Correlation Analysis of Transient Vibration Signals for the Location Template Matching Method

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Abstract-- **The location template matching (LTM) method is a source localization method and is often applied to structural health monitoring and large-scale human–computer interface (HCI) systems. Despite the fact that the cross-correlation between measured signals resulting from impacts at different points on a structure is a key element, the essential properties of cross-correlation for implementing the LTM method have not been studied yet. In this paper, two practical properties of the cross-correlation between two transient vibration signals are investigated: the number of vibration modes contained in a measured signal and variation of the correlation coefficient value with respect to the distance between two neighboring impact points. Two interesting results are obtained through extensive computer simulations and experiments. First, if the number of vibration modes contained in a measured signal is more than three or four, it is sufficient for the LTM method. This is a useful result because any unnecessarily high-frequency components can be filtered out to increase the signal-to-noise ratio of a measured signal. Second, the correlation coefficient value does not decrease monotonically as the distance between two neighboring impact points increases, but it fluctuates considerably, thereby indicating that there is an optimum grid spacing range for the LTM method.**

Index Term-- **Correlation coefficient, Cross-correlation, Location template matching (LTM), Vibration modes, Transient signals.**

1. INTRODUCTION

The location template matching (LTM) method is a source localization method that is especially useful when a structure is subjected to an impulsive excitation. A typical application of the LTM method is a large-scale human–computer interface (HCI) system where the location of a finger tap on a touch panel needs to be identified [1-3]. The LTM method is simple to use, does not require the knowledge of the dynamic behavior of a structure, and usually requires only one measurement sensor. The principle of the LTM method is briefly described below. As depicted in Figure 1, many grids are defined on a plate and a measurement sensor is arbitrarily fixed on the plate.

An impact is applied on each intersecting point of the grids (p_1, p_2, …, p_n); the response signal due to each impact is then measured and stored as a reference signal in a database. It should be noted that the measured response signals are transient in nature, and the database containing the reference signals is called the location template. After the database has been constructed, if a new impact is applied arbitrarily on the plate, the newly measured vibration signal is compared with all the reference signals in the location template. Then, the impact location is estimated by finding the best-matching reference signal in the location template. If the new impact is not applied exactly at one of the intersecting points, the LTM method will localize the closest intersecting point.

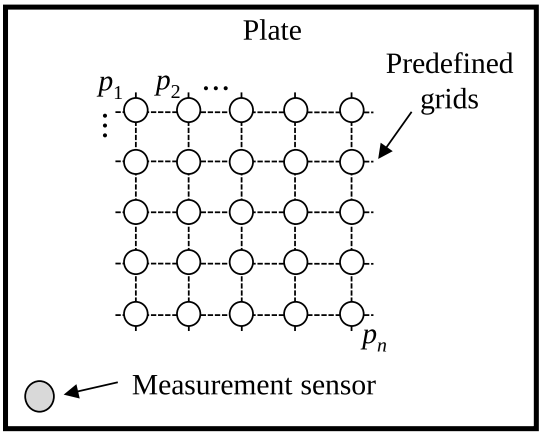

Fig. 1. A plate with predefined grids and a measurement sensor

For the purpose of comparison between measured and reference signals, the correlation coefficient (or the normalized cross-correlation function) is most widely used as a measure of similarity because of its simplicity. The correlation coefficient is defined as

$$\rho_{xy} = \frac{Cov(x, y)}{\sigma_x \sigma_y} \tag{1}$$

where $Cov(x, y)$ is the covariance between signals $x(t)$ and $y(t)$, and σ_x and σ_y are the standard deviations of the corresponding signals. Although the correlation-based LTM method is simple to use, its accuracy in localizing an impact source is not high, especially when the grid spacing (the distance between neighboring intersecting points in Figure 1) is small. Several other sophisticated techniques have been developed to increase accuracy, such as infinite impulse response filter modeling [4], a linear predictive coding-based algorithm [5], the time-frequency method [6], a method using multiple measurement sensors [7], and a group-delay-based method [8]. However, in practice, none of these methods alone may be successfully applicable, especially for a large plate with hundreds of reference signals, owing to their complexity and relatively longer computational time. Thus, for a large human-computer interface, a combination of the correlation coefficient and one of the aforementioned techniques may be suitable. Such a combination can be implemented in the following way. The interface panel is first divided into several smaller regions. The correlation-based LTM method is then applied to identify the region in which the impact source falls; other sophisticated methods are subsequently applied to the appropriate region.

Accordingly, understanding the properties of the correlation coefficient between two transient vibration signals is necessary for applying the LTM method successfully. In this paper, the following two practically important properties are investigated: the effect of the number of vibration modes contained in the measured signals, and the value of the correlation coefficient with respect to the grid spacing (i.e., the distance between two impact points on a structure). These properties are examined on the basis of computer simulations and experiments conducted using a simple cantilever beam structure. In the following section, extensive simulation results based on the theoretical model of a cantilever beam are presented. Then, in section 3, experimental results to be used for verifying the simulation results are presented. Finally, concluding remarks are presented in section 4.

2. CORRELATION ANALYSIS

2.1 Cantilever beam model and transient vibration signals

In this section, a brief description of transient vibration signals used for correlation analysis is given. The signals are generated on the basis of the theoretical model of a cantilever beam shown in Figure 2. The dimensions of the cantilever beam are 300mm × 20mm × 5mm. Its material properties are as follows: density, 1299 kg/m^3; elastic modulus, 2×10^9 N/m^2; and loss factor, 0.01. These dimensions and material properties are chosen to be identical to those of the real cantilever beam used for experiments discussed in section 3. In Figure 2, p_i denotes the impact points; the measurement point is chosen to be at 10mm from the beam tip.

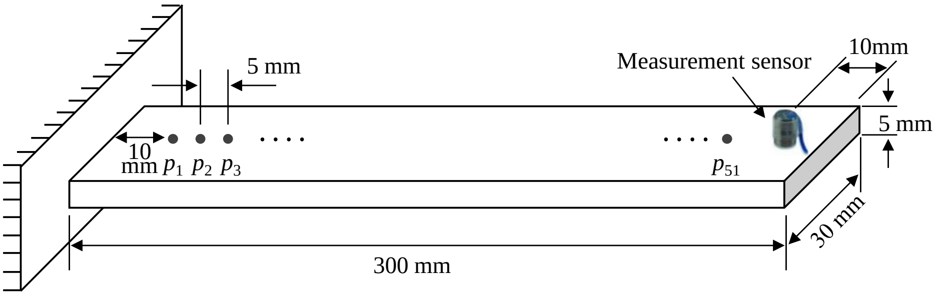

Fig. 2. Cantilever beam model

The frequency and impulse response functions for the cantilever beam can be expressed as suggested in an earlier study [9]; the expressions are given below:

$$G(j\omega) \approx \sum_{n=1}^{N} \frac{\phi_n(x_s)\phi_n(x_a)}{M_n\left[(\omega_n^2-\omega^2)+j2\zeta\omega_n\omega\right]} \tag{2}$$

$$g(t) \approx \sum_{n=1}^{N} \frac{\phi_n(x_s)\phi_n(x_a)}{M_n\omega_n\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n\sqrt{1-\zeta^2}\,t \tag{3}$$

where M_n is modal mass, ζ is damping ratio, and $\phi_n(x)$ is the mode shape function. Moreover, x_s and x_a are the locations of the measurement and excitation points on the beam, respectively. Impulse response signals are generated using Eq. (3) for each excitation at point p_i, and these are used as transient vibration signals in the correlation analysis. A total of 51 excitation points (p_1, p_2, …, p_{51}) are considered in the analysis, the distance between any two points being 5 mm. The number of modes contained in a signal is controlled by N in Eq. (3). As an example, the frequency and impulse response functions for $N = 9$ and excitation at p_1 are shown in Figure 3(a) and 3(b), respectively.

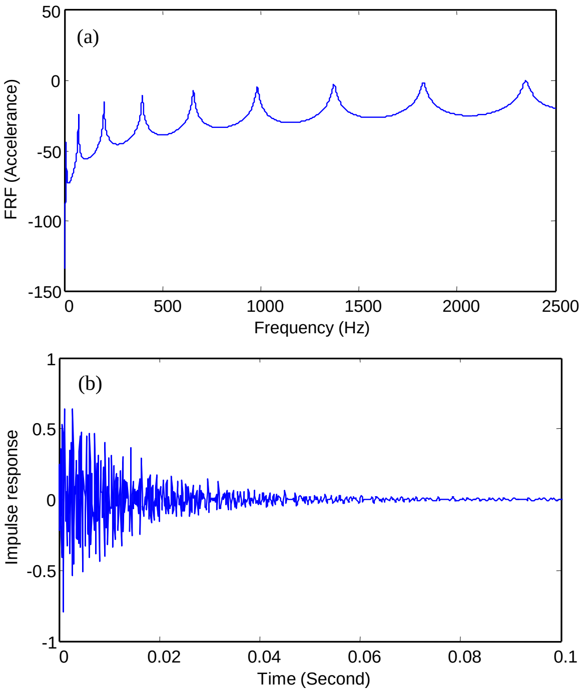

Fig. 3. Response to excitation at p_1 (10 mm from the root of the beam): a) frequency response function and b) impulse response function

2.2 Effect of number of vibration modes

The required number of vibration modes contained in a signal is an important factor for implementing the LTM method effectively. If many vibration modes are necessary, the required sampling rate of the measurement must be high. This implies more data to handle, and thus, more computational time is needed. Moreover, high-frequency components are generally prone to measurement noise. Therefore, a low number of vibration modes is highly preferable. However, if the number of vibration modes is too small, measured responses at different excitation points may not be distinguishable on the basis of correlation coefficients. For example, if there is only one vibration mode, the impulse response functions corresponding to two different excitation points can be easily written using Eq. (3) as $g_1(t) \approx A_1 e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$ and $g_2(t) \approx A_2 e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$. These equations are simply scaled versions of each other, i.e., the only difference between them is the magnitude. In this case, the correlation coefficient given by Eq. (1) becomes unity, i.e.,

$$\rho_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} = \frac{E\left[g_1(t) g_2(t)\right]}{\sqrt{E\left[g_1^2(t)\right] E\left[g_2^2(t)\right]}} = \frac{A_1 A_2 E\left[e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t\right]}{A_1 A_2 E\left[e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t\right]} = 1 \tag{4}$$

where the operator $E[\ ]$ denotes expectation or ensemble average. This result indicates that a reasonable number of vibration modes must be contained in the measured signals to distinguish them from each other; yet, a large number of vibration modes should be also avoided.

An analytic expression such as Eq. (4) is generally not obtainable for a high number of modes. Even if an analytic form is derived for a particular number of modes, it may be too complicated to comprehend. Thus, in this section, computer simulations are described that are used to examine the effect of the number of modes on the correlation coefficient.

Two excitation points p_5 and p_{25} are chosen on the beam shown in Figure 2; they are located at distances of 30 mm and 130 mm, respectively, from the root of the beam. The corresponding impulse response signals are generated using Eq. (3). The value of the correlation coefficient between the impulse response signals is then calculated for the number of vibration modes from $N = 1$ to $N = 9$. The results are shown in Figure 4.

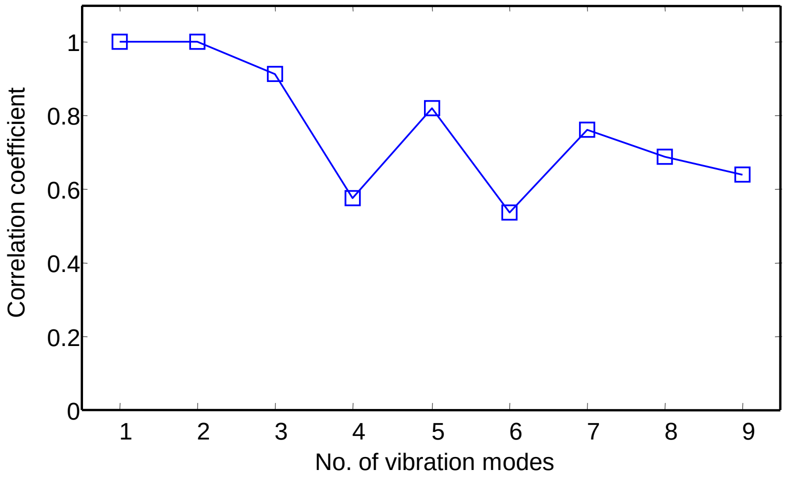

Fig. 4. Values of correlation coefficient with respect to the number of vibration modes contained in impulse response signals

As expected from Eq. (4), the correlation coefficient value is highest (i.e., 1) when there is only one vibration mode. The correlation coefficient shows a decreasing tendency as the number of vibration modes increases, though it does not decrease monotonically. If the number of vibration modes is greater than three, the correlation coefficient value is relatively very low (<0.8) compared with those in the cases of a lower number of modes.

Because any two impact points are well separated (100 mm apart), if the LTM method is applicable, the value of the correlation coefficient between two corresponding impulse response signals must be low. Thus, it may be concluded that values greater than three or four for the number of vibration modes may be sufficient for the LTM method, although a larger number of modes may result in greater accuracy.

2.3 The effect of the grid spacing

For the LTM method, the grid spacing (or the distance between neighboring impact points in Figure 1) is the most important element. If the grid spacing is too small, the correlation coefficient between two impulse response signals corresponding to neighboring impact points can be too high to distinguish between the signals. On the contrary, it may be expected that the correlation coefficient value decreases as the distance between two impact points increases. Although this may be a reasonable proposition, it is necessary to examine it in detail in order to properly use the correlation-based LTM method. Thus, in this section, the relation between the correlation coefficient value and the distance between two impact points is investigated.

As before, all the signals are generated using Eq. (3) with the number of vibration modes $N = 9$. Excitation point p_{19} (located at a distance of 100 mm from the root of the beam) is chosen as the reference point, and the corresponding impulse response signal is chosen as the reference signal. A total of 299 impact points 1 mm apart from each other are selected between 1 mm and 299 mm from the root of the beam. Impulse response signals resulting are then generated by impacts at these points. They are then compared with the reference signal on the basis of the correlation coefficient. The resulting correlation coefficients are shown in Figure 5.

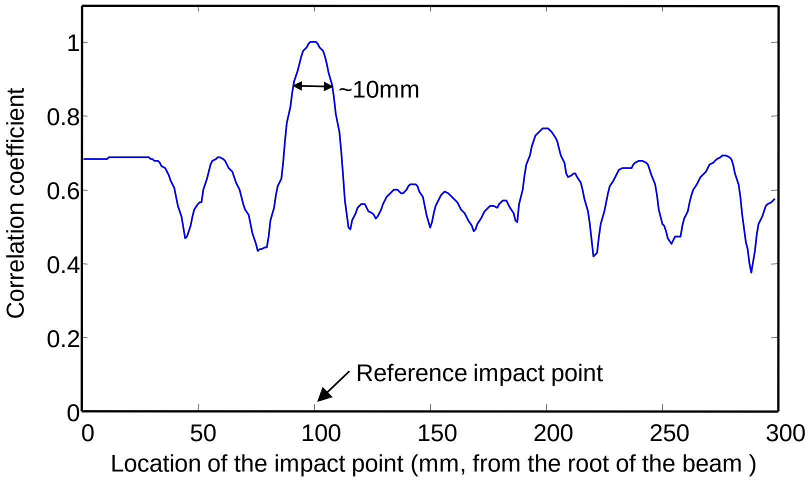

Fig. 5. Correlation coefficients with respect to the distance between two impact points; the reference impact point is located at a distance of 100 mm from the root of the beam

In Figure 5, an interesting property can be observed – the correlation coefficient value does not decrease uniformly as the distance between two neighboring impact points increases, but it fluctuates irregularly, forming the main robe at the reference impact point and many side robes. This implies that a large distance between impact points does not guarantee a low correlation coefficient value. In other words, for the LTM method, wrong classification of the impact location is possible if the grid spacing is too large. It should be noted that the maximum value of the side robes is close to 0.8. Consequently, the grid spacing must be chosen such that the correlation coefficient between the signals corresponding to two neighboring impact points must be well above 0.8 but not close to unity. The correlation coefficient value satisfying this condition may be chosen as about 0.9 in this particular example. The width of the main robe at the value of 0.9 is approximately 10 mm, as shown in Figure 5. Thus, the optimal grid spacing may be regarded as 10 mm, i.e., the relative length is 1/30 with respect to the beam length.

Similar results are found for different reference impact points. As an example, Figure 6 shows the correlation coefficients when excitation point p_{29} (located at a distance of 150 mm from the root of the beam) is chosen as the reference point. In this figure, although the maximum value of the side robes is lower than that in Figure 5, the optimal grid spacing is found to be equal, i.e., a relative length of 1/30 with respect to the beam length may be regarded as the optimum distance between two impact points for the correlation-based LTM method.

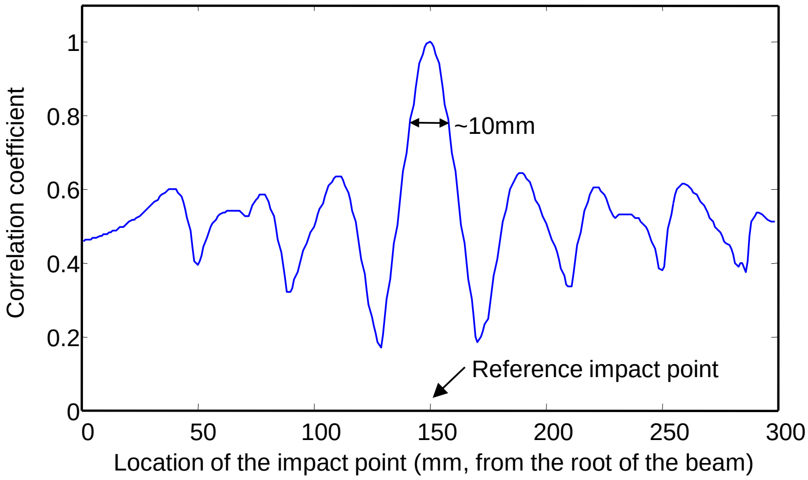

Fig. 6. Correlation coefficients with respect to the distance between two impact points; the reference impact point is located at a distance of 150 mm from the root of the beam

3. EXPERIMENTS

3.1 Description of experiments

In order to verify the results of the correlation analysis described in the previous sections, extensive experiments are performed using a plastic cantilever beam shown in Figure 7. This cantilever beam has the same material properties and dimensions as the one used for the computer simulations described in section 2. Moreover, the same excitation and measurement points are used. That is, a measurement accelerometer sensor (Model M352C66, PCB Piezotronics) is attached at a distance of 10 mm from the beam tip, and a total of 51 excitation points (p_1, p_2, …, p_{51}) 5 mm apart from each other are considered, e.g., p_1 and p_{51} are located at distances of 10 mm and 260 mm, respectively, from the root of the beam. A miniature impact hammer (Model 086E80, PCB Piezotronics) is used to excite the beam at these specified impact points, and the impact force signal and acceleration response signal are measured with a sampling rate of 5 kHz. The impact hammer has a sensitivity of 22.5 mV/N with a non-linearity less than 1%, and the accelerometer has a sensitivity of 100 mV/g with a non-linearity less than 1% in the frequency range between 0.5 Hz to 10 kHz.

Accordingly, a total of 51 sets of frequency and impulse response functions are obtained. As an example, the frequency and impulse response functions for excitation at p_1 (10 mm from the root of the beam) are shown in Figure 8(a) and 8(b), respectively. Upon comparing Figure 8(a) with Figure 3(a), it is noted that the experimental frequency response function has more vibration modes than that based on the theoretical cantilever beam model, especially in the high-frequency region. This may be due to torsional vibration modes that are not considered in the theoretical model. Nevertheless, it is demonstrated that the experimental beam has a sufficient number of vibration modes for the correlation analysis.

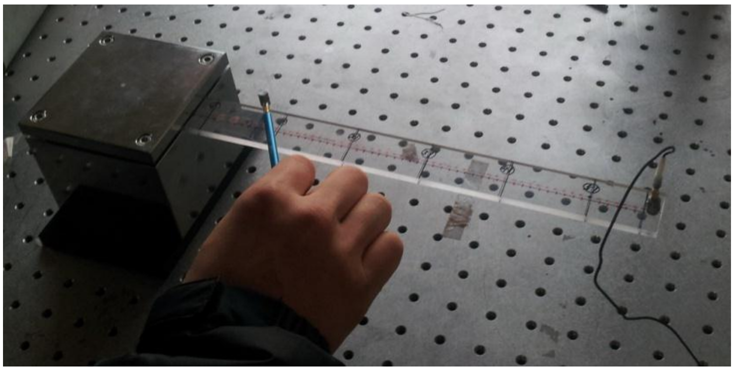

Fig. 7. Experiments with a miniature impact hammer

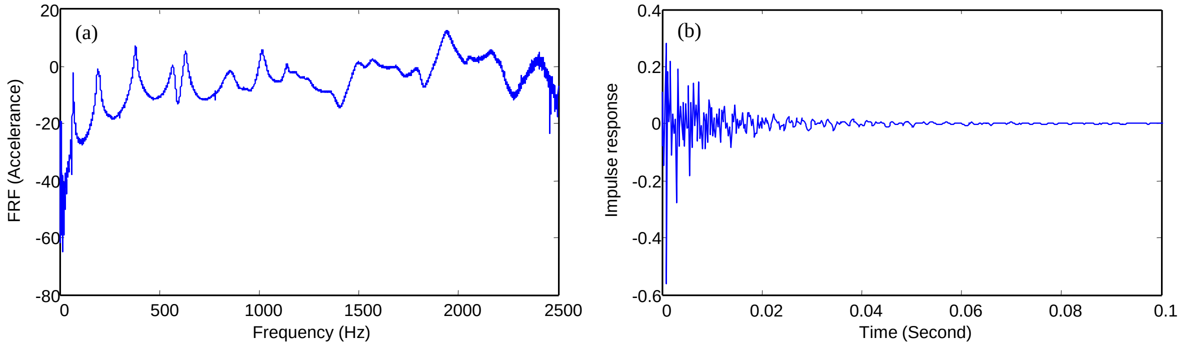

Fig. 8. Response to excitation at p_1 (10 mm from the root of the beam): a) frequency response function and b) impulse response function

3.2 Effect of grid spacing

Herein, the effects of the number of vibration modes and grid spacing are investigated by adopting the same procedures as those used in the computer simulations. First, in order to examine the effect of the number of vibration modes, the experimental impulse response functions are filtered through an appropriate low-pass filter the cut-off frequency of which covers the specified number of modes only.

Two excitation points p_5 and p_{25} are chosen, and the value of the correlation coefficient between the corresponding impulse response signals is calculated for the number of vibration modes from 1 to 9. The results are shown in Figure 9. Unlike the simulation results, the correlation coefficient value is not unity when there is only one vibration mode. This may be due to the fact that the effect of higher modes cannot be completely removed by the low-pass filter. Apart from this, results similar to the simulation ones are obtained; the decreasing tendency of the correlation coefficient as the number of vibration modes increases is also observed. If the number of modes is greater than three, the correlation coefficient value is relatively very low (<0.6) compared with those in the cases of a lower number of modes. This result confirms that values greater than three or four for the number of vibration modes may be sufficient for the correlation-based LTM method.

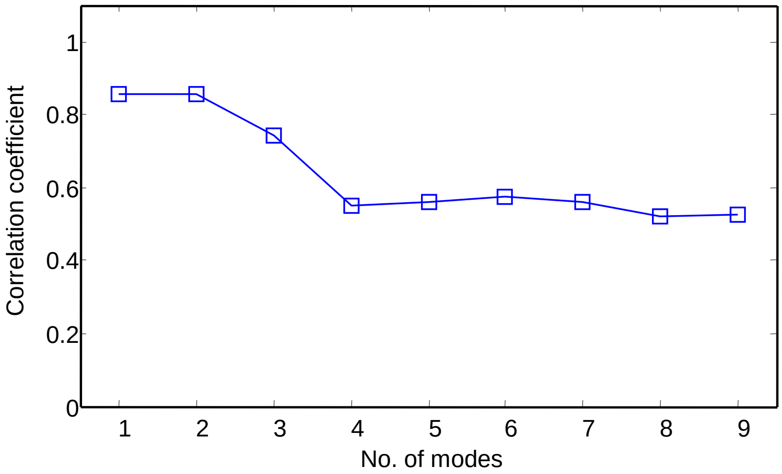

Fig. 9. Correlation coefficients with respect to the number of vibration modes contained in impulse response signals

Next, the effect of the grid spacing is examined, i.e., the correlation coefficient value with respect to the distance between two neighboring impact points is investigated. In this case, a low-pass filter is not used. The impulse response function due to excitation at point p_{19} (located at a distance of 100 mm from the root of the beam) is chosen as the reference signal. Then, the impulse response function due to each impact at the specified points (a total of 51 impact points) is compared with the reference signal. The resulting correlation coefficients are shown in Figure 10.

As shown in Figure 10, the results are similar to the simulation ones, i.e., the correlation coefficient value does not decrease uniformly as the distance between two neighboring impact points increases, but it fluctuates irregularly, forming the main robe at the reference impact point and many side robes. This confirms the simulation results, i.e., there is an optimum grid spacing range for the LTM method; further, the optimum grid spacing may be chosen as the width of the main robe, where the width is measured at the midpoint between the maximum values of the main robe and side robes. Thus, in this case, the optimum grid spacing may be chosen as 20 mm, i.e., the relative length is 1/15 with respect to the beam length. Although this value is greater than that suggested by the simulation results, taking into account measurement noise and the presence of different vibration modes, it may be considered that overall properties suggested by the experimental results agree well with those suggested by the simulation results.

Similar results are observed for other reference impact points. As an example, Figure 11 shows the correlation coefficients when excitation point p_{29} (located at a distance of 150 mm from the root of the beam) is chosen as the reference point. This further confirms that there is an optimum grid spacing range for the correlation-based LTM method. Although the optimum grid spacing may be different depending on the structure being used, the results presented in this paper suggest that the grid spacing must be chosen carefully on the basis of extensive correlation analysis before implementing the correlation-based LTM method.

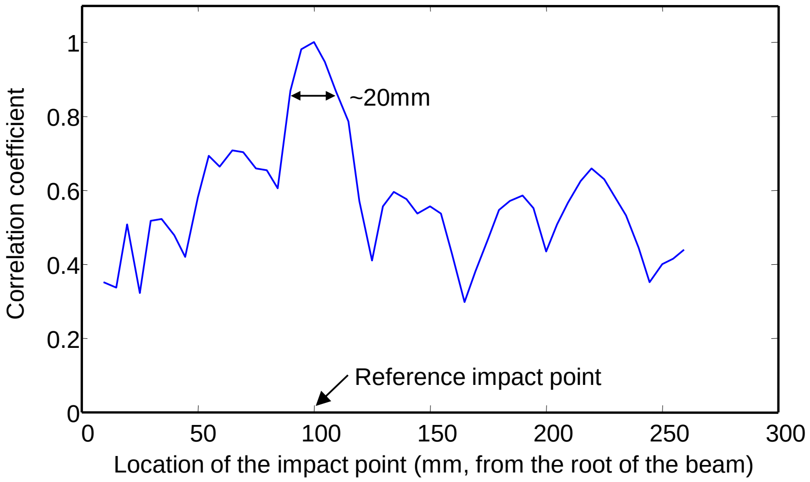

Fig. 10. Correlation coefficients with respect to the distance between two impact points; the reference impact point is located at a distance of 100 mm from the root of the beam

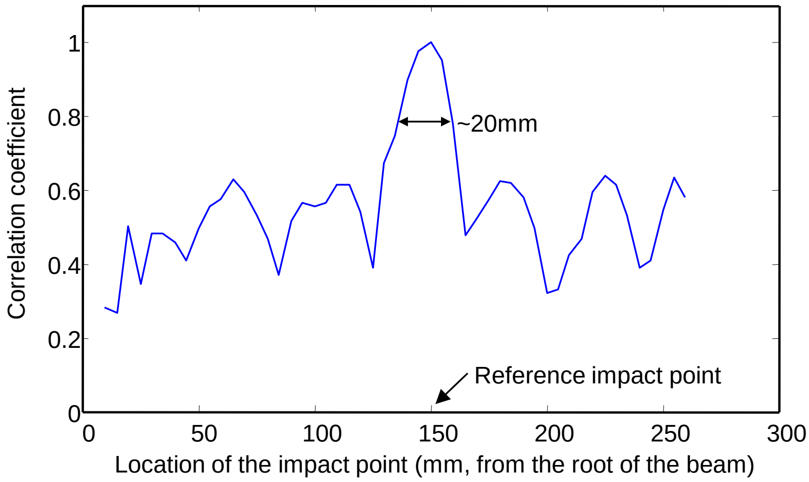

Fig. 11. Correlation coefficients with respect to the distance between two impact points; the reference impact point is located at a distance of 150 mm from the root of the beam

4. CONCLUSIONS

In this paper, two properties of correlation coefficients between two transient vibration signals are investigated; these properties are practically important for the correlation-based LTM method. The effect of the number of vibration modes contained in a measured signal is first considered; the variation of the correlation coefficient value with respect to the grid spacing is then examined. A few interesting results are obtained through extensive computer simulations, and these are verified by experiments.

If the number of vibration modes contained in a measured signal is more than three or four, it may be sufficient for the LTM method. This is a practically useful result because the lower the number of modes the lower the sampling rate, the lower the sampling rate the lower the amount of data to handle, and hence, a computationally efficient algorithm can be constructed. Moreover, because high-frequency components are prone to measurement noise, the signal-to-noise ratio of a measured signal can be improved by filtering out any unnecessarily high-frequency components.

With regard to the effect of the grid spacing, the correlation coefficient does not simply decrease as the distance between two neighboring impact points increases, but it fluctuates rather irregularly, thereby forming a main robe at the reference impact point and many side robes. This implies that there is an optimum grid spacing range for the correlation-based LTM method. Because the optimum grid spacing can be different for different structures, there is no easy way to find this analytically. In particular, the experimental results suggest that the grid spacing must be chosen carefully on the basis of extensive correlation analysis before implementing the correlation-based LTM method.

ACKNOWLEDGEMENTS

This research was supported by the Basic Science Research Program through a National Research Foundation of Korea (NRF) grant funded by the Ministry of Education (2013R1A1A4A01005104).

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