Dynamic Response of Homogeneous and Film/Substrate Systems Subjected to Low Velocity Impact

Bong Hwan Kim[1], Woo Bong Lee[2*], and Kook Chan Ahn[1]
[1]Department of Automotive Engineering, Gyeongnam National University of Science and Technology, Jinju 52725, Korea
[2]Graduate School, Department of Automotive Engineering, Gyeongnam National University of Science and Technology, Jinju 52725, Korea
*Corresponding author. Telp +82-55-751-3641, Fax.: +82 -55-751-3649
e-mail: bnnd123@gmail.com

***Abstract*--** **The impact response of homogeneous and film/substrate systems subjected to low velocity impact is studied by the coded finite element program. To model and simulate impact response, a effective finite element approach based on the Sun's higher-order beam finite element, Kurapati's generalized power law and Dharani's film model is proposed. The verification of the numerical model is conducted by using energy balance model and wave propagation model, and the present finite element results show a good agreement with two open literature results. Film of layered system may eventually be protected from impact damage and layered system is more impact resistant than homogeneous system, whereas we can see that the variation of film thickness of layered system does not affect so much on impact responses. In addition, the maximum stress by this impact analysis of thin soft film over hard substrate is observed at the interface of film and substrate unlike occurring right underneath of the indentor in static analysis, that is, the interface between two layers by low velocity impact is subject to more failure risk than the other layer.**

***Index Term*--** **, Film / substratesystem, Homogeneous system, Layered system, Dynamic response, Generalized power law**

1. INTRODUCTION

The homogeneous medium like monolithic glass is the most commonly available a single lite of glass. Large shards with sharp edges will be produced when the glass breaks. When penetration and collapse of the glass structure may occur, the failure stress of the homogeneous glass is also relatively low. Due to its brittle nature, glass can often be considered the weakest link of a system. If glass brakes, it does so in large, sharp-edged projectiles, which can hurt or even kill people subjected to the danger. Therefore glass is often laminated. A film is placed at the opposite side of which an impact is most likely to happen. Upon impact the glass shards are pushed into the film and are thereby prevented to injure anyone in the direct environment. There are also situations where indoor film cannot be applied such as when access to the interior of a building or structure is limited or prohibited.

Exterior window film for screen protection and outside weatherable film are the perfect application for situations where physical limitation properties prohibit the installation of traditional indoor window film. Exterior window film has been developed for resisting blast and impact loading and the main purpose of the film is to provide absorption to the impact, which puts less stress on the actual glass. When layered media (film/substrate system) are subjected to an impact that caused by a sufficient heavy and fast impactor, it will break. However, unlike the homogeneous material that fails in a brittle manner, layered media can reduce the number of dangerous flying fragments as many fragments will be adhered by the film layer. Hence, the risk of injuries of people can be significantly reduced. At the same time, the film layer can act as a barrier avoiding penetration. Another advantage of layered media over the homogeneous material is that it is possible to reduce the weight of the glass of the same total thickness.

Layered media have been increasingly used in applications such as microelectronics, optoelectronics and protective coatings on engineering structures. The low-load/low-depth indentation has recently been used to characterize the mechanical properties of films and multilayers. In spite of their advantages, however, the efficient application of layered media is limited, because of the difficulties in their strength calculations at the stage of their design. Foreign object like a small stone thrown into the windows shall give an impact to architectural glass. For optimal design of layered media that minimizes property damage is required a thorough understanding of the impact behaviors of film/substrate systems subjected to dynamic impact.

The dynamic responses of isotropic materials and composite laminates subjected to transient dynamic loading have been studied in terms of analytical, numerical and experimental works[1-3]. When the beam is applied to impact loading, the elastic waves generated in the beam are short wavelength vibration modes. Sun and Huang[1] developed a higher order beam finite element with six degrees of freedom for the dynamic response of elastic isotropic beams subjected to impulsive loadings. This higher order beam finite element showed to be more efficient than the conventional element with four degrees of freedom.

Local deformations in the contact zone are not modeled with beam and plate theories since those theories usually assume that the structure is inextensible in the transverse direction. However, in many cases, local indentation has a significant effect on the contact force history and must be accounted for in the analysis. The contact phenomenon is recognized as being rate independent for most laminated composite materials and statically determined contact laws are used by most investigators[1-3]. During the loading and unloading processes of the impact, the contact force *F* has been related to the indentation by Hertzian contact law and the modified Hertzian contact law. However, when a thin film is deposited on a substrate, the deformation and stress field in the resultant layered materials becomes much more complex. The classical Hertz contact law is no longer valid in characterizing the load-depth response for the indentation of a layered material. In recent, Kurapati[4] suggested that a generalized power law (load-displacement curve) in layered media vary with the film thickness and modulus. The validity of this generalized power law has been validated with the testing data generated from FEM (ABAQUS).

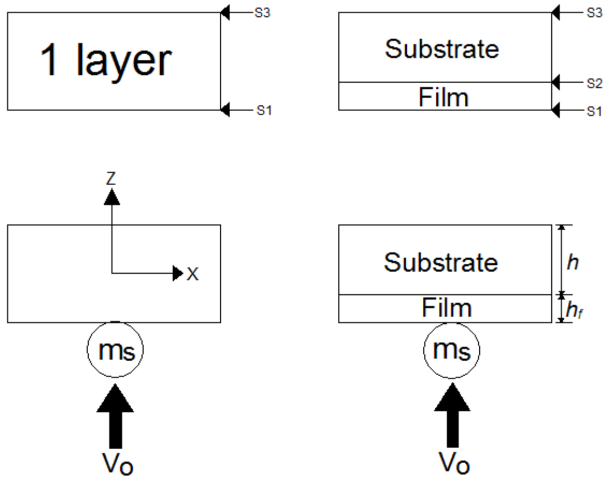
A series of paper on impact of laminated glass for architectural has been published by Dharani and his coworkers[5-10]. In several earlier studies[5,6] on laminated architectural glazing, the film has been traditionally modeled as linear-viscoelastic. The most recent works[7-10] on laminated glass have shown that the film can be modeled as linear elastic.

Therefore, in the present study, a finite element approach based on Sun's higher-order beam finite element, Kurapati's generalized power law and Dharani's film model is proposed and simulated a coded FEM program for low-velocity impact analysis of the homogeneous and layered media.

The verification of a numerical approach in conjunction with the modified Hertzian contact law has been conducted on impact behaviors of laminated composites and laminated glass by using energy balance model and wave propagation model[11-14]. For re-verifying this code in conjunction with a generalized power law, it is compared with the results by the modified contact law on impact behavior of homogeneous medium. And then the effect of thickness of film and substrate due to small mass impact is investigated. That is, the dynamic responses such as the time histories for contact force, deflection of target, displacement of impactor, energy, strain and stress during impact event are obtained and compared with each other between the homogeneous and layered media.

2. THEORETICAL BACKGROUND

Consider the homogeneous and layered media consisting of a single layer and multiple layers with substrate thickness h and film thickness h_f subjected to transverse impact by a steel ball of radius R with initial impact velocity V_0, as shown in Figure 1.



(a) (b)

Fig. 1. Schematic diagram of low velocity impact of (a) homogeneous and (b) layered(film/substrate) systems.

The purpose of this study is to investigate impact induced responses through the homogeneous and layered beams. We assume a low velocity impact such that the glass ply does not fracture. Therefore, a higher-order beam theory with six degrees of freedom is used to analyze on impact response of these beams. The element displacement function is taken as

$$v = a_1 + a_2 x + a_3 x^2 + a_4 x^3 + a_5 x^4 + a_6 x^5 \qquad (1)$$

where v is the transverse displacement and a_i are constant coefficients. The three degrees of freedom at each node are the transverse displacement v, the rotation θ and the curvature k. The coefficients a_i in Equation (1) can be replaced by the six generalized nodal displacements at the two end nodes and, as a result, the displacement function can be alternatively expressed in terms of the nodal displacements.

For contact force and indentation relation, a generalized power law by fitting data generated using a wide range of film/substrate properties is given as follows

$$F = CE_2\delta^p \qquad (2)$$

where F is contact force and δ the indentation. CE_2 is contact stiffness. C and p are material constant and power, and defined by

$$C = 10\ \hat{}\{C_0 + C_1(\log(E_1/E_2)) + C_2(\log(E_1/E_2))^2$$

$$+ C_3(\log(E_1/E_2))^3\} \qquad (3)$$

where

$$C_0=0.9892-0.02725(h/R)+0.068268(h/R)^2-0.03375(h/R)^3$$

$$C_1=0.2939+1.7397(h/R)-1.9264(h/R)^2+0.77333(h/R)^3$$

$$C_2=-0.11509+0.064477(h/R)+0.00013037(h/R)^2-0.013021(h/R)^3$$

$$C_3=0.038188-0.15565(h/R)+0.18009(h/R)^2-0.072568(h/R)^3$$

and

$$p=10\ \hat{}\{p_0+p_1(\log(E_1/E_2))+p_2(\log(E_1/E_2))^2+p_3(\log(E_1/E_2))^3+p_4(\log(E_1/E_2))^4\} \qquad (4)$$

where

$$p_0=0.18478-0.010549(h/R)+0.0081517(h/R)^2-0.0017708(h/R)^3$$

$$p_1=-0.11958+0.15183(h/R)-0.082706(h/R)^2+0.013552(h/R)^3$$

$$p_2=-0.0000081967-0.081956(h/R)+0.11148(h/R)^2-0.047344(h/R)^3$$

$$p_3=0.0089133-0.0059241(h/R)-0.010213(h/R)^2+0.0081615(h/R)^3$$

$$p_4=0.0006394+0.0070813(h/R)-0.010569(h/R)^2+0.0045875(h/R)^3$$

In Equation (3) and (4), E_1 and E_2 are elastic modulus of the layer and substrate material. h and R are layer thickness and radius of the indenter, respectively. Equation (2) indicates that for the indentation of any elastic film-substrate system, the resultant load-displacement response follows a general power-law relation that is defined by the normalized film modulus (E_1/E_2) and the normalized film thickness (h/R).

Glass and film/glass used as homogeneous and layered media in this study, respectively, are widely used in many engineering applications (vehicles, aircraft, buildings and electronic etc.). The simple applications of this material have the shape of a glass beam panels. In case of impact of a hard projectile, impact responses are expected to occur in the impact zone where direct contact of the projectile and the glass takes place. Thus, it is very important to estimate accurately the contact force and its history.

The relaxation modulus $G(t)$ for a linear viscoelastic material such as film (PET and PVB) is generally given in the form

$$G(t)=G_\infty+(G_0-G_\infty)e^{-\beta t} \qquad (5)$$

where G_∞ is the long time shear modulus, G_0 is the short time shear modulus and β is the decay factor. Since the impact duration is in the range of milliseconds, the stress relaxation modulus $G(t)$ changes very little during impact. In this short time, film behaves like a solid glassy material. The linear elastic treatment of film not only facilitates a closed-form solution but also results in a significant reduction in computational time. In the time durations for low velocity impact problems, the difference in stresses obtained by treating film as linear viscoelastic and linear elastic is less than 2%**[8]**. The most recent works[7-10] have shown that film can be modeled as linear elastic by using the short term shear modulus for a transient response. The Young's modulus E_p and the Poisson's ratio ν_p for film are given in terms of short term shear modulus $G = G_0$ and bulk modulus K as

$$E_p=9KG_0/(3K+G_0) \qquad (6)$$
$$\nu_p=(3K-2G_0)/(6K+2G_0)$$

In this study, therefore, film and glass will be modeled as a linear elastic material. The governing equation of this structures dynamic behavior is given by the Hamilton's principle in the following form

$$[M]\{\ddot{u}\}[K]\{u\}=\{F\} \qquad (7)$$

where $[M]$ and $[K]$ are the mass and stiffness matrix of the beams, respectively. $\{u\}$ and $\{\ddot{u}\}$ are the displacement and acceleration vector, respectively. $\{F\}$ is the equivalent of external load, which includes the impact force.

In order to get numerical solution on the impact responses of homogeneous and layered media, we adopt another equation of a generalized power law, Newton's second law for the dynamic equation of the impactor and Newmark's integration scheme for solving the dynamic equations of the target and the impactor for each time step inluding Equation (7). Similar simulating process were described in detail in Ref. [13,14].

3. NUMERICAL INVESTIGATION

A higher-order beam finite element is conducted for the study of the dynamic response of homogeneous and layered beams with variable thickness of film and substrate due to low-velocity impact.

It is applied to a generalized contact law that both loading and unloading process are treated as elastic because the glass is a brittle material. The beams are assumed to be impacted at the centre by a steel ball impactor with diameter 12.7mm and initial impact velocity 10m/s. The models are simply supported on both side edges. The material properties of target and impactor for simulation are shown in Table 1.

Table 1. Material properties of target and impactor for simulation.

Materials		Properties
Target	Film	$G_0 = 1Gpa$, $G_\infty = 0.69GPa$ $\rho = 1100kg/m^3, \beta = 12.6s^{-1}$ $h_f = 0.2,0.4mm$
	Substrate	$w \times l = 100 \times 600mm$ $E = 70GPa$, $v = 0.23$ $\rho = 2440kg/m^3$ $h = 4,6,8mm$
Impactor		$E = 200GPa$, $v = 0.29$ $\rho = 7800kg/m^3$ $R = 6.35mm$ $V_0 = 10m/s$

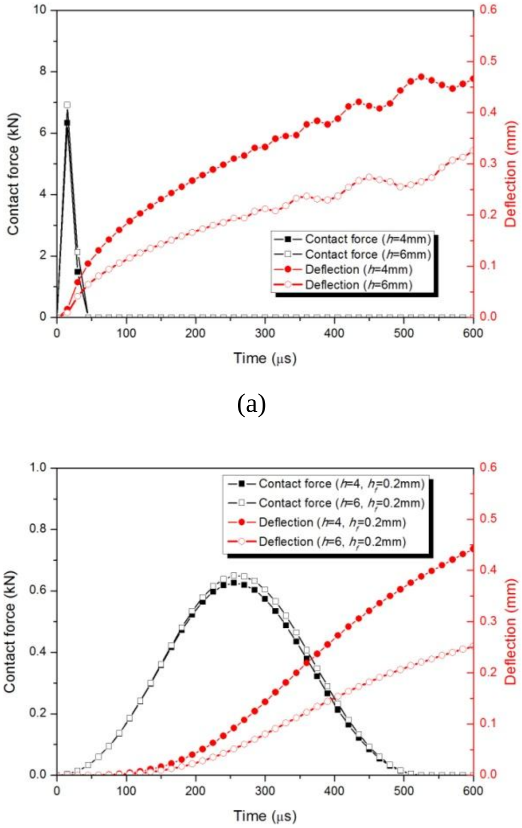
4. RESULTS AND DISCUSSION

Figure 2 shows the histories of contact force and deflection for homogeneous and layered system with various thicknesses of film and substrate obtained from the present finite element analysis at velocity 10m/s. From Figure 2, the maximum contact forces for homogeneous and layered system occur at around 15μs and 255μs after the initial impact, respectively. In order to verify this coded finite element program, the present finite element analysis is compared with an energy balance model that the maximum contact force and the contact duration can be estimated, and wave propagation method that the maximum contact force can be predicted simply as shown in Table 2.

Table 2 shows that maximum contact forces between present study and wave propagation model are agreed well to each other, however, energy balance model tends to overestimate maximum contact force after the onset of impact. Abrate[11] showed that this trend in energy balance model occurs since it does not account for wave propagation. And also results of contact duration between present study and energy balance model are agreed well to each other. Therefore, it is shown that this approach can be applied to response problem of homogeneous system with h_f=0.0mm in layered system due to small mass impact. Table 3 depicts the maximum contact force and contact duration due to variable thickness of film and substrate by this coded program.

Table 2. Comparison of max. contact force and contact duration of homogeneous system between the present study, energy balance model and wave propagation model

System		Homogeneous		
Substrate thickness		4	6	8
Max. contact force (N)	Present	6340	6920	7180
	Energy balance model	8300	8300	8300
	Wave propagation model	6360	6930	7190
Contact duration (μs)	Present	38.5	38.0	37.5
	Energy balance model	37	37	37



(a)

(b)

Fig. 2. Comparison of the histories of contact force and deflection by the present study of (a) homogeneous (h=4, 6mm) and (b) layered system (h=4, 6mm and h_f =0.2mm).

From Figure 2, Table 1 and 2, it can be seen that the maximum contact force in homogeneous system is about ten times larger than that of layered system and the deflection in homogeneous system much larger than that of layered system, but contact duration in homogeneous system is much smaller than that of layered system. That is, we can see that the higher the thickness of substrate at the same film thickness, the larger the magnitude of contact force but the smaller the magnitude of deflection. And also, Table 3 shows that thickness of film at the same substrate thickness has no significant effects on contact force and contact duration.

Table 3. Results of contact force and contact duration of layered system by the present study.

System	Film /substrate thickness (h_f/h, mm)	Max. contact force (N)	Contact duration (μs)
		Present	Present
Layered	0.2/4	626	540
	0.4/4	623	540
	0.2/6	650	540
	0.4/6	647	540
	0.2/8	660	540
	0.4/8	657	540

Relationship of contact force and indentation at homogeneous (h=4, 6mm) and layered (h=4, 6 and h_f=0.2, 0.4mm) systems can be depicted by the curve shown in Figure 3. The contact force is assumed to approach to elastic behavior in the unloading process (A-O curve) after it passes the maximum value of the indentation in the loading process (O-A curve). All of the work done by the impactor on the system in the loading process is the kinetic energy. From Figure 3, it can be seen that the corresponding power p=1.5 of homogeneous system calculated by a generalized power law is consistent with the Hertzian equation but p=2.11 or 2.06 of layered system not consistent. Results of contact stiffness and power by the present study are depicted in Table 4. We can see that contact law of two systems is dependent on existance of film than the thickness of film and substrate from Figure 3 and Table 4. And also, contact stiffnesses of homogeneous system are much larger than those of layered system. This means that layered system is much more impact resistant than homogeneous system.

Next, the present numerical results are compared with the wave propagation theory for verification of this coded program. Figure 4 shows the dynamic strain histories at the points (0, 30, 90mm apart from the centre) on the surface S3 which is opposite to the impacted surface in the homogeneous and layered media at velocity 10m/s. From Figure 4, the strains in the homogeneous medium are larger than those of layered media during the impact event because the film has less stiffness than substrate.

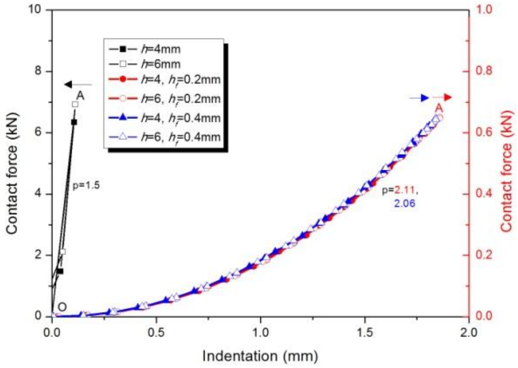
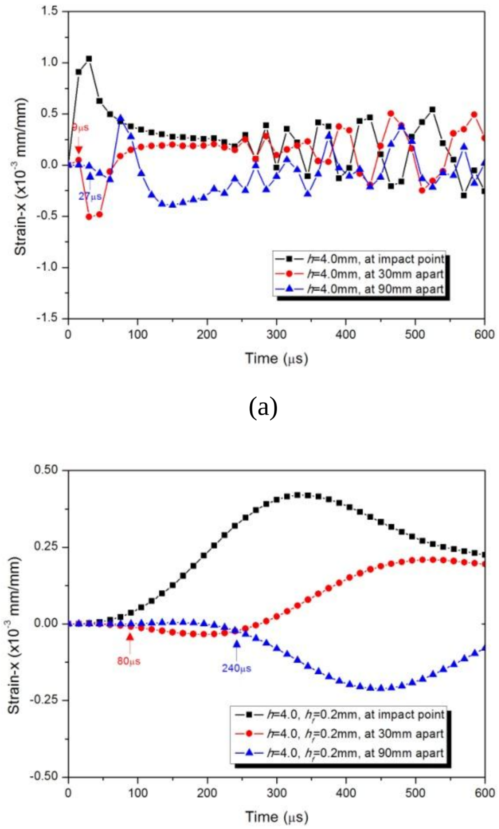


Fig. 3. Relationship of contact force and indentation of homogeneous (h=4, 6mm) and layered system (h=4, 6mm and h_f =0.2, 0.4mm).

Table 4. Results of contact stiffness and power by Equation (3) and (4).

Substrate thickness (h, mm)	Film thickness (h_f, mm)	Contact stiffness (CE_2, N/mm^p)	Power (p)
4	0.0	0.186E6	1.5
	0.2	0.176E3	2.11
	0.4	0.183E3	2.06
6	0.0	0.186E6	1.5
	0.2	0.176E3	2.11
	0.4	0.183E3	2.06
8	0.0	0.186E6	1.5
	0.2	0.176E3	2.11
	0.4	0.183E3	2.06

In Figure 4.(a), the first dynamic strain responses of homogeneous medium at 30mm and 90mm apart from the impact point occur at around 9μs and 27μs after the initial impact, respectively. From these results of the dynamic strain responses, the transverse wave velocity becomes 3333m/s. The transverse wave velocity of homogeneous medium with a single layer by wave propagation theory is 3394m/s. From this comparison, the present coded program can be verified by good coincidences between each other. From Figure 4(b), transverse wave velocity of layered media with multiple is predicted 375m/s because the first dynamic strain responses at 30mm and 90mm apart from the impact point occur at around 80μs and 240μs after the initial impact, respectively. We can predict that transverse wave velocity of homogeneous medium is much faster than that of layered media. Theoretical comparison on transverse wave velocity of layered media needs to be reviewed later again with other researcher's paper if it can be found.



(a)

(b)

Fig. 4. The dynamic strain histories of (a) homogeneous (*h*=4mm) and (b) layered system *h*=4mm and h_f =0.2mm) at impact point on surface S3.

Figure 5 shows contact force-deflection curves on the homogeneous and layered systems at impact velocity 10m/s. It shows that the thickness of substrate at identical film thickness affects so much on the relation of contact force and deflection but the thickness of film at identical substrate thickness has no significant effect, and the maximum deflection does not occur at the maximum contact force. It shows a typical wave-controlled impact that the contact force and beam deflection are never in phase[15].

The numerical results for impactor velocity and energy histories in two systems are given in Figure 6. The velocity and energy at the time zero are the initial velocity and energy of impactor at which the impactor hits the target. Velocity curves of Figure 6(a) decrease and take negative values and remain constant by time. These negative values represent rebound velocity of the impactor. Minimum kinetic energy in Figure 6(b) occurs when velocity is zero. At these curves, the lowest tip of the curve shows minimum kinetic energy and the end of curve that remain constant shows the rebound energy. And, also, the energy difference between initial energy and rebound energy becomes absorbed energy by target.

Figure 7 shows relationship of energies and thickness of film and substrate. It can be seen that from Figure 7, the rebounded energy of layered system is larger than that of homogeneous system at each substrate thickness, and the thicker the thickness of substrate of homogeneous and layered systems, the larger the rebounded energy is and the smaller the absorbed energy become. In addition, the thickness of film at identical substrate thickness does not affect so much on the rebounded and absorbed energies.

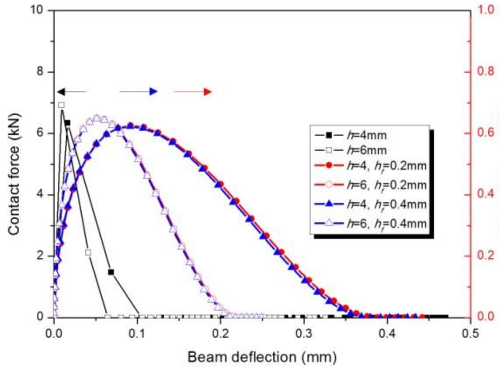
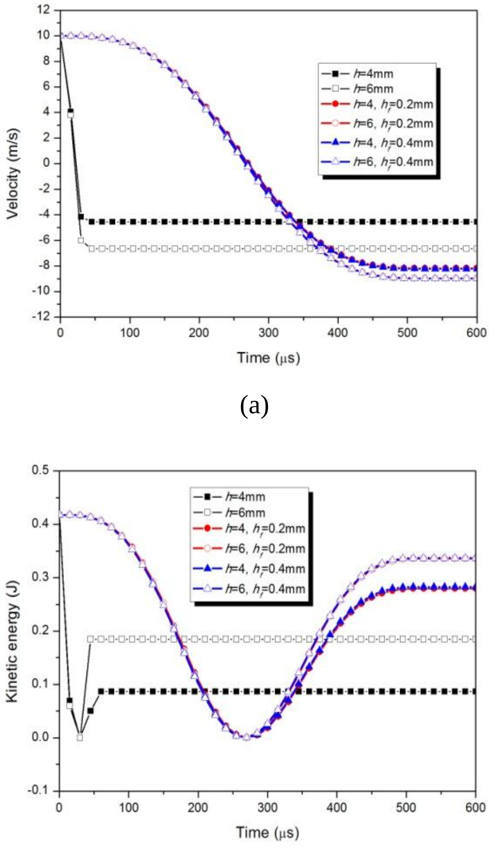


Fig. 5. Relationship of contact force and beam deflection of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm).



(a)

(b)

Fig. 6. The (a) velocity and (b) energy histories of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm).

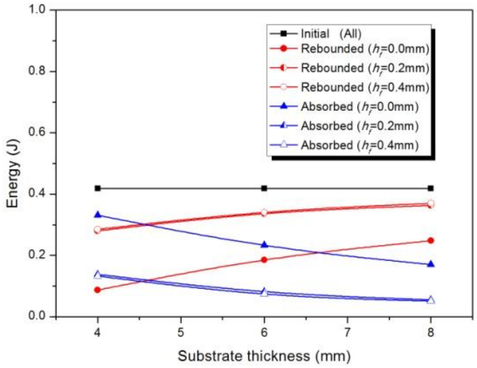


Fig. 7. Relationship of energies and thickness of film and substrate.

Figure 8 depicts relationships of contact force and ball displacement of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm). The rigid impactor hitting the deformable target involves loading and unloading processes. The loading process is represented by curve O-A, whereas the unloading process is represented by curve A-O. The energy of the impactor is the product of contact force by displacement, thus, the area under these closed curves represents the loading and unloading energy phase, respectively. If energy loss is negligible, the area under the loading curve O-A represents the initial energy (E_i), which is the same as the kinetic energy at the start of impact. The area under the unloading curve A-O represents the rebound energy (E_r) to the impactor. The absorbed energy (E_a) absorbed during the impact is E_i minus E_r and is shown in the loop area of the contact force-displacement curve O-A-O.

In this study, the absorbed energy is attributed wholly to crushing of the target because the impactor is not damaged. From Figure 8, two closed loops of kinetic energy-displacement curve for layered system with the thickness of the identical substrate and the different film, that is, [*h*=4, h_f=0.2mm and *h*=4, h_f=0.4mm] and [*h*=6, h_f=0.2mm and *h*=6, h_f=0.4mm] have approximately same area, respectively. It may be concluded that the existence of film thickness between homogeneous and layered system has significant effect but the thickness of film in layered system does not affect so much on the absorbed energy of target like impact behaviors of Fig. 7 by small mass impact.

Figures 9 and 10 show the dynamic stress histories through each layer for homogeneous and layered systems at impact point and 30mm apart from impact point. S2 and S1 represent the impacted surface of homogeneous and layered systems, respectively. S2 in layered system means surface on the substrate and S3 the opposite surface of impact in two systems as shown in Figure 1. It is shown that stresses in x-direction in *h*=4mm of two systems from Figures 9 and 10 are larger than those of *h*=6mm. In addition, stresses on the impacted surface S1 of layered system approach to zero and is smaller than those on the substrate surface S2. Kurapati[4] was shown that the maximum stress in static analysis for the soft film over the hard substrate occurs right underneath the indentor, whereas the value by this impact analysis is observed at the interface of thin film and substrate. Hence the interface is prone to more failure risk than the other layer.

Figures 11 and 12 show the variation of strains and stresses through the thickness of homogeneous and layered system at impact point and 30mm apart from impact point. All strain components in Figure 11 vary linearly through the thickness and they are independent of the material variations through the thickness, whereas the variation of stress through the thickness of layered system shows its discontinuity due to a significant difference in the modulus values between film and substrate. Hence, film prevents substrate of layered system from damage by reducing strain and stress to zero unlike homogeneous system, whereas homogeneous system advances damage from surface S1 to S4 rapidly.

Figure 13 depicts relation of stress and thickness of film and substrate on each layer of homogeneous and layered systems. When the thickness of substrate in homogeneous system increases, stress on impacted surface S2 decreases, whereas in case of layered system, stress on impacted surface S1 is approximately zero independent of film thickness. In addition, when the thickness of substrate in layered system increases, stress on the surface S2 tends to decrease independent of film thickness. All stresses on the bottom surface S3 is reversed those on the surface S2 of two systems. Hence, it can be seen that stress occurred by low velocity impact depends on substrate thickness of two systems but film thickness in layered system does not affect so much on impact stress.

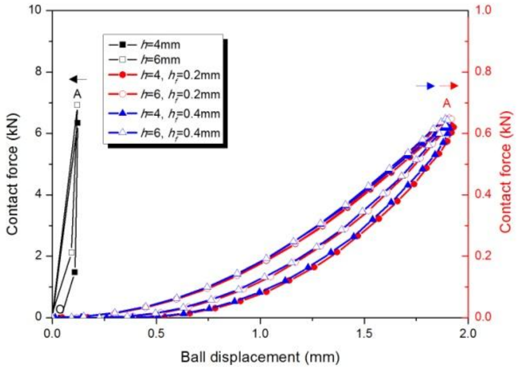
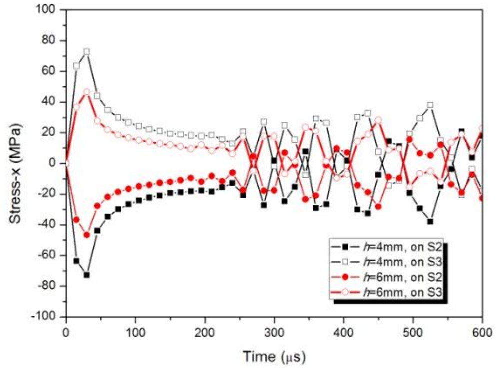
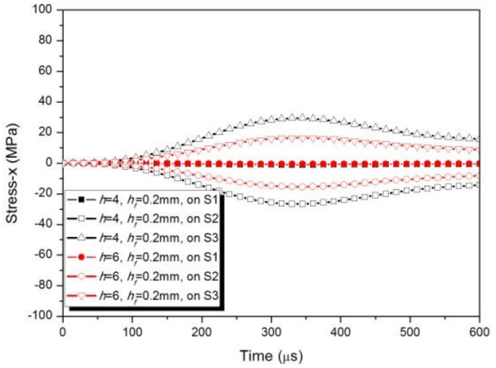


Fig. 8. Relationship of contact force and ball displacement of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm).

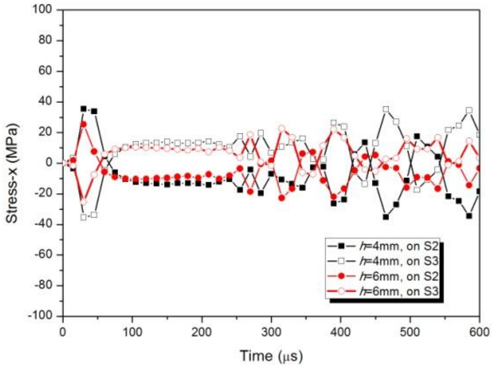


(a)

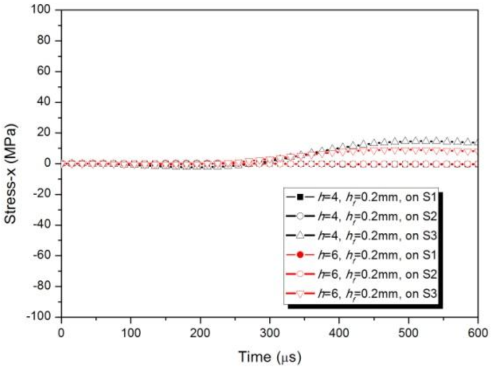


(b)

Fig. 9 Dynamic stress histories through the layer of (a) homogeneous (*h*=4, 6mm) and (b) layered system (*h*=4, 6mm and h_f =0.2mm) at impact point.

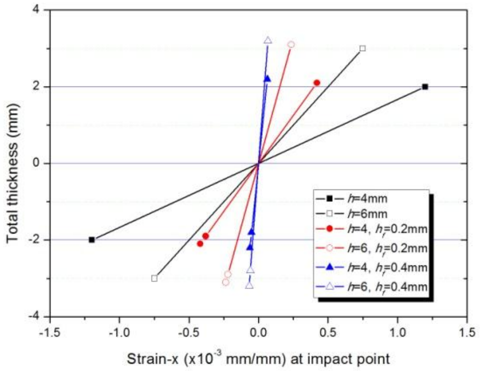


(a)

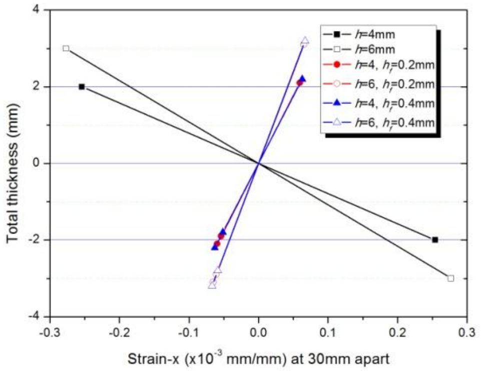


(b)

Fig. 10 Dynamic stress histories through the layer of (a) homogeneous (*h*=4, 6mm) and (b) layered system (*h*=4, 6mm and h_f =0.2mm) at 30mm apart.

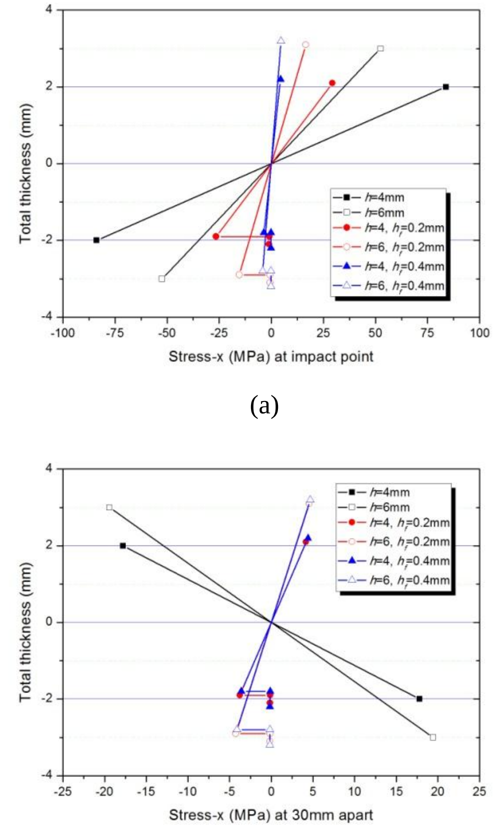


(a)



(b)

Fig. 11 Variations of strain through the layer of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm) at (a) impact point (b) 30mm apart.



(a)

(b)

Fig. 12 Variations of stress through the layer of homogeneous (*h*=4, 6mm) and layered system (*h*=4, 6mm and h_f =0.2, 0.4mm) at (a) impact point (b) 30mm apart.

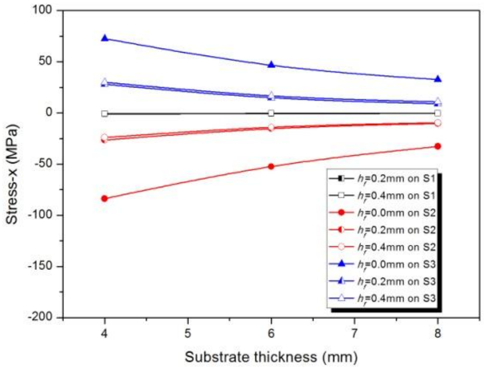


Fig. 13. Relationship of stress and thickness of film and substrate on each layer of homogeneous and layered systems.

5. CONCLUSION

In the present study, a effective finite element approach based on Sun's higher-order beam finite element, Kurapati's generalized power law and Dharani's film model for the impact responses of homogeneous and layered systems under low velocity impact is proposed and the corresponding finite element program is coded. Numerical results using the program are compared with those of energy balance model and wave propagation theory, and are verified by a good agreement among these results. The impact responses such as impact energy, wave propagation, strain and stress etc. of homogeneous and layered systems are additionally analyzed and compared with each other. From the present numerical results, it can be concluded that a generalized power law and film model applied is very effective on prediction of dynamic responses of homogeneous and layered systems.

Impact responses such as contact force, energy, wave propagation, strain and stress in homogeneous system is more sensitive than those of layered system of the same substrate thickness and prone to more failure risk. This means that film of layered system may eventually be protected from impact damage and layered system is more impact resistant than homogeneous system. But we can see that the variation of film thickness of layered system does not affect so much on impact responses. In addition, the maximum stress by this impact analysis is observed at the interface of thin film and substrate unlike occurring right underneath of the indentor in static analysis, that is, the interface between two layers by low velocity impact is subject to more failure risk than the other layer. The results of this research may be used a guide in making some preliminary design considering impact for film/substrate composed of multilayer with different material properties in the future.

ACKNOWLEDGEMENT

This work was supported by Gyeongnam National University of Science and Technology Grant-2015.

REFERENCES

[1] C. T. Sun, and S. N. Huang, Transverse impact problems by higher order beam finite lement, *Computers & Structures*, 5 (1975) 297-303.

[2] S. H. Yang, and C. T. Sun, Indentation law for composite laminates, *ASTM STP* 787, (1982) 425-449.

[3] Tan, T. M., and Sun, C. T., 1985, "Use of Statical Indentation Laws in the Impact Analysis of Laminated Composite Plates", J. of Applied Mechanics, Trans. ASME, 52, pp. 6-12.

[4] S.N.V.R.K. Kurapati, Y. C. Lu and F. Yang, "Indentation Load-Displacement Relations for the Spherical Indentation of Elastic Film/Substrate Structures", CMC (Computers, Materials & Contina), 20(10), pp. 1-17, 2010.

[5] L. R. Dharani, and F. S. Ji, Dynamic analysis of normal impact of occupant head on laminated glass, *SAE Paper, No. 980862*, (1998).

[6] L. R. Dharani, F. S. Ji, R. A. Behr, J. E. Minor, and P. A. Kremer, Breakage prediction of laminated glass using the sacrificial ply design concept, *ASCE J. Archit. Eng.*, 10(4) (2004) 126–135.

[7] J. Wei, and L. R. Dharani, Response of laminated architectural glazing subjected to blast loading, *Int. J. Impact Eng.*, 32 (2006) 2032-2047.

[8] S. Zhao, L. R. Dharani, L. Chai, and S. D. Barbat, Analysis of damage in laminated automotive glazing subjected to simulated head impact, *Eng. Fail. Anal.*, 13 (2006) 582-597.

[9] S. Zhao, L. R. Dharani, and X. Liang, Analysis of damage in laminated architectural glazing subjected to blast loading, *Adv. Struct. Eng.*,11 (2008) 129-134.

[10]M. S. Shetty, J. Wei, L. R. Dharani, and D. S. Stutts, Analysis of damage in laminated architectural glazing subjected to wind loading and winddborne debris impact, *Buildings*, 3 (2013) 422-441.

[11]Abrate. S., 2001, "Modeling of Impacts on Composite Structures," Composite Structures, 51, pp. 129-138.

[12]I. M. Daniel, T. Liber, and R. H. LaBedz, Wave propagation in transversely impacted composite laminates, *Experimental Mechanics*,(1979) 9-16.

[13]B. H., Kim, K. C. Ahn, and C. W. Lee, Low Velocity Impact Behaviors of a Laminated Glass, *Smart Science*, 2(4) (2014) 209-213.

[14]K. C. Ahn, The Dynamic Effect of PVB Interlayer Thickness on Laminated Glass Window, *Int. J. of Engineering & Technology IJET-IJENS*, 2(4) (2015) 33-40.

[15]R. Olsson, Closed form prediction of peak load and delamination onset under small mass impact, *Composite Structures*, 59(3) (2003) 314-349.