = ([1.6000,1.8500], [1.4000,1.7500])$$

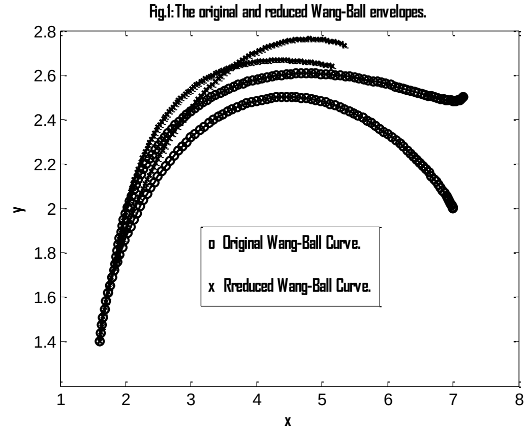
$$[\beta_1^-, \beta_1^+] = ([2.0800,2.3900], [2.5100,2.8900])$$

$$[\beta_2^-, \beta_2^+] = ([2.3650,2.5800], [2.8100,3.0150])$$

$$[\beta_3^-, \beta_3^+] = ([5.1600,5.3500], [2.6400,2.7350])$$

Simulation results in Figure (1) shows the envelopes of the original interval WB curve and the reduced interval WB curve, respectively.

Fig.1:The original and reduced Wang-Ball envelopes.



IV. CONCLUSIONS

In this paper, the interval WB curve degree reduction algorithm has been proposed using conversion matrices between Bezier and WB curves and Bezier degree reduction matrix. The four fixed Kharitonov's polynomials (four fixed WB curves) associated with the original interval WB curve are obtained. These four fixed WB curves are transformed into four fixed Bezier curves. The degree of the four fixed Bezier curves is reduced based on the matrix representations of the degree reduction process. The process of degree reductions k times are applied to the four fixed Bezier curves of degree n to obtain the four fixed Bezier curves of degree $n-k$ without changing their shapes. The four fixed reduced Bezier curves are converted into WB curves of the same degree. Finally the reduced interval WB control points are obtained from the four fixed reduced WB control points. Comparing two bitmapped images in image processing and visualization needs to be compared from their pixels by matching pixel-by-pixel. Consequently, it takes a lot of computational time while the comparison of two vector-based images is significantly faster. Sometimes these raster graphics images can be approximately converted into the vector-based images by various techniques. After conversion, the problem of comparing two raster graphics images can be reduced to the problem of comparing vector graphics images. Hence, the problem of comparing pixel-by-pixel can be reduced to the problem of polynomial comparisons. In computer aided geometric design (CAGD), the vector graphics images are the composition of curves and surfaces.

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O. Ismail (M'97–SM'04) received the B. E. degree in electrical and electronic engineering from the University of Aleppo, Syria in 1986. From 1987 to 1991, he was with the Faculty of Electrical and Electronic Engineering of that university. He has an M. Tech. (Master of Technology) and a Ph.D. both in modeling and simulation from the Indian Institute of Technology, Bombay, in 1993 and 1997, respectively. Dr. Ismail is a Senior Member of IEEE. Life Time Membership of International Journals of Engineering & Sciences (IJENS*)* and Researchers Promotion Group (RPG).His main fields of research include computer graphics, computer aided analysis and design (CAAD), computer simulation and modeling, digital image processing, pattern recognition, robust control, modeling and identification of systems with structured and unstructured uncertainties. He has published more than 60 refereed journals and conferences papers on these subjects. In 1997 he joined the Department of Computer Engineering at the Faculty of Electrical and Electronic Engineering in University of Aleppo, Syria.

In 2004 he joined Department of Computer Science, Faculty of Computer Science and Engineering, Taibah University, K.S.A. as an associate professor for six years.

He has been chosen for inclusion in the special 25th Silver Anniversary Editions of Who's Who in the World. Published in 2007 and 2010.

Presently, he is with Department of Computer Engineering at the Faculty of Electrical and Electronic Engineering in University of Aleppo.