Investigating the effect of Volume Fraction on the Stress Intensity Factor for Composite Plate with Central Crack

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***Abstract*–** **In this research, a stress intensity factor (SIF) mode-I of unidirectional composite plate with central crack is calculated with volume fraction of unidirectional fiber effect. The stress intensity factor (SIF) of composite plate calculated by using finite element method (FEM) and compare the results with mesh free method (MFree) for different volume faction of unidirectional fiber of composite plate. The stress intensity factor of composite plate evaluated with different volume fraction of unidirectional fiber as (14.4%, 20.2%, 26%, 37.5%, 49%, and 60.4%), for composite plate with dimensions of plate (104 mm*40mm) and different central crack length as (3, 8, 16, 24, 32, 40, 48 and 56 mm). The stress intensity factor of composite plate evaluated with applying constant displacement in y-direction at the end of composite plate, and then, evaluated the stress intensity factor due to stress resulted from the displacement applied on the end of plate. The results shown When the crack length and volume fraction of fiber increase, the changing of SIFs gives a good explanation for the fracture phenomena in composite materials which in turn answer the question of "How the crack is propagate in composite material". And, a comparison between the FEM and MFree method is made to study the advantages and disadvantages of each method and to choose the suitable method for calculating SIF for composite plate. Generally, there is a good agreement between the FEM and MFree method but there is a slightly difference between them. This slight difference happens due to the calculation technique of SIF and distribution of nodes used in each method.**

***Index Term*—** **Stress Intensity Factor (SIF), Central Crack, Composite Plate, Longitudinal Composite Material, Volume Fraction, Finite Element Method (FEM), ANSYS, MFree Method, Fracture Machine.**

I. INTRODUCTION

Composite materials are widely used in the aerospace industry along with various applications in many other industries. The use of composites in the aerospace industry and the other engineering applications has increased dramatically since the 1970s. The primary benefits that composite components can offer reduced weight and simplification in assembly. The other advantage of composites over the steel alloys is their resistive against corrosion. So, investigating on composite materials attracts many researchers in recent years **[1-6]**.

Preventing failure of composite material systems has been an important issue in engineering design. The two types of physical failures that occur in laminated composite structures and interact in complex manner are interalaminar and interlaminar failures. Interalaminar failure is manifest in micro-mechanical components of the lamina such as fiber breakage, matrix cracking, and debonding of the fiber- matrix interface. Generally, aircraft structures made of fiber reinforces composite materials are designed such that the fibers carry the bulk of the applied load. Interlaminar failure such as delamination refers to debonding of adjacent lamina.

The more general applications of fiber reinforcement invariably involve multidirectional reinforcement to address directional variability in the loading of fiber-reinforced materials. Unidirectional reinforcement usually results from a highly specific functional requirement in the use of fiber reinforcement, where the dominant loading direction can be identified a priori. In spite of this limitation, the study of unidirectional reinforced materials provides useful insight into micro-mechanical features that can influence the load transfer between the fibers and the matrix. In its fabricated condition, a fiber-reinforced material is expected to be defect free. The notion of a defect free fiber-reinforced material is largely a matter of definition, since even perfect fiber reinforcement can exhibit micro-mechanical defects during curing processes and under certain loading and environmental conditions resulting from localized loads, extreme temperatures and impact. The integrity of a fiber-reinforced material can therefore be influenced by the development of features such as fiber pullout, fiber breakage, fiber–matrix interface delamination, matrix fracture, matrix void growth, matrix damage, etc. The influence of damage and defects on the structural integrity of fiber-reinforced materials was discussed several decades ago by a number of researchers including **Aveston and Kelly [7]**, **Beaumont and Harris [8]**, **Backlund [9] and Bowling and Groves [10]**.

Computation of notch stress intensity factors (SIFs) for composite materials is important, since cracks are often initiated at this location. Labossiere and Dunn presented a procedure to calculate mode I and II notch SIFs in anisotropic media using the path-independent H-integral **[11]**. Qian used a contour integral in conjunction with the finite element method to evaluate the SIFs at the bi-material wedge **[12]**. Banks-Sills and Ishbir used a conservative integral based on the Betti reciprocal principle to obtain SIFs for a bimaterial notch **[13]**. Kumagai and Shindo described an experimental and analytical study on the cryogenic fracture behavior of CFRP-woven laminates under tension with a sharp notch **[14].** Ju et al. used image correlation experiments to find composite notch SIFs **[15]**.

A number of methods have been used for the determination of stress intensity factors **[16-21]**.One of these methods is a numerical method like Green's function, weight functions, boundary collocation, alternating method, integral transforms, continuous dislocations MFree and finite elements methods.

A Stress Intensity Factor (SIF) Mode I is calculated by **Alansari et. al. [22]**, when the fiber volume fraction is (37.5%), using the finite element method (FEM) (ANSYS) and MFree method for different types of longitudinal fiber reinforced composite plate. In this paper, the stress intensity factor Mode I , for centre cracked composite plate with different fiber volume fraction, were calculated using MFree Method and finite elements method(using ANSYS software) in order to study the effect of fiber volume fraction on the value of stress intensity factor.

II. Numerical Investigation

The numerical investigation of stress intensity factor of central crack composite plate with volume fraction of unidirectional fiber effect included two method, first, finite element method and second, Mfree method, as,

1. The Finite Element Method

Several researches, using the finite element method, were done in order to calculate the stress intensity factor (SIF) Mode I or studying the transfer stress phenomena between fiber and matrix in longitudinal composite plate. Some of these researches used ANSYS software like Aslantas **[23]**, Chandwani et al **[24]**, Chin **[25]** and Al-Ansari [21,26]. Generally, all these researches used the model proposed by Phan **[27]** in order calculate the stress intensity factor Mode-I. Since the plate is composite and the fiber volume fraction varies, therefore, the number of fibers and the fibers distribution will change in each fiber volume fraction. The number and dimensions of fibers and the dimensions of matrix will be summarized in Table I.

Table I
The Numbers and Dimensions of Reinforcement Fiber and Resin Matrix.

Volume Fraction of Fiber (V_f)	***Material***	***Number***	***Diameter or Width (mm)***	***Height or Length (mm)***
14.4%	***Fiber***	***5***	***3***	***40***
	Matrix	***6***	***14.833***	***40***
20.2%	***Fiber***	***7***	***3***	***40***
	Matrix	***8***	***10.375***	***40***
26%	***Fiber***	***9***	***3***	***40***
	Matrix	***10***	***7.7***	***40***
37.5%	***Fiber***	***13***	***3***	***40***
	Matrix	***14***	***4.643***	***40***
49%	***Fiber***	***17***	***3***	***40***
	Matrix	***18***	***2.944***	***40***
60.6%	***Fiber***	***21***	***3***	***40***
	Matrix	***22***	***1.864***	***40***

The element (PLANE82) was used in finite element model (ANSYS Software) in order to calculate the stress intensity factor (SIF) Mode I. The meshing criteria, that deals with the number of elements around the crack tip and limited by Phan [27], was used (i.e. Create the Concentration Key-point (Crack Tip)). In addition to meshing criteria, the elements, created by this criteria, must be lies in the same material. Also, the material, used to calculate the stress intensity factor (SIF), must be changed into material containing the crack tip.

2. The Meshless (MFree) Methods

Eight regular nodal configurations will be formed for a quarter model of the problem. For the sake of illustration, Fig.(1-a) exhibits the nodal distribution for one model where the width of the plate is (52 mm) and the height is (20 mm). The Meshless Local Petrov-Galerkin (MLPG) method formulation requirements for each of the predefined configurations involve the following,

1- For the Moving Least Squares (MLS) approximation and for each influence node, it is suggested to use the linear basis and the quartic spline weight function over circular local support domains (Ωw) [29].
2- The test function constructed over a local sub-domain (Ωt) of size equals to the nodal spacing using the same quartic spline weight function.
3- (8*8) of Gaussian integration points are used in each local sub-domain for the domain integral and (9) Gaussian integration points are used for the boundary integral.

The essential boundary conditions are enforced using (α=1*1016 Pa/m) as the penalty factor [28].

The Meshless Local Petrov-Galerkin "MLPG" method was used to build the FORTRAN program in order to calculate the stress intensity factor (SIF) Mode I for central cracked composite plate. The number of nodes in this model was up to 2500 node depending on the crack length. The Fortran program can be explain by the flowchart shown in Fig.(1-b)(for more details see Ref. **[22] and [29]**).

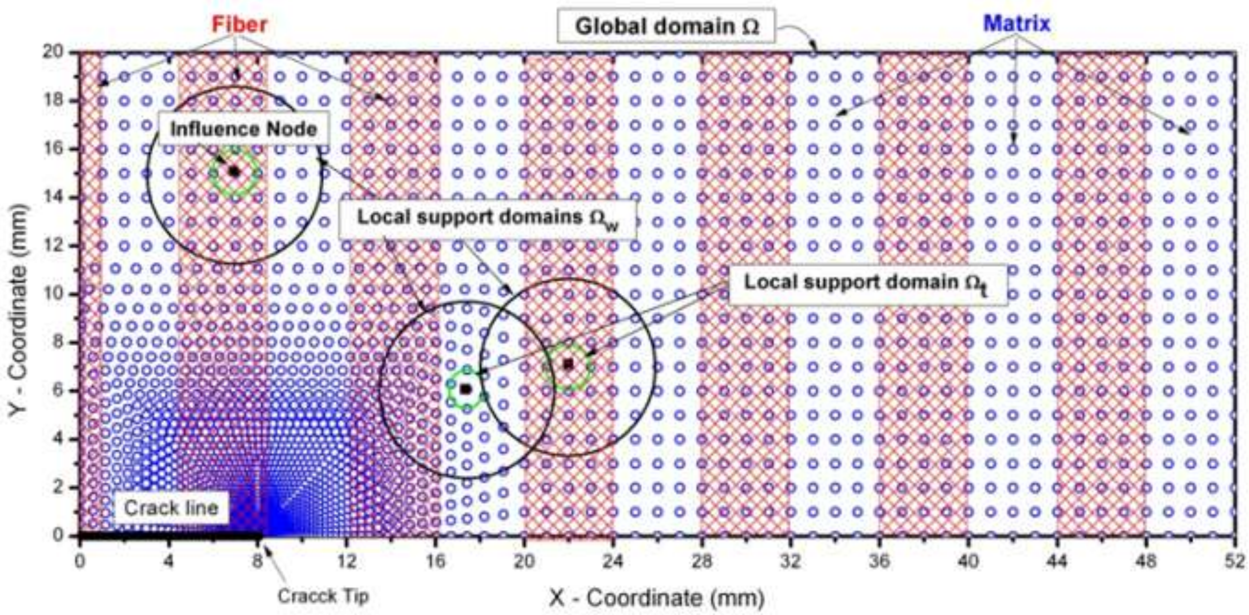

Fig. (1-a). MFree Modeling for Composite Material (The Nodal Distribution for the Global Domain)[22,29]

III. Results and Discussion

The properties of fiber, properties of matrix, volume fraction of fiber, dimensions of fiber, dimensions of matrix, crack dimensions and dimensions of plate are important parameters in the studying of composite materials fracture. The following parameters are used in this paper,

1- Properties of Fiber and Matrix

The properties of fiber and matrix , which are used in this paper, were:

- Fiber Material= Glass Fiber.
- Modulus of Elasticity of Fiber = 83.5 MPas.
- Poisson Ratio of Fiber = 0.3.
- Specific Gravity of Fiber = 2.5.
- Matrix Material = Epoxy.
- Modulus of Elasticity of Matrix = 3.3MPas.
- Poisson Ratio of Matrix = 0.4.
- Specific Gravity of Matrix = 1.25.

2- Dimensions of Plate

The Dimensions of composite plate are:

- The width of the Plate = 104 mm.
- The height of the Plate = 40 mm

3- Crack Length (2a)

In this work, eight crack length (2a) values were used (i.e. eight different configurations of ANSYS model and MFree model). The crack lengths (2a) were (3, 8, 16, 24, 32, 40, 48 and 56) mm.

Constant displacement of 3mm in y-direction is used as applied load on the composite plate in all the above cases.

In this work, three important parameters were studied. These parameters are the effect of volume fraction of fiber, the effect of crack length (crack propagation) and the comparison between the FEM and MFree method.

1- Effect of the Volume fraction of fiber

When the volume fraction of fiber changes, the number of fibers will be change and the distribution of fibers will be change too. This make the position of crack tip change for the same crack length, when the distribution of fibers changes and this effect directly on the value of stress intensity factor (SIF). Fig.(2) shows the variation of stress intensity factor (SIF) with the increasing of crack length when volume fraction of fiber is (14.4%). It is obviously to see rapture in the results curves, this rupture happened because of the change in the position of crack tip and type of material. The same behavior can be seen in Fig.(3-7) when volume fraction of fiber is (20.2, 26, 37.5,49 and 60.6)%. The maximum number of peaks appears when volume fraction of fiber is (37.5%) because the interference between the volume fraction of fiber and the chosen crack length. If the crack length increases with small increment the maximum number of peaks must be appear when volume fraction of fiber is (60.6%) due to increase the number of fiber. In order to explain this behavior, see Fig.(8-15),which describe the variation of stress intensity factor (SIF) with volume fraction changes when the crack length is (3,8,16,24,32,40,48 and 56) mm respectively. From Fig.(9-15), the peaks appear in any figure means the crack tip lies in fiber region and the other means the crack tip lies in matrix region.

2- Effect of the crack length (crack propagation)

Fig.(16-31) shows the variation of the displacement in Y-direction calculated using Finite Element Method and MFree Method for different crack lengths when the volume fraction of fiber 37.5% which gives good results for explanation. When the crack length increases, the number of points which has maximum displacement in Y-direction will be increase (i.e. the red region) and this means the maximum opening displacement will be increase too. In other word, the stress intensity factor (SIF) Mode I will be increase. This is true when the plate is not composite plate. If the plate is

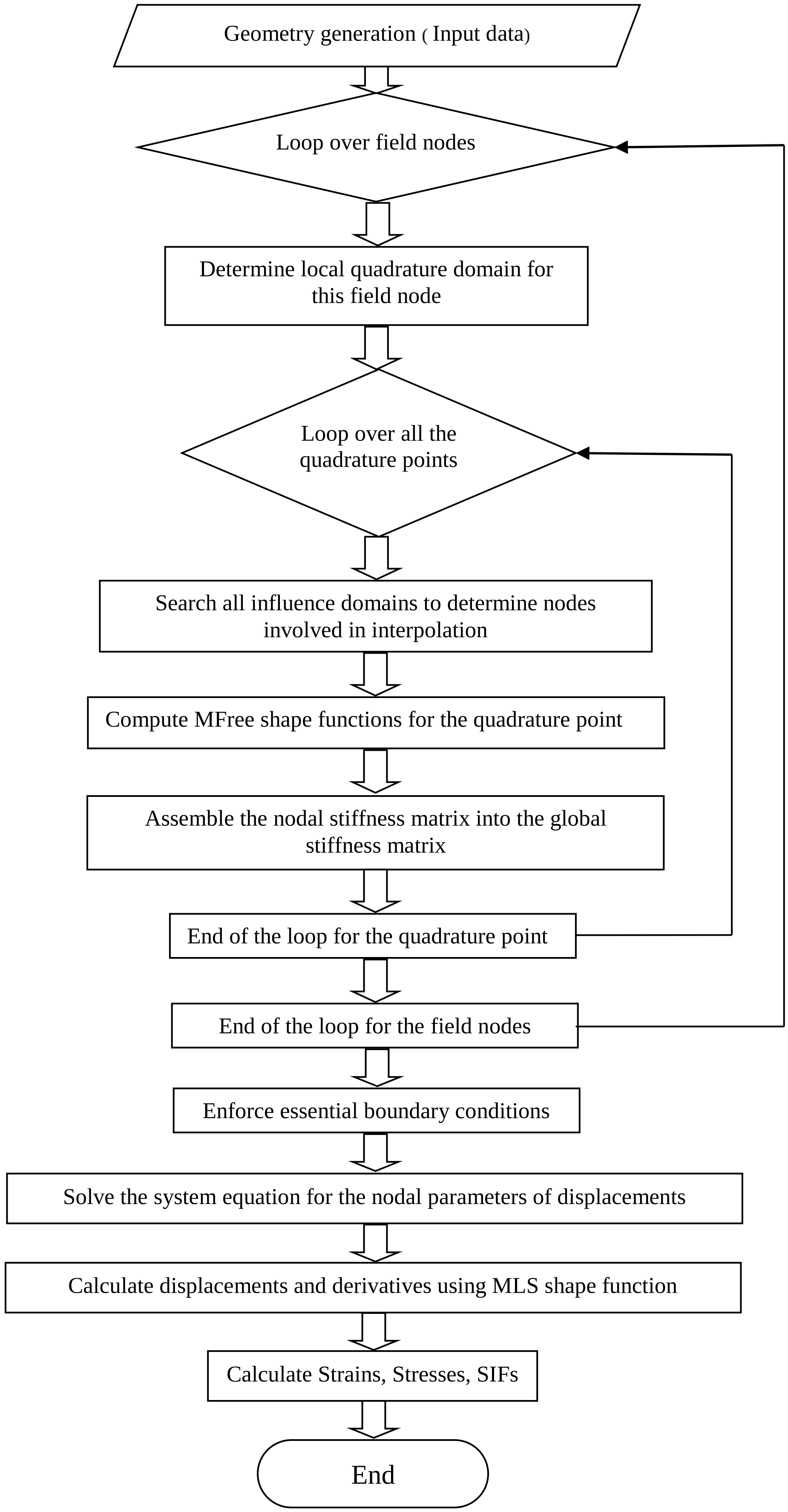

Fig.(1-b). Flowchart of MLPG Method Program composite plate, the stress intensity factor (SIF) depends on the position of crack tip in addition to the properties of fiber and matrix and crack length. When any fiber or/and matrix break, the redistribution in displacement and stress will happened. This make the crack go toward the point has the smallest stress intensity factor or create a new crack or debond between fiber and matrix.

3- The comparison between the FEM and MFree method,

From all Figures, the comparison between the Finite Element method and MFree method can be done and the following remarks can be noted,

a. In Fig.(9-15), there is a good agreement between Finite Element results and MFree results. But, in Fig.(8) when the crack length is (0.003 m), there is a difference between them. This difference due to the technique used in calculating the SIF. In MFree method, the technique used for calculating SIFs depends on the stress field straightly ahead of the crack tip [22], hence, it shows drawback when the crack tip lies on the line between fiber and matrix in contrast to the J-Integral technique used in ANSYS method. However, such drawback can be dominated by using larger support domains in the MFree formulation near the crack tip.

b. In Fig.(2-15) except Fig.(8), some points can be appear as a higher or smaller than the other point. For example in Fig.(3), the stress intensity factor calculating by MFree method ,when the crack length is (0.024 m), is higher than that calculating by ANSYS. While the value of stress intensity factor calculating by ANSYS, when the crack length is (0.056 m), is higher than that calculating by MFree method. This occurs because

- The technique used in calculating the SIF.
- The number of nodes in MFree method is about (2500) nodes while the number of nodes in ANSYS is about (40000) nodes.
- The Distribution of nodes in MFree method differs than that in Finite Element method. In MFree method, the nodes distribute in the region around the crack tip with ratio differ than other regions while, in ANSYS, the nodes distribute in the region around the crack tip and in the regions around the interface lines between fiber and matrix [22].

c. The difference in the formulation and solution techniques used in MFree and FEM affects the load transferring between composite strips in each method which caused relatively small unagreement in the obtained results. This appears clearly in the displacement in Y- direction shown in Fig.(16-31).

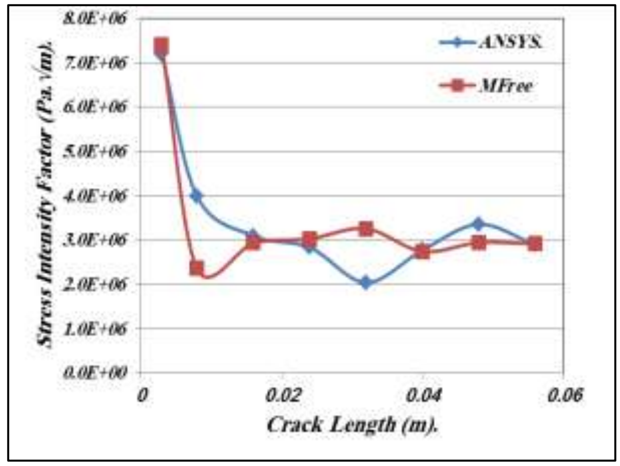

Fig. 2. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (14.4%).

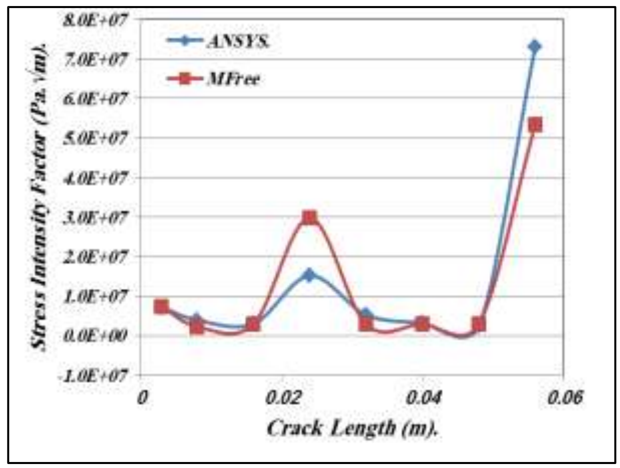

Fig. 3. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (20.2%).

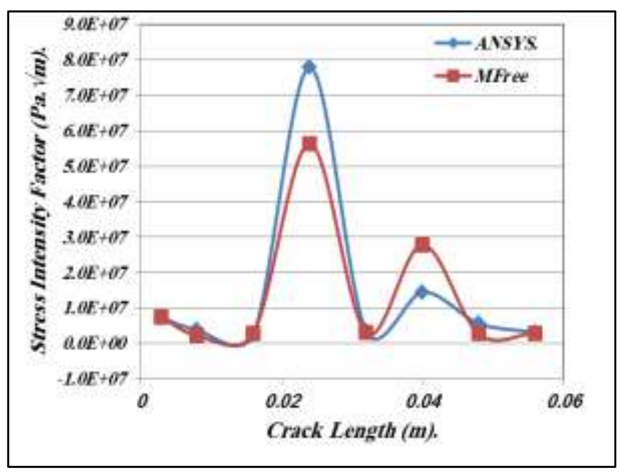

Fig. 4. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (26%).

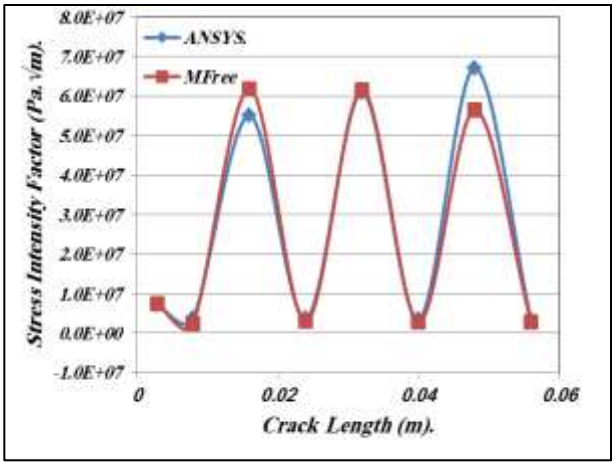

Fig. 5. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (37.5%).

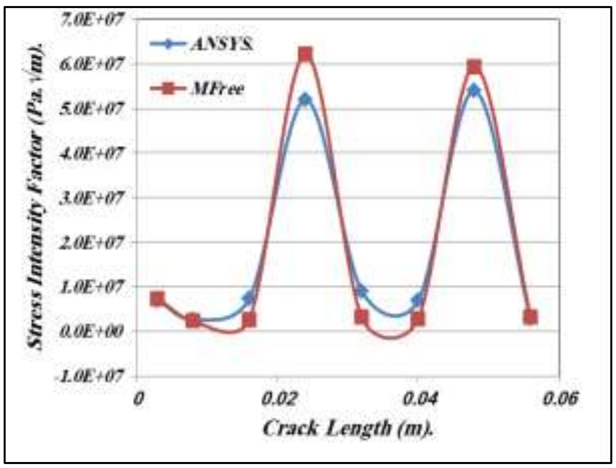

Fig. 6. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (49%).

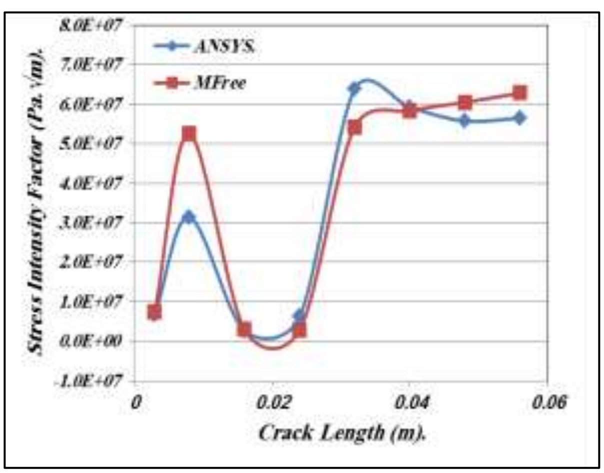

Fig. 7. Variation of Stress Intensity Factor with the Increasing of Crack Length When Volume Fraction of Fiber is (60.6%).

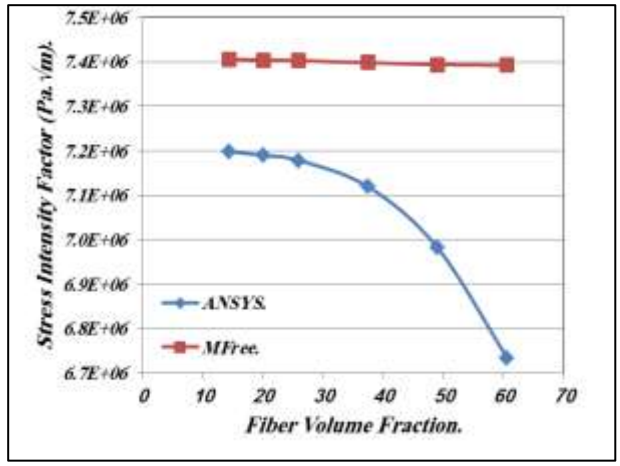

Fig. 8. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.003 m).

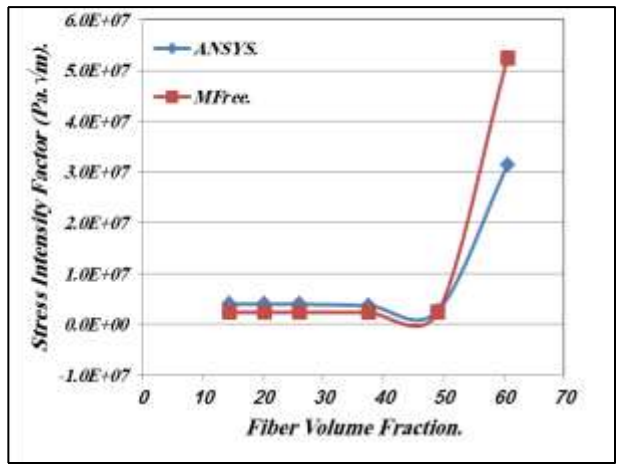

Fig. 9. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.008 m).

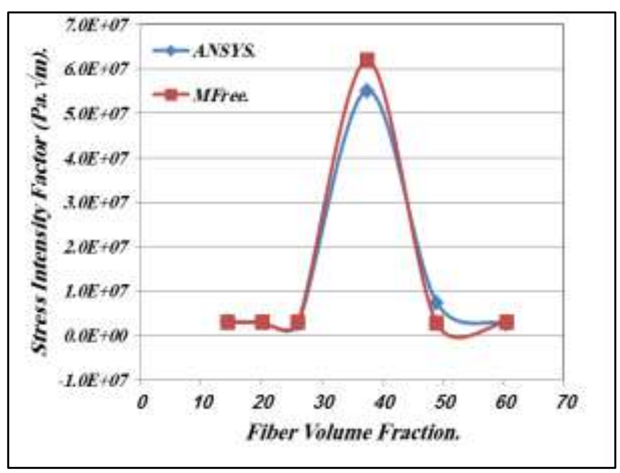

Fig. 10. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.016 m).

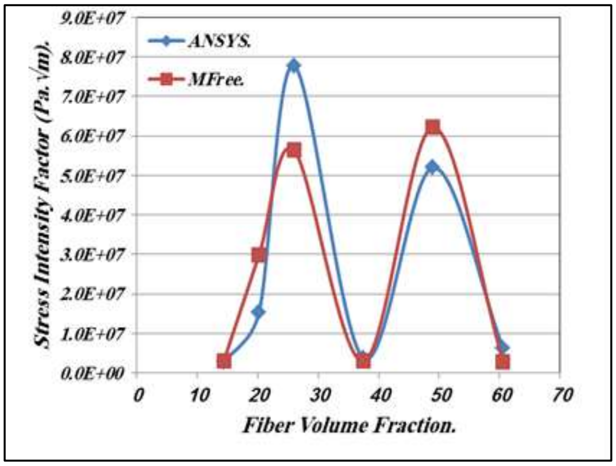

Fig. 11. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.024 m).

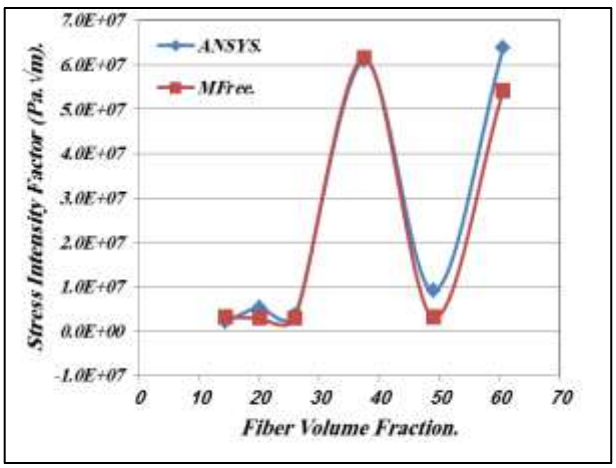

Fig. 12. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.032 m).

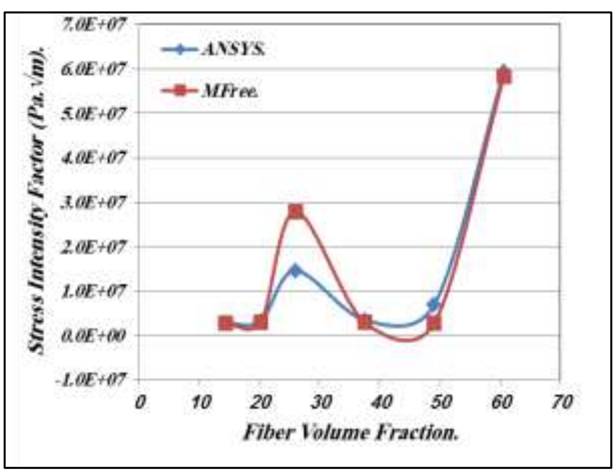

Fig. 13. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.040 m).

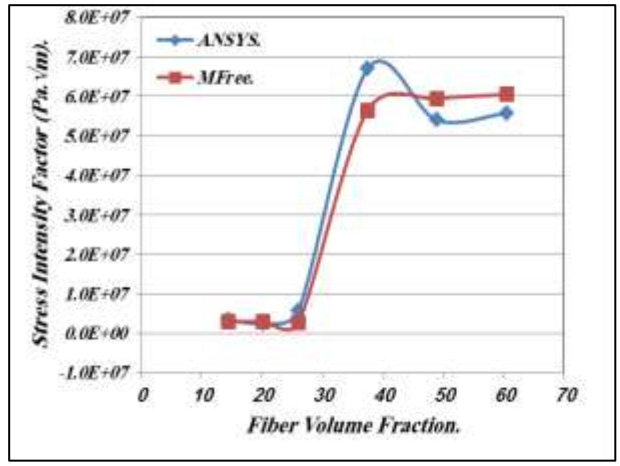

Fig. 14. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.048 m).

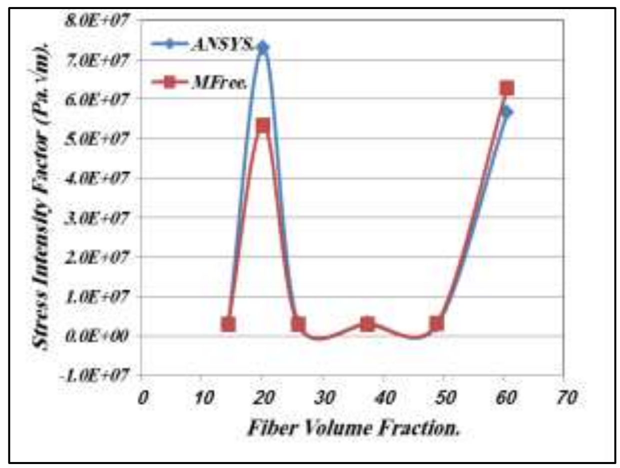

Fig. 15. Variation of Stress Intensity Factor with the Increasing of Fiber Volume Fraction When Crack Length is (0.056 m).

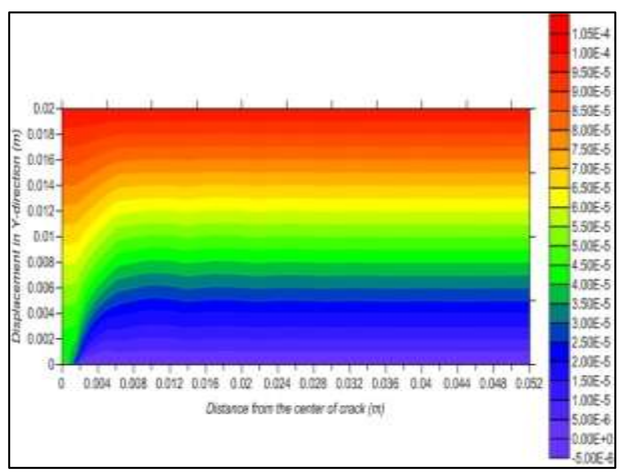

Fig. 16. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.003 m).

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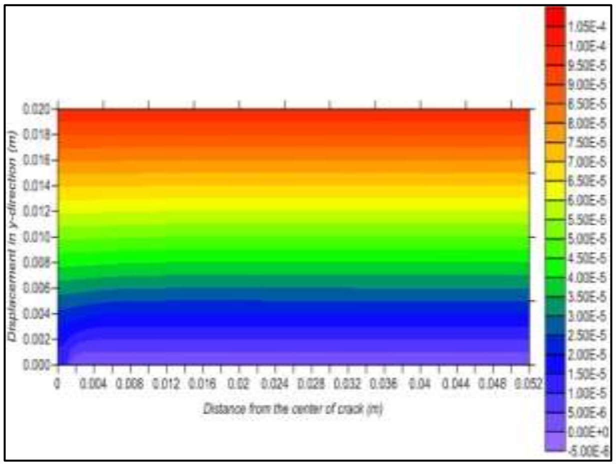

Fig. 17. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.003 m).

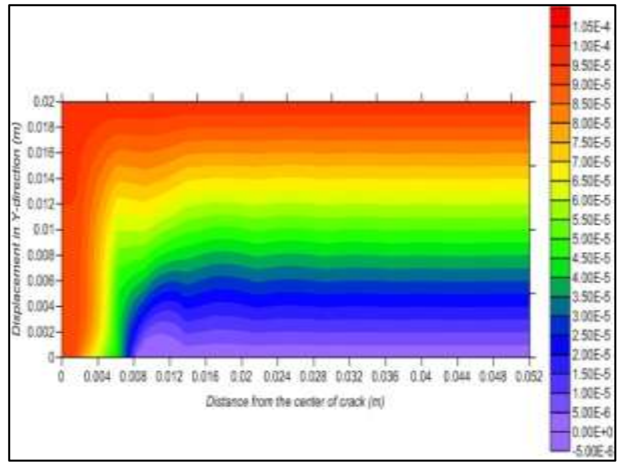

Fig. 20. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.016 m).

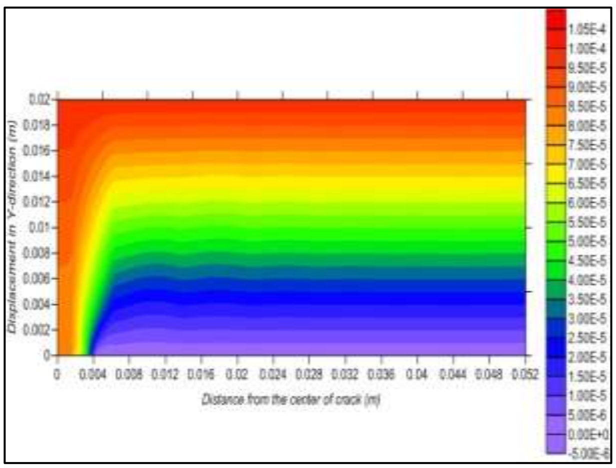

Fig. 18. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.008 m).

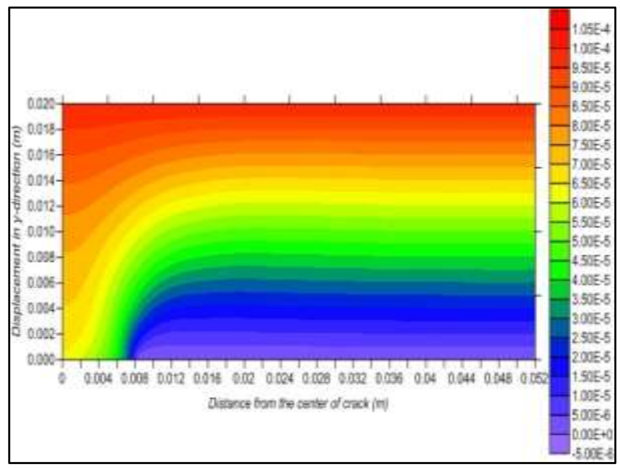

Fig. 21. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.016 m).

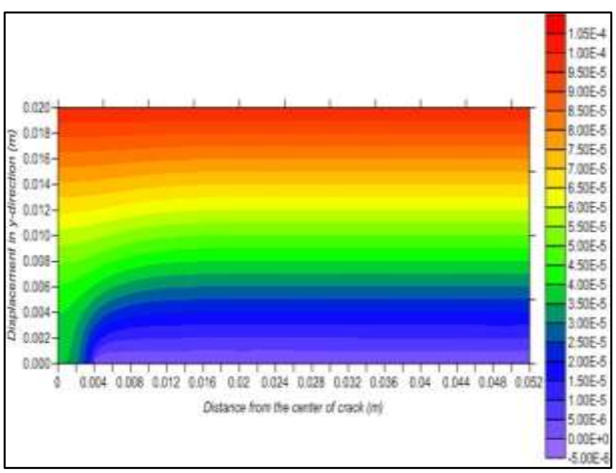

Fig. 19. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.008 m).

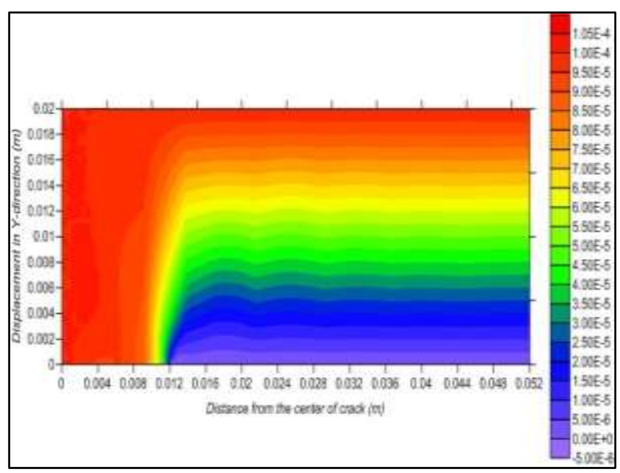

Fig. 22. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.024 m).

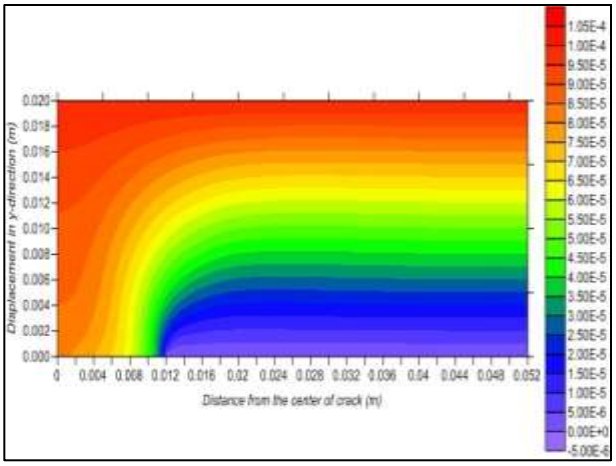

Fig. 23. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.024 m).

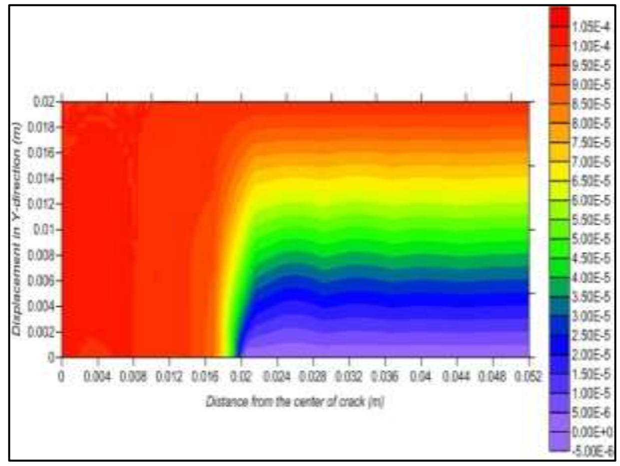

Fig. 25. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.04 m).

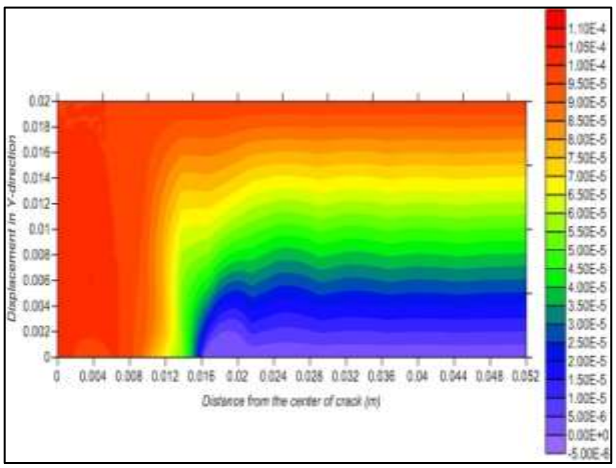

Fig. 24. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.032 m).

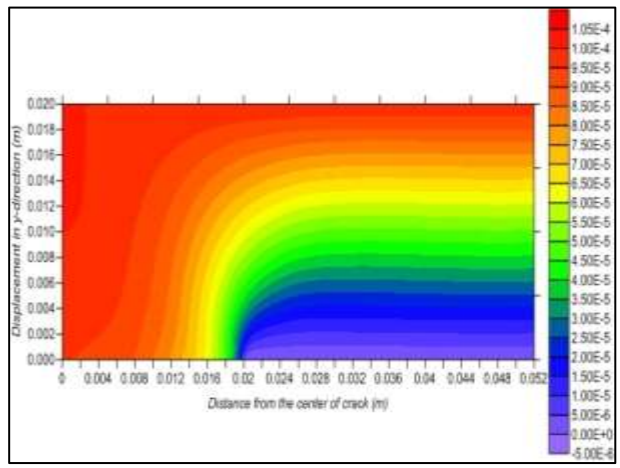

Fig. 27. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.04 m).

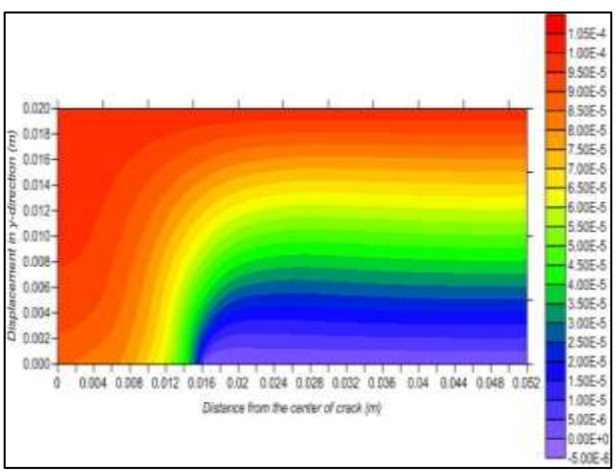

Fig. 25. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.032 m).

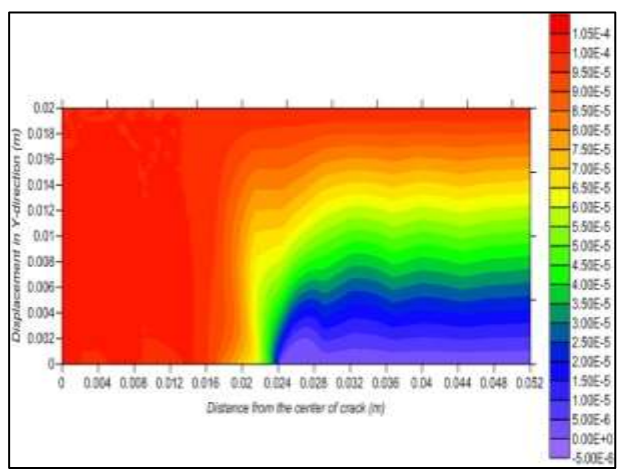

Fig. 28. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.048 m).

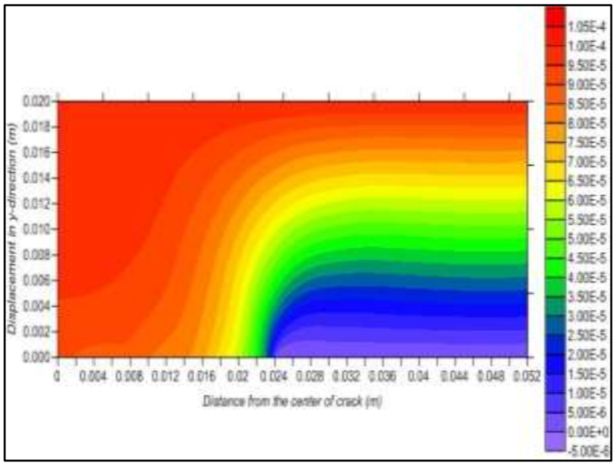

Fig. 29. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.048 m).

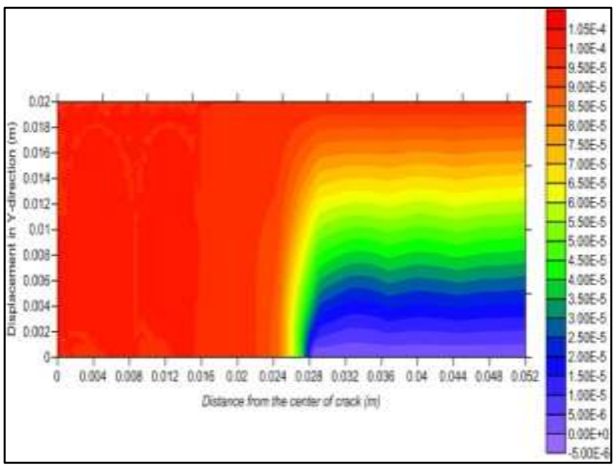

Fig. 30. Contour of Displacement in Y-Direction Calculating by FEM When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.056 m).

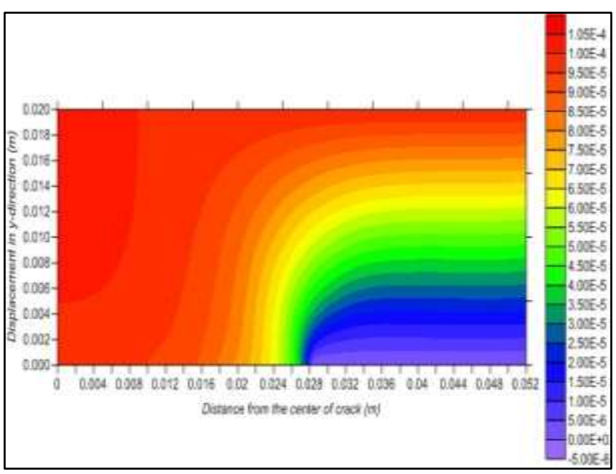

Fig. 31. Contour of Displacement in Y-Direction Calculating by MFree Method When the Fiber Volume Fraction is (37.5%)and Crack Length is (0.056 m).

IV. CONCLUSION

The following points can be concluded,

1. A comparison made between numerical method with using finite element method and numerical results from MFree method of central crack unidirectional composite plate, show a good approximation with maximum error between finite element method and MFree method results about (1.9%).
2. Depending on the position of crack tip and the volume fraction of fiber (i.e. distribution of fibers and matrices), the value of stress intensity factor will be changed. Hence, the value and distribution of SIFs will be changed.
3. The changing of SIFs, when the crack length and volume fraction of fiber increase, gives a good explanation for the fracture phenomena in composite materials which in turn answer the question of "How the crack is propagate in composite material".
4. The smooth contours of displacement in Y-direction using MLPG method shows the potentiality of MFree Methods in getting results which usually depends on the radius of local equilibrium domain and the radius of the MLS shape functions and relatively the number and distribution of nodes.
5. Referring to the above conclusion in (3), the transmitted mutual effects in composite materials can be easily treated even near/on the interfacing lines depending on local support domains.
6. Finally, crack orientation and type of applied load in longitudinal and laminate composite plate can be studied in future work to describe their effects in the stress intensity factor.

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