Centrifugal and Thermal Influence on the Dispersion of Surface Waves Propagating on a Thermopiezoelectric Half-Space

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***Abstract*— The paper presents an analysis of the thermal and Centrifugal effects on the behavior of the acoustic waves propagating in a half space thermopiezoelectric material surface in the framework of linear thermopiezoelectricity including Coriolis and centrifugal forces. The secular piezothermoelastic generalized equations are formulated and mathematical boundary conditions in closed and isolated form are derived. The characteristics of surface waves propagating in generalized piezothermoelastic solid half-space and their dependence upon geometric and physical parameters have been investigated. It is shown that the Rayleigh and Bleustein-Gulyaev waves, depending of thermal and physical properties of material, can be suppressed by rotation, and are generally dispersive. However, like in piezoelectric half space, there is certain abnormal dispersion for small rotation perturbations and the Coriolis and centrifugal forces on dispersion are generally of the same order. The pre-heating temperature of the material, thermomelastic and pyroelectric coupling effects have not to be neglected. Using PZT-6B as a thermopiezoelectric model material, Numerical and results are generated, simulated and commented.**

***Index Term*— Thermopiezoelectric material, Acoustic waves, Thermal effect, Centrifugal effect.**

I. INTRODUCTION

SURFACE wave propagation in piezoelectric solids has been used with great success by the telecommunication industry and used in wireless transmission and reception technology for colour television sets, cell phones and global positioning systems. Wave propagation in piezoelectric media has various applications in the fields of aerospace, mechanical and civil engineering. Surface wave propagation in a piezoelectric plate is used to achieve the time delay effect in acoustic devices.
The frequency vibration equations of thermo-piezoelectric crystal plate have derived by Mindlin [1]. Lothe et al. [2] are demonstrated that surface wave solution for non-rotating piezoelectric solids may not be unique. There are generally, two surface waves namely Rayleigh wave and Bleustein - Gulyaev wave owing to the original work of Bluestein [3] and Gulyaev [4]. Tiersten et al. [5] studied thickness vibrations of piezoelectric plates and also discussed wave propagation in an infinite piezoelectric plate. Yang et al. [6] have studied the thickness vibrations of rotating piezoelectric plate. Lao [7] derived gyroscopic effect in surface acoustic waves. Tiersten et al. [8], [9] have studied frequency shifts due to rotation have been used to make gyroscopes, i.e., angular rate sensors. Fang et al. [10], [11] have analyzed rotation-perturbed surface acoustic waves propagating in piezoelectric crystals and the Rotation sensitivity of waves propagating in a rotating piezoelectric plate. Different problem of rotating media has been investigated in the last years. Sharma et al. [12] studied the propagation characteristics of Rayleigh waves in transversely isotropic piezothermoelastic materials and extended the study to generalized piezothermoelastic half space. Propagation of surface acoustic waves in pre-stressed piezoelectric structures have studied by Panait [13], Liu et al. [14], [15], Qian et al. [16], [17], Jin et al. [18], Su et al. [19] and Du et al. [20], [21].

In this paper the effects of rotation and temperature on the dispersion of the Rayleigh and Bleustein - Gulyaev waves over a thermopiezoelectric half-space are studied. The equation of motion, including both Coriolis and centrifugal accelerations, the constitutive equations for linear thermo-piezoelectricity, and the temperature equation are formulated. The surface of the half space is subject to stress free, electrically shorted/charge free, thermally insulated, and rotating about a three symmetry axis. The half-space is cut along a plane normal to one of these axes. The deriving secular equations for non-rotation and rotation cases, in open and closed circuit electric surface conditions, are explicitly deduced and discussed. The curves of phase velocity, attenuation, and longitudinal normalized components of mechanical displacement in function as rotation, and initial temperature are plotted and commented. The effects of rotation, initial temperature of the medium, the pyroelectric coupling on the dispersion of Rayleigh wave are analyzed and discussed.

II. PROBLEM FORMULATION

The current formulation is based on the linear theory of thermopiezoelectricity in which the equations of linear elasticity are coupled to the charge equations of electrostatics and to temperature through the piezoelectric, pyroelectric, and thermoelastic constants. Due to the assumption of a linear theory, both mechanical and electric body forces and couples are neglected in the derivation. The model assumes infinitesimal deformations and strains. We consider a linear thermo-piezoelectric body that occupies the half space, denoted by $x_2 \leq 0$ in the Cartesian frame shown in Fig.1. To study the rotation effect, we let the body rotates about the x_2-axis at a constant rate $\boldsymbol{\Omega}$.

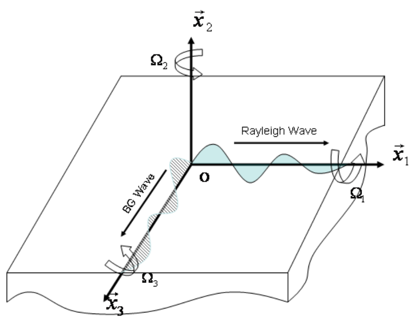

Fig.1. Rayleigh and BG waves propagation on x_2 rotating thermopiezoelectric half space.

Introducing the effects of force of Coriolis and centrifugal force, the equation of motion is given by

$$T_{ij,j} + f_i - (2\Omega_j \times \dot{u}) - (\rho\,\Omega^2 j \times (j \times u)) = \rho \ddot{u}_i \qquad (1)$$

where T_{ij} is the symmetric stress tensor, f_i represents the mechanical body force, the terms $2\Omega_j \times \dot{u}$ and $\rho\Omega^2 j \times (j \times u)$ are respectively the Coriolis and centrifugal forces.

The electrical response is described by the electrostatic equation for the conservation of electric flux relative to Coulomb's law and are given by

$$D_{i,i} = 0 \qquad (2)$$

Assuming that there is no heat source in the half space surface, the Temperature equation can be

$$(\rho c_0 / \theta_0)\dot{\theta} + \lambda_{ij}\dot{\gamma}_{ij} + p_i \dot{E}_i = -\kappa_{ij}\theta_{,ij} \qquad (3)$$

The Constitutive equations for linear thermospiezo-electricity are given by

$$T_{ij} = c_{ijkl}\gamma_{kl} - e_{kij}^{T} E_k - \lambda_{ij}\theta \qquad (4)$$

$$D_m = e_{mkl}\gamma_{kl} + \varepsilon_{mk} E_k + p_i \theta \qquad (5)$$

Where E is the electric vector field, D the dielectric displacement, γ the linearized strain tensor and θ is the temperature. The matrices c, e, λ, ε and p are respectively, elastic, piezoelectric, thermal, dielectric and pyroelectric tensors. c_0 is the specific heat per unit mass, the subscript T indicates transpose. The elastic module c_{ijkl} measured at constant electric field, the piezoelectric module e_{mkl} and the dielectric permittivity ε_{mkl} have the following symmetry properties, respectively

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}$$
$$e_{mkl} = e_{kml}\ ,\ \ \varepsilon_{mk} = \varepsilon_{km}$$

The strain γ, the electric field intensity vector E, and the heat flux q_i are related respectively to the displacement vector, the electric potential ϕ, and temperature θ through the following

$$\gamma_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \qquad (6)$$

$$E_i = -\Phi_{,i} \qquad (7)$$

We are interested in waves propagating along a thermally isolated surface that is electroded and free of mechanical loads, and hence, for $x_2 = 0$, the following boundary conditions are considered

$$T_{21} = T_{22} = T_{23} = 0\ ,\ \ \Phi = 0\ ,\ \ \theta = 0 \qquad (8)$$

To seek surface wave's solutions, we require that both the displacement and electric potential diminish with depth, i.e.

$$\lim_{x_2 \to -\infty} u = 0\ ,\ \lim_{x_2 \to -\infty} \Phi = 0\ ,\ \lim_{x_2 \to -\infty} \theta = 0 \qquad (9)$$

We now write the constitutive relations to thermopiezoelectric crystals of tetragonal system with point group 4mm, and we note that the polycrystalline thermo-piezoelectric ceramics are of the same symmetry. Placing the x_3 axis along the four-fold axis, and the compressed matrix notation.

$$[T] = [T_{11}, T_{22}, T_{33}, T_{13}, T_{23}, T_{12}]$$
$$[\gamma] = [\gamma_{11}, \gamma_{22}, \gamma_{33}, \gamma_{13}, \gamma_{23}, \gamma_{12}]$$
$$[u] = [u_1, u_2, u_3]^T, [x] = [x_1, x_2, x_3]^T$$
$$[D] = [D_1, D_2, D_3]^T, [E] = [E_1, E_2, E_3]^T$$

respectively, elasticity, dielectric, pyroelectric, piezoelectric, thermal conductivity and stress temperature matrices are given by:

$$[c]=\begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

$$[\varepsilon]=\begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix},\ [p]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p_3 \end{bmatrix}$$

$$[e]=\begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{13} & e_{13} & e_{33} & 0 & 0 & 0 \end{bmatrix}$$

$$[\kappa]=\begin{bmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{bmatrix},\ [\lambda]=\begin{bmatrix} \lambda_{11} & 0 & 0 \\ 0 & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{bmatrix}$$

We notice here that the material homogeneity is assumed in this analysis. Considering a wave propagating along the x_1-direction, and for the upper half-space, the three mechanical displacements, temperature, and electric field solutions can be written as

$$u_{1,2,3}=A_{1,2,3}\,e^{k\,\eta x_2}e^{i\,k\,(x_1-vt)} \quad (10)$$

$$\theta=A_4\,e^{k\,\eta x_2}e^{i\,k\,(x_1-vt)} \quad (11)$$

$$\Psi=i\,A_5\,e^{k\,x_2}e^{i\,k\,(x_1-vt)} \quad (12)$$

Here i is the unit imaginary number, the amplitudes A_j, A_4 are complex, and A_5 is taken real. The decay constant (or attenuation factor) η is complex and his real component must be positive ($\mathrm{Re}(\eta)>0$) to satisfy the conditions in equations (9). the frequency ω, and the wave number k are positive and they are related to the phase velocity by $\omega = k\,v$. We can introduce (Bluestein, 1968) a new variable Ψ defined by

$$\Psi=\Phi-\frac{e_{15}}{\varepsilon_{11}}u_3 \quad (13)$$

The equation of motion (1) and Coulomb's law (2) in terms of displacements u_i, temperature θ, and function Ψ are given by

$$\begin{aligned} &c_{11}u_{1,11}+c_{12}u_{2,21}+c_{66}(u_{1,22}+u_{2,12})-\rho\ddot{u}_1 \\ &-2\rho\Omega\dot{u}_3+\rho\Omega^2 u_1-\lambda_{11}\theta_{,1}=0 \end{aligned} \quad (14)$$

$$c_{66}(u_{1,21}+u_{2,11})+c_{12}u_{1,12}+c_{11}u_{2,22}-\rho\ddot{u}_2-\lambda_{11}\theta_{,2}=0 \quad (15)$$

$$\bar{c}_{44}(u_{3,11}+u_{3,22})-\rho\ddot{u}_3+2\rho\,\Omega\,\dot{u}_1+\rho\,\Omega^2 u_3=0 \quad (16)$$

$$\frac{\rho c_0}{\theta_0}\dot{\theta}+\lambda_{11}(u_{1,1}+u_{2,2})-\kappa_{11}(\theta_{,11}+\theta_{,22})=0 \quad (17)$$

$$\Psi_{,11}+\Psi_{,22}=0 \quad (18)$$

With correspondence, the boundary conditions in (8) become

$$T_6=T_{12}=c_{66}(u_{1,2}+u_{2,1})=0 \quad (19)$$

$$T_2=T_{22}=c_{12}u_{1,1}+c_{11}u_{2,2}-\lambda_{11}\theta=0 \quad (20)$$

$$T_4=T_{23}=\bar{c}_{44}u_{3,2}+\mathrm{e}_{15}\,\Psi_{,2}=0 \quad (21)$$

$$\phi=\Psi+\frac{e_{15}}{\varepsilon_{11}}u_3=0 \quad (22)$$

with

$$\bar{c}_{44}=c_{44}+\frac{e_{15}^2}{\varepsilon_{11}}u_3\ ,\ c_{66}=(\mathrm{c}_{11}-\mathrm{c}_{12})/2$$

III. DISPERSION EQUATIONS AND BOUNDARY CONDITIONS

Substitution of equations (10), (11) and (12) into equations (15), (16), (17), (18) and (19) leads to the following complex eigenvalue problem

$$\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & 0 & d_{24} \\ d_{31} & 0 & d_{33} & 0 \\ d_{41} & d_{42} & 0 & d_{44} \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix}=0 \quad (23)$$

where

$$d_{11}=\alpha_{11}+v^2\left(\frac{\Omega^2}{\omega^2}\right);\ d_{11}=i\,\frac{\rho\,c_0}{k\,\theta_0}v+\kappa k^2\left(\eta^2-1\right);$$

$$d_{12}=-d_{21}=i\,\eta\left(v_1^2-v_2^2\right);d_{13}=-d_{31}=2iv^2\left(\frac{\Omega}{\omega}\right);$$

$$d_{14}=-i\,\lambda_{11}/\rho k\ ;d_{41}=\lambda_{11}v\ ;d_{42}=-i\,\lambda_{11}\,\mathrm{v}\eta\,;$$

$$d_{33}=\alpha_{33}+v^2\left(\frac{\Omega}{\omega}\right)^2;d_{24}=-\eta\lambda_{11}/\dot{\rho}k$$

With

$$\alpha_{11}=v_2^2\eta^2-v_1^2+v^2\ ;\ \alpha_{22}=v_1^2\eta^2-v_2^2+v^2$$

$$\alpha_{33}=v_3^2(\eta^2-1)+v^2$$

Here (Ω/ω) is the rotation rate between the rotation of body and the wave frequency, and v is the phase velocity. The longitudinal wave speed v_1, and the shear wave speeds v_2 and v_3 are respectively defined by

$$v_1=(c_{11}/\rho)^{1/2}\ ;\ v_2=(c_{66}/\rho)^{1/2}\ ;v_3=(\bar{c}_{44}/\rho)^{1/2}$$

For simplification, we take

$$\alpha_{12}=\alpha_{21}=\left(v_1^2-v_2^2\right)\ ;\ \alpha_{13}=\alpha_{31}=2v^2\left(\frac{\Omega}{\omega}\right)$$

$$\beta_0=\left(\rho c_0/k^2\theta_0\right);\ \beta=\kappa k\left(\eta^2-1\right)/\rho v\ ,\ \alpha=-\lambda_{11}/\rho k$$

For equation (23) to have non-zero solutions, the determinant of the coefficient matrix must vanish

$$A=\begin{vmatrix}\alpha_{11} & i\alpha_{12}\eta & i\alpha_{13} & i\alpha\\ i\alpha_{21}\eta & \alpha_{22} & 0 & \alpha\eta\\ -i\alpha_{13} & 0 & \alpha_{33} & 0\\ -\alpha vk & i\alpha vk\eta & 0 & i\beta_0 vk+\beta\end{vmatrix}=0 \qquad (24)$$

This leads to the following polynomial equation of degree four for $\eta^{2.}$ The roots of this polynomial equation are generally complex.

$$\begin{aligned}&\alpha_{33}\left[i\alpha^2\left(\alpha_{22}-\eta^2\alpha_{11}-2\eta^2\alpha_{12}\right)+\right.\\&\left.\beta\left(\alpha_{11}\alpha_{22}-\eta^2\alpha_{12}^2\right)\right]=4v^4\left[\beta\alpha_{22}-i\alpha^2\eta^2\right]\left(\frac{\Omega}{\omega}\right)^2-\\&v^2\{\beta[\ \alpha_{22}\left(\alpha_{11}+\alpha_{33}\right)+\eta^2\alpha_{12}+v^2\alpha_{22}\left(\frac{\Omega}{\omega}\right)^2\ ]+\\&i\alpha^2\left[\left(\eta^2\alpha_{11}+\alpha_{33}+2\alpha_{12}-\alpha_{22}+v^2\alpha_{22}\left(\frac{\Omega}{\omega}\right)^2\right)\ \right]\}\left(\frac{\Omega}{\omega}\right)^2\end{aligned} \qquad (25)$$

The above dispersion equation (25) is fourth order in $\eta^{2,}$ and indicates that the frequency ω and the phase velocity v are generally dependents unless $\Omega = 0$, i.e., the surface waves are generally dispersive due to the rotation effect. We note that on the right-hand side of this dispersion relation, the first term results from the Coriolis force and the second term is due the centrifugal force. They depend of temperature coefficients, material properties and are of the same order in terms of the ratio Ω/ω which a small parameter. Because it is very difficult to resolve this polynomial equation algebraically, we assume now that it has four distinct complex roots and we later verify this assumption numerically. This leads to the following new representation for the wave solutions

$$u_{1,2,3}=\sum_{j=1}^{4}C_jA_{1,2,3}^{(j)}e^{k\eta j\,x_2}e^{ik(x_1-vt)} \qquad (26)$$

$$\theta=\sum_{j=1}^{4}C_jA_4^{(j)}e^{k\eta j\,x_2}e^{ik(x_1-vt)} \qquad (27)$$

$$\Psi=i\,C_5e^{k\,x_2}e^{ik(x_1-vt)} \qquad (28)$$

Where $A^{(j)}$ are the eigenvectors corresponding to the eigenvalues η^2_j. The complex magnitudes C_j and the real magnitude C_5 are to be determined. Substituting the equation (26), (27) and (28) into the boundary conditions (8) yields to

$$\sum_{j=1}^{4}[\ \eta_jA_1^{(j)}+i\,A_2^{(j)}\ ]C_j=0$$

$$\sum_{j=1}^{4}[\ i\,c_{12}A_1^{(j)}+c_{11}\eta_jA_2^{(j)}-\frac{\lambda_{11}}{k}A_4^{(j)}\ ]C_j=0$$

$$\sum_{j=1}^{4}[\ \bar{c}_{44}\eta_jA_3^{(j)}\ ]C_j+e_{15}C_5=0$$

$$e_{15}\sum_{j=1}^{4}A_3^{(j)}C_j+\varepsilon_{15}C_5=0\ ;\ \sum_{j=1}^{4}A_4^{(j)}C_j=0$$

Whish can be regrouped as follows

$$\begin{bmatrix}b_{11} & b_{12} & b_{13} & b_{14} & 0\\ b_{21} & b_{22} & b_{23} & b_{24} & 0\\ \bar{c}_{44}A_3^{(1)}\eta_1 & \bar{c}_{44}A_3^{(2)}\eta_2 & \bar{c}_{44}A_3^{(3)}\eta_3 & \bar{c}_{44}A_3^{(4)}\eta_4 & e_{15}\\ e_{15}A_3^{(1)} & e_{15}A_3^{(2)} & e_{15}A_3^{(3)} & e_{15}A_3^{(4)} & \varepsilon_{11}\\ A_4^{(1)} & A_4^{(2)} & A_4^{(3)} & A_4^{(4)} & 0\end{bmatrix}\times \qquad (29)$$

$$\left[C_1\,C_2\,C_3\,C_4\,C_5\right]^T=0$$

where

$$b_{1j}=\eta_jA_1^{(j)}+i\,A_2^{(j)}\ ,$$

$$b_{2j}=i\,c_{12}A_1^{(j)}+c_{11}\eta_jA_2^{(j)}-\frac{\lambda}{k}A_4^{(j)}$$

For equation (29) to have non-zero solutions, the determinant **B** of complex coefficients matrix must vanish

$$\begin{vmatrix}b_{11} & b_{12} & b_{13} & b_{14} & 0\\ b_{21} & b_{22} & b_{23} & b_{24} & 0\\ \bar{c}_{44}A_3^{(1)}\eta_1 & \bar{c}_{44}A_3^{(2)}\eta_2 & \bar{c}_{44}A_3^{(3)}\eta_3 & \bar{c}_{44}A_3^{(4)}\eta_4 & e_{15}\\ e_{15}A_3^{(1)} & e_{15}A_3^{(2)} & e_{15}A_3^{(3)} & e_{15}A_3^{(4)} & \varepsilon_{11}\\ A_4^{(1)} & A_4^{(2)} & A_4^{(3)} & A_4^{(4)} & 0\end{vmatrix}=0 \quad (30)$$

The real and imaginary components of the above complex determinant have a polynomials forms, and they depend in v and k. Therefor to vanish the equation (30) and determinate the phase velocity *v*, we have to verify that the real and/or imaginary parts must vanish.

IV. SURFACE WAVE SOLUTION

A. Non rotation case

As a special case we first determine the dispersion relations for non-rotating body with the poling direction along x_3 axis. In this case u_1 and u_2 are coupled with θ and u_3 is coupled with Φ, but the two groups do not couple each other. We denote first by $\bar{\eta}^2$ the non-rotation case eigenvalues. To determine the non-rotation values of the surface wave speeds and the eigenvalues, we set Ω = 0 in equation (25) and we obtain fourth order equation in $\bar{\eta}^2$, i.e.

$$\left\{\bar{\eta}^2 - 1 + \frac{v^2}{v_3^2}\right\}\left\{\bar{\eta}^2 - 1 + \frac{v^2}{v_2^2}\right\}\left\{\bar{\eta}^2 - \left[r_1 \pm \frac{\sqrt{2}}{2}\left(\left(r_2^2 + s_2^2\right)^{\frac{1}{2}} + r_2\right)^{\frac{1}{2}}\right] + i\left[s_1 \pm \frac{\sqrt{2}}{2}\left(\left(r_2^2 + s_2^2\right)^{\frac{1}{2}} - r_2\right)^{\frac{1}{2}}\right]\right\} = 0 \tag{31}$$

where

$$r_1 = \frac{v^2}{2v_1^2} - 1 \quad , \quad s_1 = \frac{\beta_0 - \alpha^2}{2\beta} \ ,$$

$$r_2 = \left(\frac{\beta_0 - \alpha^2}{2\beta}\right)^2 + \frac{v^4}{4v_1^4} \ ,$$

$$s_2 = \frac{4\alpha^2 v_1^2 - v^2(\beta_0 + \alpha^2)}{2\beta v_1^2}$$

One notes here that the two conjugates complex roots depend upon thermal coefficients and *v*. The two real distinct roots remain the same of the piezoelectric media. Each of the above four roots represents an elementary mode of the half space. For positives values of *k* and *v*, those roots which are real or complex must have positive real parts to ensure decay into the thermopiezoelectric media, while imaginary roots must also be positive for propagation away from the surface. In this case the phase velocity founded must verify the following conditions:

$$r_1 = \pm\frac{\sqrt{2}}{2}\left(\left(r_2^2 + s_2^2\right)^{\frac{1}{2}} + r_2\right)^{\frac{1}{2}} > 0$$

$$s_1 = \pm\frac{\sqrt{2}}{2}\left(\left(r_2^2 + s_2^2\right)^{\frac{1}{2}} - r_2\right)^{\frac{1}{2}} > 0 \tag{32}$$

$$v < v_1 \,; v < v_2$$

We note that the above inequalities are taken in compte numerically in the iterative procedure used for resolution, i.e., to plot the dispersion curves, we make sure that the Rayleigh wave velocity *v*, and the number wave *k* verify the above conditions.

In our numerical analysis, we choose the following eigenvectors for non-rotation case:

for $\eta = \bar{\eta}_{1,4}$:

$$A_1^{(1,4)} = \bar{\beta}^{(1,4)} \ ; \ A_2^{(1,4)} = i\,\bar{\eta}_{1,4}\bar{\beta}^{(1,4)} \ ; \ A_3^{(1,4)} = 0 \ ;$$

$$A_4^{(1,4)} = \alpha\left(1 + \bar{\eta}_{1,4}^2\right)$$

for $\eta = \bar{\eta}_2$:

$$A_1^{(2)} = i\ \bar{\eta}_2 \quad ; A_2^{(2)} = 1 \ ; \ A_3^{(2)} = 0 \ ; \ A_4^{(2)} = 0$$

for $\eta = \bar{\eta}_3$:

$$A_1^{(3)} = 0 \ ; \ A_2^{(3)} = 0 \ ; \ A_3^{(3)} = 1 \ ; \ A_4^{(3)} = 0$$

where $\bar{\beta}^{(1,4)} = i\,\beta_0 - \beta(\bar{\eta}_{1,4}^2 - 1)$

For non-trivial solution the determinant (30) must vanish. This leads to the following boundary condition decoupled equations

$$\left[\bar{c}_{44}\eta_3\varepsilon_{11} - e_{15}^2\right] = 0 \tag{33}$$

and / or

$$\left[1 + \eta_2^2\right]\left\{ i\,c_{12}\left[\bar{\beta}^{(4)}A_4^{(4)} - \bar{\beta}^{(1)}A_4^{(1)}\right] + i\,c_{11}\left[\eta_1^2\,\bar{\beta}^{(4)}A_4^{(4)} + \eta_4^2\,\bar{\beta}^{(1)}A_4^{(4)}\right] + \frac{\lambda_{11}}{k}\left[\left(A_4^{(4)}\right)^2 - \left(A_4^{(1)}\right)^2\right]\right\} = 0 \tag{34}$$

The first equation is real and permit to determine the non-rotation Bleustein-Gulyaev wave speed $\bar{v}_B$ and the second is complex and gives the non-rotation Rayleigh wave $\bar{v}_R$. We return now to study numerically the existence of phase velocity *v* (or of frequency *ω*) and their dependence upon temperature coefficients using PZT-6B as a model material whose relevant properties (Setter and Colla, 1993), (Xu, 1991) are listed below:

$c_{11} = 16.8 \times 10^{10}$ (N/m^2) , $c_{12} = 6.0 \times 10^{10}$ (N/m^2) ,

$c_{44} = 2.71 \times 10^{10}$ (N/m^2) , $c_{66} = (c_{11} - c_{12}) / 2$,

$e_{15} = 4.6$ (C / m^2) , $\varepsilon_{11} = 36\times 10^{-10}$ (F / m) ,

$\rho = 7600$ (Kg / m^3) , $c_0 = 420$ ($J\,Kg^{-1}\,K^{-1}$) ,

$\lambda_{11} = \lambda_{22} = 2.016 \times 10^6$ ($Pa\,K^{-1}$) ,

$\kappa_{11} = \kappa_{22} = 1.2$ ($W\,m^{-1}K^{-1}$) ,

$p_3 = 3.7\times 10^{-4}$ ($C\,K^{-1}m^{-2}$)

The wave $\bar{v}_B$ can be found explicitly by replacing the expression of $\bar{\eta}_3$ in the equation (33):

$$\bar{v}_B = \left(1 - \frac{e_{15}^4}{\varepsilon_{11}^2 \bar{c}_{44}^2}\right)^{0.5} v_3 < v_3 \quad (35)$$

We can notice, in this relation, the absence of pyroelectric and thermomechanical coefficients, i.e. there is no influence of thermo-mechanical coupling on this wave speed. We find here that the Bleustein-Gulyaev wave speed has a same value as in the piezoelectric half space studied by Jiang et al. [10]. Hence the velocity can be determined easily from equation (35) we find for our material $\bar{v}_B = 2316.5\, m/s$ corresponding to a single eigenvalue $\bar{\eta}_3^2 = 0.02124$.

The equation (34) allows to determine the Rayleigh wave $\bar{v}_R$ by replacing the eigenvalues $\bar{\eta}_1(v,k)$ and $\bar{\eta}_4(v,k)$ in the two correspondent's equations representing the real and imaginary components. This leads to

$$R\,\mathrm{e}(\bar{B}) = 4\bar{\beta}^2 v^4 k^2 + 4v_1^2 v^2\left(\bar{\beta}_0^2 - 2\bar{\beta}_0\bar{\alpha}^2 + \bar{\alpha}^4\right) - 16\bar{\beta}^2 v_1^2 k^2 = 0 \quad (36)$$

and/or

$$Im(\bar{B}) = 10\rho\left(\bar{\beta}_0 + \bar{\alpha}^2\right)v^2 - \bar{\beta}_0\left(c_{11} + c_{12}\right) + 4\lambda_{11}\bar{\alpha} + 4\lambda_{11}\bar{\alpha}^2 = 0 \quad (37)$$

where $\bar{\alpha} = \lambda_{11}/\rho$, $\bar{\beta}_0 = c_0/\theta_0$, $\bar{\beta} = \kappa/\rho$, and $\bar{B}$ is the determinant expressed as function of non-rotation eigenvalues $\bar{\eta}_j^2$.

By respecting the conditions in equation (32), we will verify the existence of the Rayleigh wave v_R depending only on complex roots $\bar{\eta}_1$ and $\bar{\eta}_4$. In this purpose, we replace the values of k and v obtained, respectively, from equations (36) and (37) in the expressions of the eigenvalues $\bar{\eta}_1^2$ and $\bar{\eta}_4^2$. Consequently, we find for our model material, for $\Omega = 0$, the Rayleigh wave speed $v_B = 2191$ *m/s.* Indeed, the existence of thermal coefficients in the above expressions let to that the Rayleigh waves will be disrupted by the thermic effect and increase or decrease relatively their dispersion.

B. Perturbation by rotation case

In this section we will study the existence of surface wave solutions when a thermo-piezoelectric body rotates. One is going to concentrate our discussion on the study of Rayleigh's wave v_R because of decoupling in the equation (34) and that the Bleustein-Gulyaev wave v_B depends only in η_3. In our numerical resolution, we use, for (j=1,...,4), the following eigenvectors:

$$A_1^{(j)} = \beta\alpha_{22}^{(j)}\left(1-\eta_j^2\right) - i\left(\beta_0\alpha_{22}^{(j)} - \alpha^2\eta_j^2\right)$$
$$A_2^{(j)} = \eta_j\left(-\alpha_{12}^{(j)}\beta_0 + \alpha^2 + i\,\alpha_{12}^{(j)}\beta\left(\eta_j^2 - 1\right)\right)$$
$$A_3^{(j)} = -\alpha_{13}^{(j)}\left(-i\,\alpha_{22}^{(j)}\beta\left(\eta_j^2-1\right) - \beta_0\alpha_{22}^{(j)} + \alpha^2\eta_j^2\right)/\alpha_{33}^{(j)}$$
$$A_4^{(j)} = \alpha\left(\eta_j^2\alpha_{12}^{(j)} - \alpha_{22}^{(j)}\right)$$

To determine the dependence of the surface wave speed upon the rotation, we use an iterative procedure: we let the rotation rate increase gradually from zero and solve equation (24) and (30) using iteration procedure. For a given real value of *k*, we choose a real trial $v_{0,}$ we determine the eigenvalues, the four roots of $\bar{\eta}_j^2\left(j=1,2,3,4\right)$ of the polynomial equation (24), solve (30) for the surface wave speed *v* and repeat these steeps until a prescribed is met $|v - v_0| < \varepsilon << 1$ or making the remaining determinant (30) vanish. Substituting the so- determined $\bar{\eta}_j^2$ and their corresponding eigenvectors results in a formally complex determinant, or equivalently two real determinants (the real and imaginary parts of the complex determinant) returns to the determination of *v* and *k*. The so-determined v, and *k* together with the given Ω/ω defines a dispersion curves.

V. RESULTS AND DISCUSSION

In this section we present a selection of numerical simulations which illustrate the effect of rotation, thermomechanical coupling, and preheating temperature on the linear dispersion spectra and the profiles versus depth of the longitudinal mechanical displacement component of generalized Rayleigh waves propagating along the x_1 direction for the thermopiezoelectric ceramic PZT- 6B. The dispersion curves below are plotted for different initial temperatures when the thermopiezoelectric body rotates for small or large ratios and with including or neglecting the centrifugal effects.

Fig.2 and Fig.3 illustrate, respectively, the variation of Rayleigh wave phase velocity as a function of large and small rotation ratios ($\Omega_{1,2,3}/\omega$) for material electroded surface about the three symmetry axis with including and neglecting the effect of centrifugal forces. Respectively, Fig.4 and Fig.5 show the variation of RW velocity for small rotation ratios ($\Omega_{1,3}/\omega$) and ($\Omega_{1,2}/\omega$) about the two symmetry axis x_1 and x_3 and x_1 and x_2 for different material initial temperatures with electrode surface (here, rotation rates are chosen arbitrarily (Ω/ω = 0.03) and (Ω/ω = 0.06)). Fig.6 represents RW velocity versus large rotation ratios ($\Omega_{1,2,3}/\omega$) about the three symmetry axis for the initial temperatures 30°C and 50°C with material electroded surface and including centrifugal forces. The changes of RW velocity function of preheating temperatures of the body (θ_0) about the three symmetry axis with the rotation ratios ($\Omega_{1,2,3}/\omega$), and taking the surface of the material electroded is illustrated on Fig.7. Fig.8 gives the RW velocity as a function of large rotation ratios ($\Omega_{1,2,3}/\omega$) about the three symmetry axis for material electroded surface and free charges with including the effect of centrifugal forces. The attenuation coefficient versus rotation rate (Ω_2/ω) about the x_2 axis for electroded and charge free surfaces is shown on Fig.9. Fig.10 shows the changes of x_1 normalized components of mechanical displacement as a function of depth (in wavelengths) for different rotation ratios (Ω/ω). From Fig.3, Fig.4 and Fig.5, we can detect an abnormal dispersion for the small rate ($\Omega/\omega < 0.033$), i.e., a RW velocity is increasing locally when the rotation rate increases. This abnormal dispersion is due to the piezoelectric, pyroelectric, and thermal coupling in the studied material. For the large rotation ratios ($\Omega/\omega < 0.04$), the wave becomes dispersive. We note also that the influence of centrifugal force on dispersion spectra may be significant on the range of rotation rate (≈12%). It is shown that the influence of preheating temperature, usually neglected in piezoelectric materials studies, on the dispersion spectra is significant. Consequently, the range of rotation ratios where the wave speed is dispersive can be controlled by choosing the suitable initial temperature. On the other hand, we can see that the wave speed decreases when the initial temperature increases. However, for the small ratios, the wave speed increases and the dispersion velocity decreases immediately when the centrifugal effect is taken into account. We observe that the effect of rotation rate on the penetration depth of the generalized Rayleigh wave is significant (Fig.10).

VI. CONCLUSION

In the present study, effects of rotation and temperature on the dispersion of the Rayleigh and Bleustein-Gulyaev waves over a thermopiezoelectric half-space are studied and commented. Theoretical and numerical analysis show that the Rayleigh and Bleustein Gulyaev waves are generally dispersive under effect of rotation. Certain abnormal dispersion for small rotation ratios exists, i.e., a wave velocity increasing locally when the rotation ratio increases. The effects of Coriolis and centrifugal forces are of the same order and that the influence of the centrifugal force is significant. The simulations for the studied thermopiezoelectric material, are shown that the thermoelastic and pyroelectric couplings influence on the Rayleigh wave dispersion have not to be neglected and that the initial temperature of the material contribute greatly on the dispersion of the Rayleigh wave. The range of rotation ratios where the wave speed is dispersive can be controlled by choosing the suitable initial temperature.

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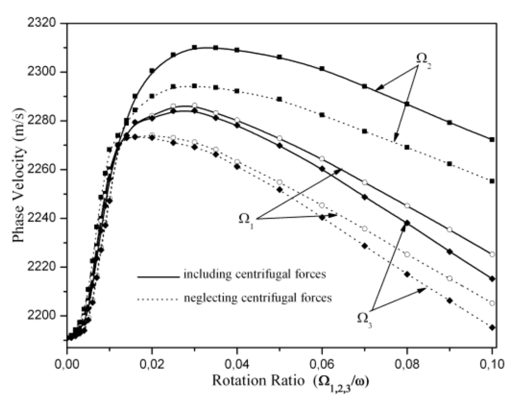

Fig. 2. Variation of RW phase velocity *vs*. Large rotation ratios ($\Omega_{1,2,3}/\omega$) about three axis with electroded surface and including/neglecting centrifugal forces.

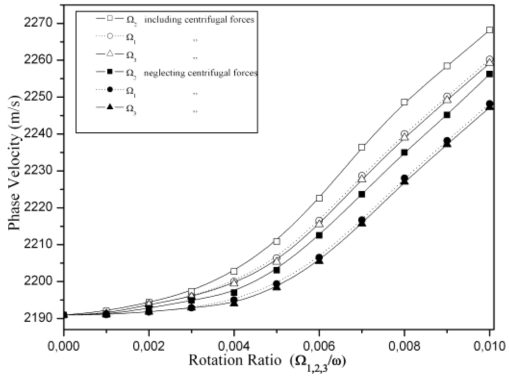

Fig.3. Variation of RW phase velocity *vs*. Small ratios ($\Omega_{1,2,3}/\omega$) for electroded surface and the three axis with including/neglecting centrifugal forces.

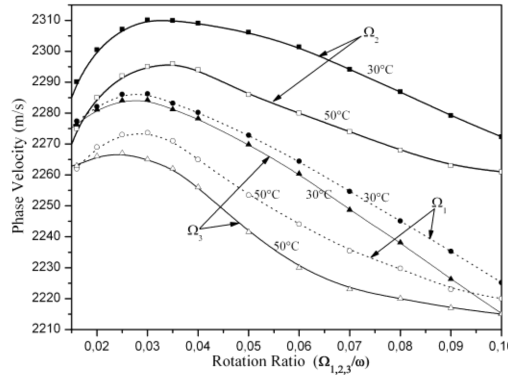

Fig.6. Variation of RW phase velocity *vs*. Large ratios ($\Omega_{1,2,3}/\omega$) about three axis for the initial temperatures 30°C and 50°C, electroded surface and including centrifugal forces.

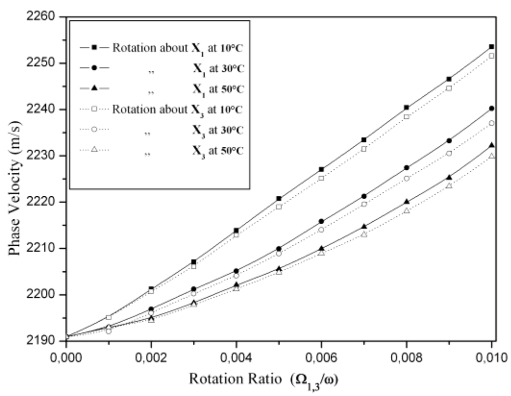

Fig.4. Variation of RW phase velocity *vs*. Small ratios ($\Omega_{1,3}/\omega$) about the x_1 and x_2 axis for different initial temperatures with electroded surface.

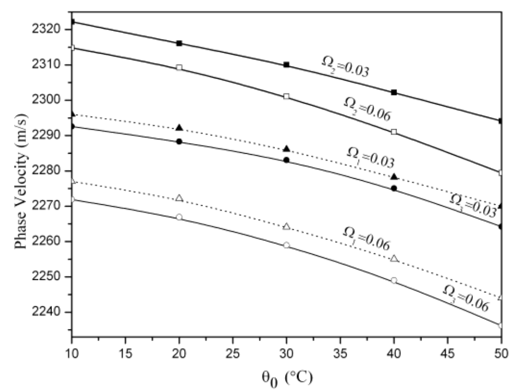

Fig.7. Variation of RW phase velocity *vs.* Preheating temperatures of the body (θ_0) about three axis with ratios ($\Omega_{1,2,3}/\omega$) and electroded surface.

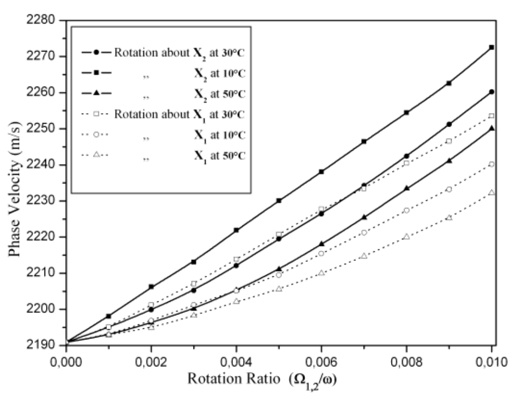

Fig.5. Variation of RW phase velocity *vs*. Small ratios ($\Omega_{1,2}/\omega$) about the x_1 and x_2 axis for different initial temperatures with electroded surface.

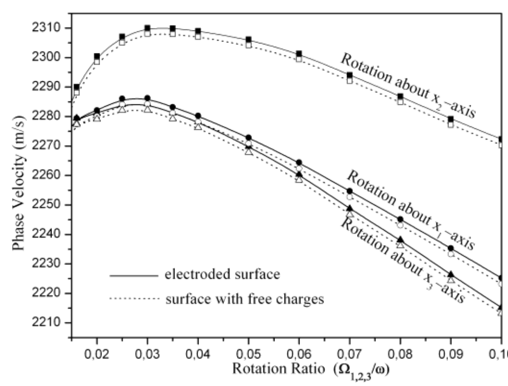

Fig. 8. Variation of RW phase velocity *vs*. Large ratios ($\Omega_{1,2,3}/\omega$) about three axis for electroded/free charges surface with including centrifugal forces.

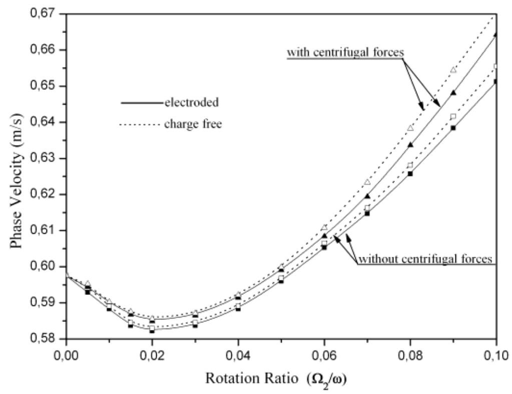

Fig. 9. Variation of attenuation coefficient *vs*. Rotation rate (Ω_2/ω) about the x_2 axis.

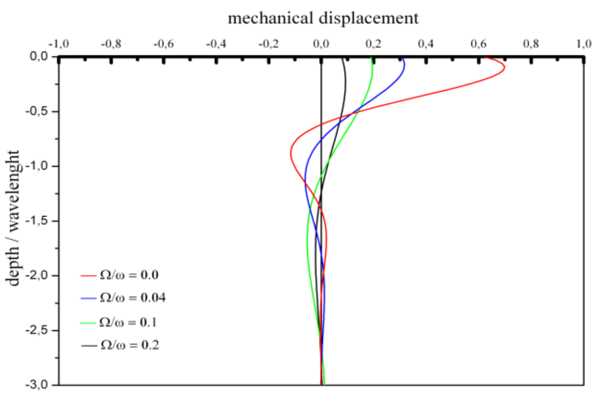

Fig. 10. Variation of x_1 normalized components of mechanical displacement *vs*. depth (in wave-lengths) for different rotation ratios (Ω/ω).