

The Effects of Film/Substrate Properties on Impact Behaviour of Layered Systems

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Abstract-- The effects of film/substrate properties on impact behaviour of layered systems subjected to low velocity impact are studied by the coded finite element program. To model and simulate impact behaviour, an effective finite element approach in conjunction with the Sun's higher-order beam theory and Kurapati's generalized power law is proposed. The mechanical properties of the substrate are fixed with elastic modulus and by changing the mechanical properties of the films, five typical film and substrate combinations, namely, $E_f/E_s=0.2, 0.5, 1, 2$ and 5 are considered. The verification of the numerical model is conducted by compared with the results of Hertzian contact law and wave propagation theory in $E_f/E_s=1.0$ layered systems i.e., homogeneous material, and the present finite element results show a good agreement with open literature results. Impact behaviours in hard film-soft substrate layered systems ($E_f/E_s=2.0$ and 5.0) are more sensitive than those of soft film-hard substrate layered systems ($E_f/E_s=0.2$ and 0.5) in the same film/substrate thickness. And, also, we can observe that in case of soft film-hard substrate ($E_f/E_s=0.2$ and 0.5) and hard film-soft substrate ($E_f/E_s=2.0$ and 5.0) layered systems, the interface and impacted surface unlike occurring in static analysis are prone to more failure risk than the other layer, respectively.

Index Term-- Layered systems, Film/substrate, Impact behaviour, Generalized power law, Finite element

1. INTRODUCTION

Layered (film/substrate) systems to coat surfaces of a substrate material with thin films have been developed for resisting blast and impact loading and the main purpose of the film is to provide absorption to the impact, which puts less stress on the actual substrate. When layered systems are subjected to an impact that caused by a sufficient heavy and fast impactor, it will break. However, unlike the homogeneous material that fails in a brittle manner, layered systems can reduce the number of dangerous flying fragments as many fragments will be adhered by the film layer. Hence, the risk of injuries of people can be significantly reduced. At the same time, the film layer can act as a barrier avoiding penetration. Another advantage of layered systems over the homogeneous material is that it is possible to reduce the weight of the substrate of the same total thickness.

Layered systems have been increasingly used in applications such as microelectronics, windows in architectural applications, windshields in automotive and protective coatings on engineering structures. The low-

load/low-depth indentation has recently been used to characterize the mechanical properties of films and multilayers. In spite of their advantages, however, the efficient application of layered systems is limited, because of the difficulties in their strength calculations at the stage of their design. Foreign object like a small stone thrown into the windows shall give an impact to architectural glass. For optimal design of layered systems that minimizes body injury and property damage during impact accident is required a thorough understanding of the impact behaviour of layered systems subjected to dynamic impact.

The dynamic responses of isotropic materials and composite laminates subjected to transient dynamic loading have been studied in terms of analytical, numerical and experimental works [1,2]. When the beam is applied to impact loading, the elastic waves generated in the beam are short wavelength vibration modes. Sun and Huang [1] developed a higher order beam theory with six degrees of freedom for the dynamic response of elastic isotropic beams subjected to impulsive loadings. This higher order beam theory showed to be more efficient than the conventional element with four degrees of freedom. Local deformations in the contact zone are not modeled with beam and plate theories since those theories usually assume that the structure is inextensible in the transverse direction. However, in many cases, local indentation has a significant effect on the contact force history and must be accounted for in the analysis. The contact phenomenon is recognized as being rate independent for most laminated composite materials and statically determined contact laws are used by most investigators [1,2]. During the loading and unloading processes of the impact, the contact force F has been related to the indentation by Hertzian contact law and the modified Hertzian contact law.

The effects of substrate on indentation behavior of layered systems have been studied in terms of numerical and experimental works [3-5]. However, when a thin film is deposited on a substrate, the deformation and stress field in the resultant layered materials becomes much more complex. The classical Hertz contact law is no longer valid in characterizing the load-depth response for the indentation of layered systems. And it is obvious that the critical indentation depends on the mechanical properties of both film and substrate, such as the ratio of the elastic modulus of film to that of substrate, E_f/E_s , and the indenter geometry. Therefore, a systematic study of the influence of E_f/E_s on the substrate effects would be very

helpful for determining the mechanical properties of films. In recent, Kurapati [6] suggested that a generalized power law (load-displacement curve) in layered systems vary with the film thickness and modulus. The validity of this generalized power law has been validated with the testing data generated from FEM (ABAQUS).

A series of paper on impact of laminated glass for architectural has been published by Dharani and his coworkers [7-9]. In several earlier studies [7,8] on laminated architectural glazing, the film has been traditionally modeled as linear-viscoelastic. The most recent works [9,10] on laminated glass have shown that the film can be modeled as linear elastic.

In the present paper, a effective impact finite element theory based on Sun's higher-order beam theory and Kurapati's generalized power law is employed to investigate the effects of film/substrate properties on impact behavior considering the mechanical properties of different combinations of thin film systems, that is, the film is either elastically softer (weaker) or harder (stiffer) than the substrate. In other words, the dynamic responses such as the time histories for contact force, deflection of target, displacement of impactor, energy, strain and stress during impact event are obtained and compared with each other between the ratio of the elastic modulus of film to that of substrate E_f/E_s .

A. 2. FINITE ELEMENT MODELING

Consider layered systems consisting of two layers with film thickness h_f and substrate thickness h_s subjected to transverse impact by a steel ball of radius R with initial impact velocity V_0 as shown in Fig. 1.

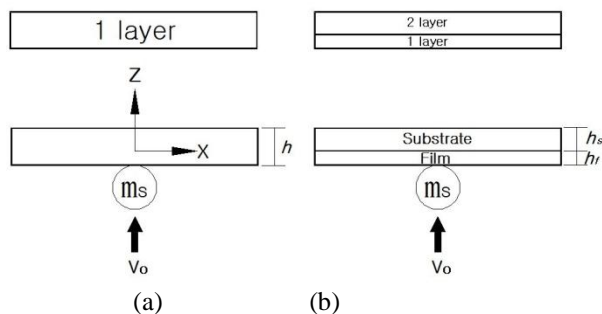


Fig. 1. Schematic diagram of low velocity impact of layered systems.

The purpose of this study is to investigate impact induced responses through the homogeneous and layered beams. We assume a low velocity impact such that the film and substrate layers does not fracture. Therefore, a higher-order beam theory with six degrees of freedom is used to analyze on impact response of these beams. The element displacement function is taken as

$$v = a_1 + a_2x + a_3x^2 + a_4x^3 + a_5x^4 + a_6x^5 \quad (1)$$

where v is the transverse displacement and a_i are constant coefficients. The three degrees of freedom at each node are the transverse displacement v , the rotation θ and the curvature k . The coefficients a_i in Eq. (1) can be replaced by the six generalized nodal displacements at the two end nodes

and, as a result, the displacement function can be alternatively expressed in terms of the nodal displacements.

For contact force and indentation relation, a generalized power law [6] by fitting data generated using a wide range of film/substrate properties is given as follows

$$F = CE_s^p \quad (2)$$

where F is contact force and δ the indentation. CE_s is contact stiffness. C and p are material constant and power, and defined by

$$C = 10^4 \{ C_0 + C_1(\log(E_f/E_s)) + C_2(\log(E_f/E_s))^2 + C_3(\log(E_f/E_s))^3 \} \quad (3)$$

where

$$C_0 = 0.9892 - 0.02725(h_f/R) + 0.068268(h_f/R)^2 - 0.03375(h_f/R)^3$$

$$C_1 = 0.2939 + 1.7397(h_f/R) - 1.9264(h_f/R)^2 + 0.77333(h_f/R)^3$$

$$C_2 = -0.11509 + 0.064477(h_f/R) + 0.00013037(h_f/R)^2 - 0.013021(h_f/R)^3$$

$$C_3 = 0.038188 - 0.15565(h_f/R) + 0.18009(h_f/R)^2 - 0.072568(h_f/R)^3$$

and

$$p = 10^4 \{ p_0 + p_1(\log(E_f/E_s)) + p_2(\log(E_f/E_s))^2 + p_3(\log(E_f/E_s))^3 + p_4(\log(E_f/E_s))^4 \} \quad (4)$$

where

$$p_0 = 0.18478 - 0.010549(h_f/R) + 0.0081517(h_f/R)^2 - 0.0017708(h_f/R)^3$$

$$p_1 = -0.11958 + 0.15183(h_f/R) - 0.082706(h_f/R)^2 + 0.013552(h_f/R)^3$$

$$p_2 = -0.000081967 - 0.081956(h_f/R) + 0.11148(h_f/R)^2 - 0.047344(h_f/R)^3$$

$$p_3 = 0.0089133 - 0.0059241(h_f/R) - 0.010213(h_f/R)^2 + 0.0081615(h_f/R)^3$$

$$p_4 = 0.0006394 + 0.0070813(h_f/R) - 0.010569(h_f/R)^2 + 0.0045875(h_f/R)^3$$

In Eq. (3) and (4), E_f and E_s are elastic modulus of the layer and substrate material. h_f and R are layer thickness and radius of the indenter, respectively. Eq. (2) indicates that for the indentation of any elastic film-substrate systems, the resultant load-displacement response follows a general power-law relation that is defined by the normalized film modulus (E_f/E_s) and the normalized film thickness (h_f/R).

Film/glass used as layered systems in this study is widely used in many engineering applications (vehicles, aircraft, buildings and electronic etc.). The simple applications of this material have the shape of a beam panels. In case of impact of a hard projectile, impact responses are expected to occur in the impact zone where direct contact of the projectile and film/glass takes place. Thus, it is very important to estimate accurately the contact force and its history.

The relaxation modulus $G(t)$ for a linear viscoelastic material such as film (PET and PVB) is generally given in the form

$$G(t) = G_\infty + (G_0 - G_\infty)e^{-\beta t} \quad (5)$$

where G_∞ is the long time shear modulus, G_0 is the short time shear modulus and β is the decay factor. Since the impact duration is in the range of milliseconds, the stress relaxation modulus $G(t)$ changes very little during impact. In this short time, film behaves like a solid glassy material. The linear elastic treatment of film not only facilitates a closed-form solution but also results in a significant reduction in computational time. In the time durations for low velocity impact problems, the difference in stresses obtained by treating film as linear viscoelastic and linear elastic is less than 2% [9]. The most recent works [9,10] have shown that film can be modeled as linear elastic by using the short term shear modulus for a transient response. The Young's modulus E_f and the Poisson's ratio ν_f for film are given in terms of short term shear modulus $G = G_0$ and bulk modulus K as

$$\begin{aligned} E_f &= 9KG_0 / (3K + G_0) \\ \nu_f &= (3K - 2G_0) / (6K + 2G_0) \end{aligned} \quad (6)$$

In this study, therefore, film and glass will be modeled as a linear elastic material. The governing equation of this structures dynamic behavior is given by the Hamilton's principle in the following form

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \quad (7)$$

where $[M]$ and $[K]$ are the mass and stiffness matrix of the beams, respectively. $\{u\}$ and $\{\ddot{u}\}$ are the displacement and acceleration vector, respectively. $\{F\}$ is the equivalent of external load, which includes the impact force. In order to get numerical solution on the impact behaviour of layered systems, we adopt a generalized power law $Eq.(2)$ for contact equation, Newton's second law for the dynamic equation of the impactor and Newmark's integration scheme for solving the dynamic equations of the target and the impactor for each time step including $Eq. (7)$. Similar simulating process were described in detail in Ref. [11].

3. NUMERICAL INVESTIGATION

A higher-order beam finite element is conducted for the study of the dynamic response of five layered beams with various ratios of elastic modulus of film and substrate due to low-velocity impact. It is applied to a generalized contact law that both loading and unloading process are treated as elastic because the glass is a brittle material.

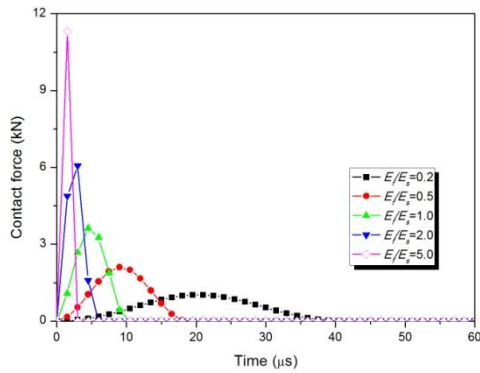
The beam with dimension of film thickness $h_f=0.4\text{mm}$ and substrate thickness $h_s=4\text{mm}$ is assumed to be impacted at the center by a spherical impactor with diameter 6.35mm and initial impact velocity 10m/s. The size of model is 100x600mm, and film and substrate are considered as isotropic elastic materials. Constant Poisson's ratios, $\alpha_f=\alpha_s=0.28$ are used for both film and substrate since the effect of Poisson ratio on indentation is very small [5]. The mechanical properties of the substrate are fixed with elastic modulus of glass, $E_s=70\text{GPa}$. By changing the mechanical properties of the films, five typical film and substrate

combinations, namely, $E_f/E_s=0.2, 0.5, 1, 2$ and 5 are considered. The film can be either weaker ($E_f/E_s < 1$) or stiffer ($E_f/E_s > 1$) than the substrate depending on the value of E_f/E_s .

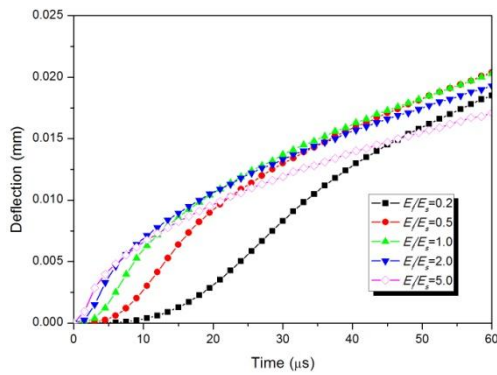
4. RESULTS AND DISCUSSION

Fig. 2 shows the histories of contact force and deflection for layered systems with various ratios of the elastic modulus of film to that of substrate E_f/E_s obtained from the present finite element analysis at velocity 10m/s. From Fig. 2, the maximum contact forces (times) for layered systems from $E_f/E_s=0.2$ to 5.0 become about 1.02kN (20 μs), 2.1kN (9 μs), 3.62kN (4.5 μs), 6.0kN (3 μs) and 11.3kN (1.5 μs), respectively. And we can see that the larger the ratio of E_f/E_s is, the larger the deflection becomes during the initial impact within 4.5 μs . Fig. 3 depicts the relation of maximum contact force, contact duration and ratio of elastic modulus E_f/E_s of layered systems. From Figs. 2 and 3, it can be seen that the maximum contact force and the deflection in $E_f/E_s=5.0$ layered systems are much larger than that of $E_f/E_s=0.2$ layered systems, whereas the contact duration in $E_f/E_s=0.2$ layered systems is much larger than that of in $E_f/E_s=5.0$ layered systems. That is, we can see that the harder the material property of film in respect to that of the substrate is at the same film/substrate thickness, the larger the maximum contact force and deflection become, but the smaller the contact duration becomes.

Relationship of contact force and indentation at layered systems can be depicted by the curve shown in Fig. 4. The contact force is assumed to approach to elastic behavior in the unloading process after it passes the maximum value of the indentation in the loading process. All of the work done by the impactor on the systems in the loading process are the kinetic energy. From Fig. 4, it can be seen that the corresponding power $p=1.5$ of $E_f/E_s=1.0$ like homogeneous system calculated by a generalized power law is consistent with the Hertzian contact law ($n=1.5$) but $p=1.1, 1.3, 1.8$ and 2.0 of layered systems not consistent. Results of contact stiffness and power by the present study are depicted in Table 1. We can see that the larger the value of E_f/E_s is, the larger the slope between the contact force and indentation becomes. This means that soft film-hard substrate layered systems ($E_f/E_s=0.2$) is much more impact resistant than hard film-soft substrate layered systems ($E_f/E_s=5.0$).



(a)



(b)

Fig. 2. Histories of (a) contact force and (b) deflection of layered systems.

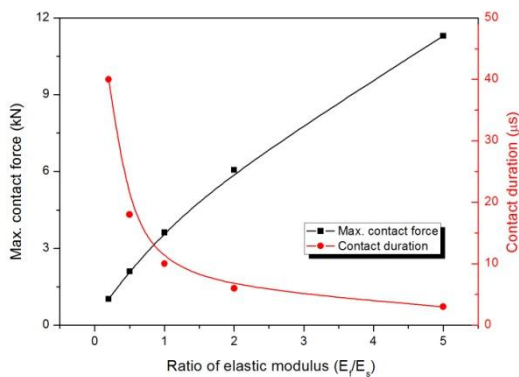


Fig. 3. Relation of max. contact force, contact duration and ratio of elastic modulus of layered systems.

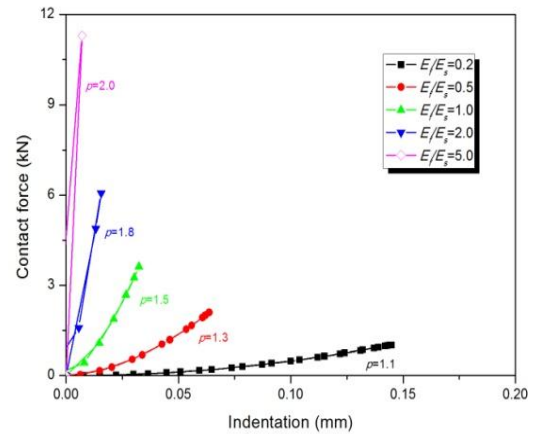


Fig. 4. Relationship of contact force and indentation of layered systems.

Table I
Results of contact stiffness and power by Eq. (3) and (4).

System	Film/Substrate				
Film thickness (h_f , mm)	0.4				
Substrate thickness (h_s , mm)	4				
Ratio of elastic Modulus (E_f/E_s)	0.2	0.5	1.0	2.0	5.0
Contact stiffness (CE_2 , N/mm ³)	0.486E5	0.274E6	0.679E6	0.133E7	0.265E7
Power (p)	2.0	1.8	1.5	1.3	1.1

Next, the present numerical results are compared with the wave propagation theory for verification on impulsive wave of this coded program. Fig. 5 shows the dynamic strain histories of $E_f/E_s=0.2, 1.0$ and 5.0 layered systems at three points (0, 30, 90mm apart from the impact point) on the surface S3 which is opposite to the impacted surface in layered systems. From Fig. 5, the magnitude and response time of the strains in the $E_f/E_s=5.0$ layered systems are larger and faster than those of $E_f/E_s=0.2$ during the impact event, respectively, because soft film has less stiffness than hard film. In Fig. 4.(b), the first dynamic strain responses of $E_f/E_s=1.0$ like homogeneous material with $h_s=4.4$ mm at 30mm and 90mm apart from the impact point occur at around $10\mu s$ and $30\mu s$ after the initial impact, respectively. From these results of the dynamic strain responses, the transverse wave velocity becomes 3333m/s. The transverse wave velocity of $E_f/E_s=1.0$ layered systems (that is, homogeneous material) by wave propagation theory [12] is 3418m/s. From this comparison, the present coded program can be verified by good coincidences between each other with 2.5% error. From Fig. 4(a) and 4(c), transverse wave velocities of $E_f/E_s=0.2$ and $E_f/E_s=5.0$ layered systems are predicted 1200m/s and 7500m/s because the first dynamic strain responses at 30mm (90mm) apart from the impact point occur at around $10\mu s$ ($30\mu s$) and $4\mu s$ ($12\mu s$) after the initial impact, respectively. We can predict that transverse wave velocity of $E_f/E_s=5.0$ layered systems is much faster than that of $E_f/E_s=0.2$ layered systems. Theoretical comparison on

transverse wave velocity of layered systems needs to be reviewed later again with other researcher's paper if it can be found.

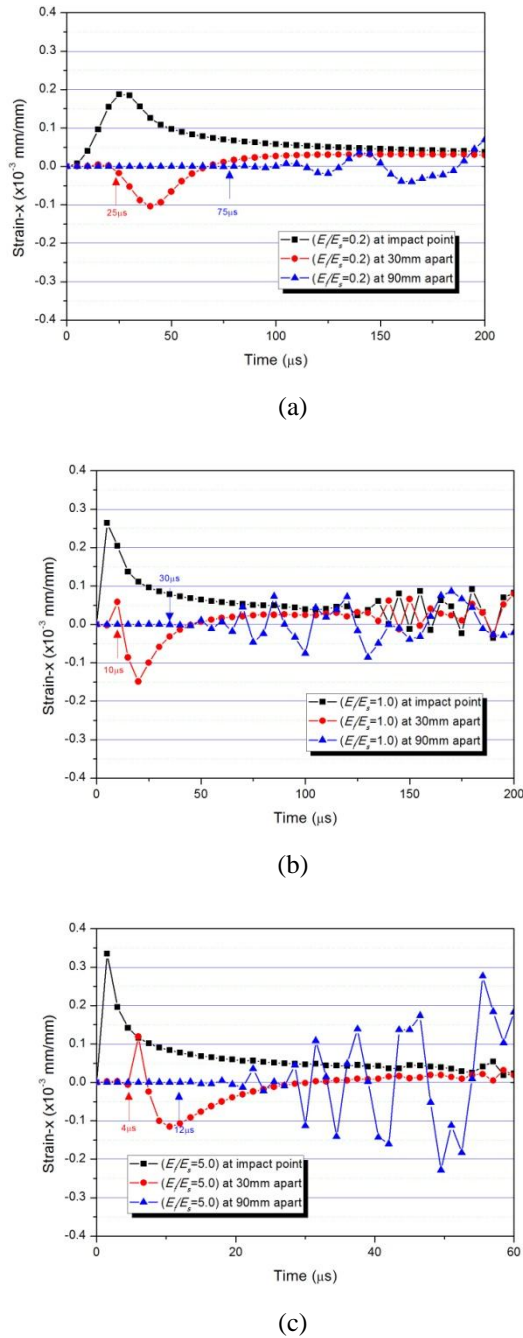


Fig. 5. The dynamic strain histories of (a) $E_f/E_s=0.2$, (b) $E_f/E_s=1.0$ and (c) $E_f/E_s=5.0$ layered systems at each point on surface S3.

Fig. 6 shows contact force-deflection curves on layered systems at impact velocity 10m/s. The maximum contact force does not occur at the maximum deflection. It shows a typical wave-controlled impact that the contact force and beam deflection are never in phase [13].

The numerical results for impactor velocity and energy histories in five layered systems are given in Fig. 7. The velocity and energy at the time zero are the initial velocity and energy of impactor at which the impactor hits the target.

Velocity curves of Fig. 7(a) decrease and take negative values and remain constant by time. These negative values represent rebound velocity of the impactor. Minimum kinetic energy in Fig. 7(b) occurs when velocity is zero. At these curves, the lowest tip of the curve shows minimum kinetic energy and the end of curve that remain constant shows the rebound energy. And, also, the energy difference between initial energy and rebound energy becomes absorbed energy by target. It can be seen that from Figs. 7 and 8, the larger the value of E_f/E_s of layered systems at identical film/substrate thickness is, the faster the rebounded response of layered systems, but the smaller the rebounded energy and then the larger the absorbed energy becomes.

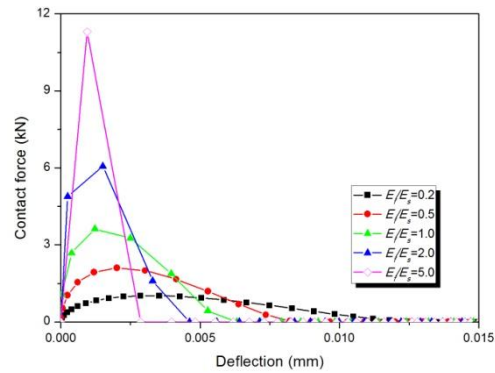


Fig. 6. Relationship of contact force and beam deflection of layered systems.

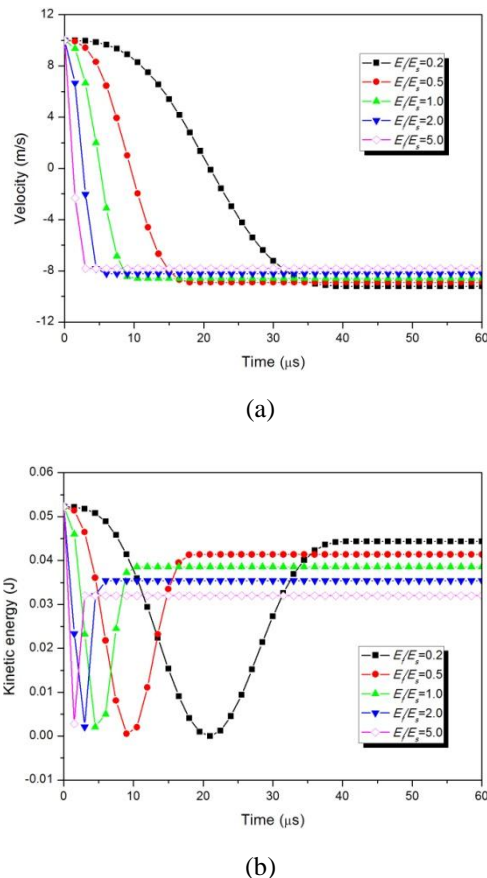


Fig. 7. The (a) velocity and (b) energy histories of layered systems.

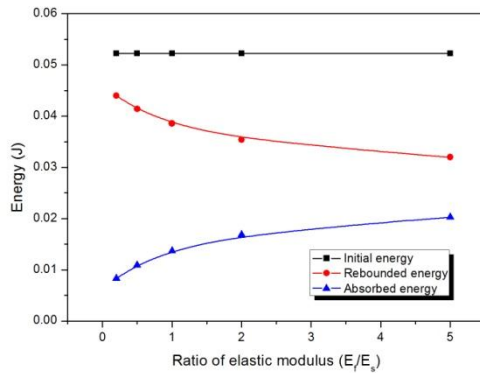
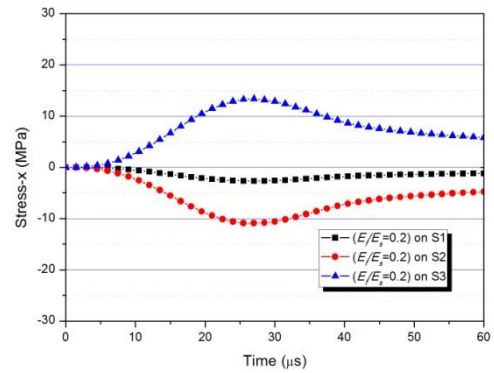


Fig. 8. Relationship of energy and ratio of elastic modulus (E_f/E_s) of layered systems.

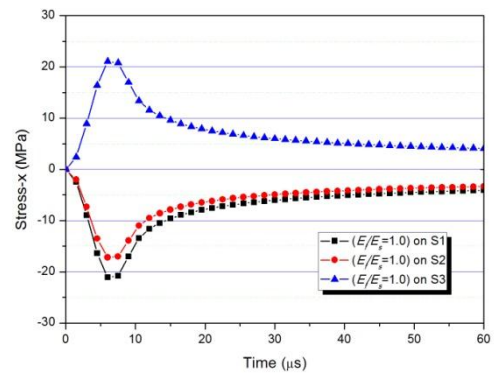
Fig. 9 show the dynamic stress histories on each layer of $E_f/E_s=0.2$, 1.0 and 5.0 layered systems. S1 and S2 in layered systems mean “the impacted surface” and “the interface between film and substrate” as shown in Fig. 1, respectively, and S3 “the opposite surface of impact”. It is shown that stresses in x-direction of $E_f/E_s=5.0$ layered systems are much larger than those of $E_f/E_s=0.2$ layered systems on three surfaces. In particular, stresses on the impacted surface S1 of $E_f/E_s=0.2$ layered systems approach to zero and is smaller than those on the substrate surface S2, however, on the impacted surface S1 in $E_f/E_s=5.0$ layered systems larger than those on the substrate surface S2. Kurapati [5] was shown that the maximum stress in static analysis for soft film-hard substrate occurs right underneath the indenter whereas for the other model which is hard film-soft substrate the value is observed at the interface of film and substrate. However, we can observe that by this impact analysis, the opposite case of static analysis is true. That is, the maximum stress for soft film-hard substrate ($E_f/E_s=0.2$) occurs at the interface of film and substrate whereas for the other model which is hard film-soft substrate ($E_f/E_s=5.0$) the value is observed right underneath the indenter. Hence for soft film-hard substrate ($E_f/E_s=0.2$ and 0.5), the interface is prone to more failure risk than the other layer, whereas for homogeneous material ($E_f/E_s=1.0$) and hard film-soft substrate ($E_f/E_s=2.0$ and 5.0), the impacted surface is prone to more failure risk than the other layer.

Fig. 10 shows the variation of strains and stresses through the layer of five layered systems at impact point. All strain components in Fig. 10(a) vary linearly through the thickness and they are independent of the material variations through the thickness, whereas the variation of stress in Fig. 10(b) shows its discontinuity due to a significant difference in the modulus values between film and substrate. In addition, film of hard film-soft substrate layered systems ($E_f/E_s=2.0$ and 5.0) prevents substrate from damage by approaching stress rapidly to zero unlike the other layered systems.

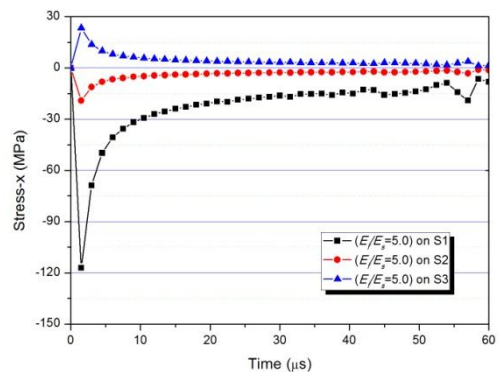
Fig. 11 depicts relationship of stress and ratio of elastic modulus on each surface of five layered systems at impact point. From Fig. 11, we can see that by the increasing the value of E_f/E_s , stress on the surface S1 shows a rapid increase in (-) value but that on the surface S2 and S3 doesn't make a big difference like that on the surface S1.



(a)



(b)



(c)

Fig. 9. Dynamic stress histories through the layer of (a) $E_f/E_s=0.2$, (b) $E_f/E_s=1.0$ and (c) $E_f/E_s=5.0$ layered systems on each surface at impact point.

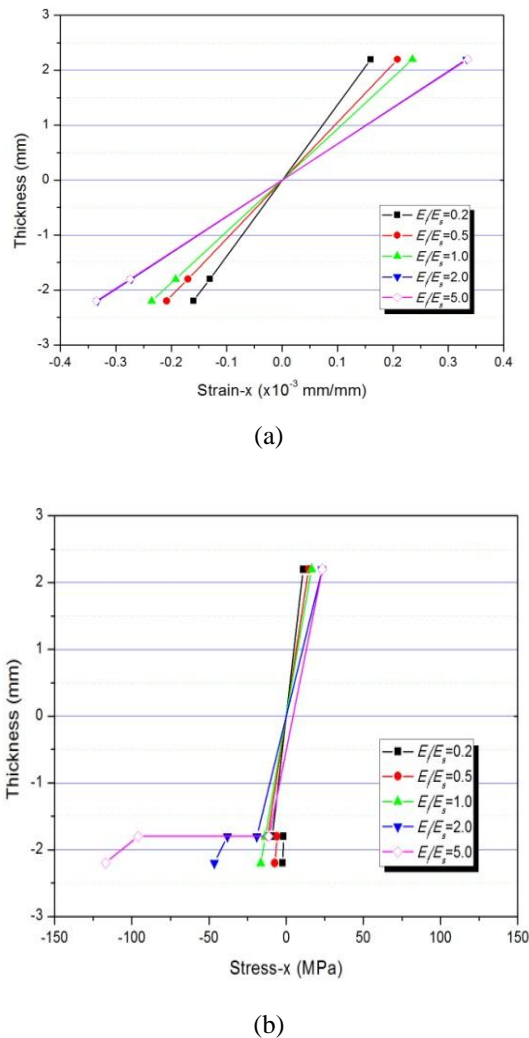


Fig. 10. Variations of (a) strain and (b) stress through the layer of layered systems at impact point.

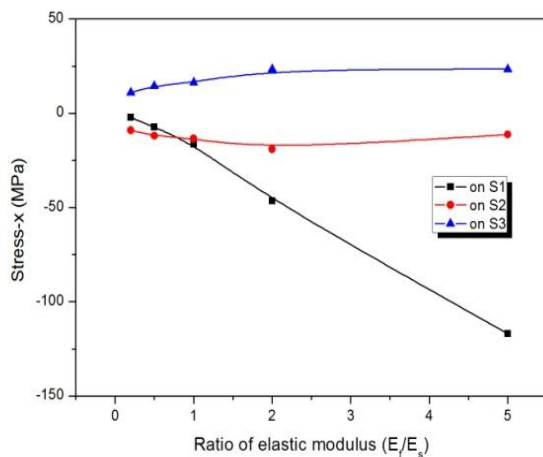


Fig. 11. Relationship of stress and ratio of elastic modulus on each surface of layered systems.

5. CONCLUSION

The effects of film/substrate properties on the impact behavior of layered systems are investigated using an effective

finite element approach based on Sun's higher-order beam theory and Kurapati's generalized power law under low velocity impact. In this work, the impact behaviour such as the time histories for contact force, deflection of target, displacement of impactor, energy, strain and stress of five typical layered systems during impact event are obtained and compared with each other between the ratio of the elastic modulus of film to that of substrate E_f/E_s . From the present numerical results, it can be seen that a generalized power law and film model applied would be very helpful for estimating the impact behaviour of layered systems.

Impact responses such as contact force, energy, wave propagation, strain and stress in hard film-soft substrate layered systems ($E_f/E_s=2.0$ and 5.0) are more sensitive than those of soft film-hard substrate layered systems ($E_f/E_s=0.2$ and 0.5) in the same film/substrate thickness. That is, this means that soft film-hard substrate layered systems may eventually be protected from impact damage and is more impact resistant than hard film-soft substrate layered systems. In addition, in case of soft film-hard substrate ($E_f/E_s=0.2$ and 0.5), the interface between the indenter in static analysis is prone to more failure risk than the other layer, while in case of homogeneous ($E_f/E_s=1.0$) and hard film-soft substrate ($E_f/E_s=2.0$ and 5.0) layered systems, the impacted surface unlike occurring at the interface in static analysis is prone to more failure risk than the other layer. The results of this research may be used as a guide in making some preliminary design considering impact behavior for film/substrate composed of multilayer with different material properties in the future.

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