A Suggested Analytical Solution for Vibration of Honeycombs Sandwich Combined Plate Structure

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***Abstract*— In this work, a suggested analytical solution for vibration analysis of honeycombs sandwich combined plate is presented. The differential equation of motion for the vibration analysis of honeycombs sandwich combined Plate is solved to evaluate the natural frequency of the plate with different design parameters . The analytical results are first calculated using the mechanical properties of honeycomb structure such as modulus of elasticity, modulus of rigidity, Poisson's ratio, and density of honeycomb structure, then finding the effect of honeycomb structural properties on the natural frequency of combine sandwich plate. The results are the natural frequencies of combine sandwich plate with different honeycomb structural dimensions effect as, length, height, thickness, and angle of regular hexagonal honeycomb structural effect. In addition to, the effects of thickness of combined plate , thickness of honeycomb structural , aspect ratio of plate, and other parameters, on the natural frequency of the plate. A comparison between analytical results obtained theoretically by solving the general equation of motion for sandwich combine plate with honeycomb structure and those evaluated with other researchers gave a good agreement, where the largest discrepancy percentage was about (2 %).**

Index Term-- **Honeycombs Sandwich,Vibration, plate**

1. INTRODUCTION

Composite materials used today are often in the form of a sandwich construction panel built up by two thin skins, also called the facings, separated by a lightweight core. The core helps to increase the moment of inertia such that the structure becomes efficient for resisting bending , twisting and buckling loads. This is why sandwich panels are being used in applications where weight saving is critical, for instance in aircraft and in portable structures Sandwich designs are also used by nature itself for instance in a human skull and in plants, **[1]**.

The honeycomb panels are aimed to form a structure which have a light weight cellular structure with high strength and stiffness/weight ratio.This is useful in carrying both inplane and out of plane loading in an effective way.The sheets can be manufactured from metals of from fibre reinforced composite materials. These types may be used in airspace applications due to their excellent properties in strength and high strength/weight ratio in addition to the low thermal conductivity [2].

Many researches were conducted to investigate the vibration and dynamic of honeycombs panels and honeycombs sandwich combined plate, as,

Harish R and Ramesh S Sharma [2]

Investigated numerically using the finite element technique and experimentally the aluminum honeycomb sandwich panels subjected to impulsive loading with different boundary conditions viz. C-F-F-F, C-F-C-F .The modal analysis was conducted in this research.Different design parameters were studied such as the effect of core height on the fundmental natural frequencie.

Jeom Kee Paik et. al. [3], presented study to investigate the strength characteristics of aluminium sandwich panels with aluminium honeycomb core theoretically and experimentally. A series of strength tests were carried out on aluminum honeycomb-cored sandwich panel specimen in three point bending, axial compression and lateral crushing loads.

C. W. Schwingshackl et. al. [4], examined several available analytic and experimental methods to determine the orthotropic material properties of honeycomb. Fifteen published sets of simple equations for the material properties were reviewed and their values calculated for a specific honeycomb aluminium core.

Vitaly Skvortsov et. al. [5], presented the elastic response of composite sandwich panels to local dynamic loading. The plane and axisymmetric formulations are considered; no overall bending is assumed. The governing equations are derived using the static Lamés equations for the core and the plate Kirchoff-Love dynamic theory for the faces.

A.Boudjemai et. al. [6],

presented a multidisciplinary investigation of satellite honeycomb structures by adopting Clamped-Free boundary conditions for the simulation of the satellite structure.The models were tested experimentally and analysed numerically using the finite element technique .

Jayasree Ramanujan et al. [7], analysed the vibration analysis of sandwich panels and a comparison of natural frequencies using various theories. A simple software has been developed for calculating the natural frequency of panels. The software was validated by comparing the natural frequency using the software package NISA and ANSYS.

G. Sakar and F. C. Bolat [8],

Investigated the vibrations of aluminum honeycomb structures. The natural frequencies and mode shapes were obtained experimentally and numerically. Different models of the honeycombs sandwich panels were fabricated for the Clamped-Free boundary conditions and the effects of upper and lower sheet thicknesses ,core material thickness , cell angle , cell diameter and foil thickness on natural frequencies and mode shapes were investigated. A numerical technique was employed using ANSYS package,

In this research , the natural frequencies of honeycombs sandwich plate with different Regular Hexagonal Honeycomb dimensions effect were evaluated and compared with those obtained numerically adopting finite element method using ANSYS Ver. 14.

2.THE SUGGESTED ANALYTICAL SOLUTION

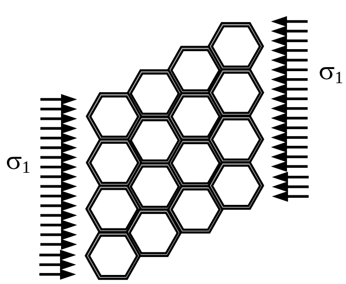
The suggested analytical solution of vibration analysis of honeycomb plate included the determination of the mechanical properties of honeycomb structure, and then, evaluation the equivalent stresses and the natural frequency of with effect of honeycomb structural parameters .

2.1 Mechanical Properties of Honeycomb Materials

The cells in a honeycomb-structure are usually hexagonal, but they can also be triangular or square or rhombic. The honeycombs can be made of different types of materials, such as metals, wood, polymer and ceramics or a combination of two or more of these materials.

Investigating the in-plane properties (transverse directions) demonstrate the failure mechanism and deformations of the cellular solid structures i.e defining the plane properties such as those shown in Fig. 1 .The stiffness and strength in x-y plane.

If the hexagonal honeycomb is regular (all angles θ are 30^o, and wall thicknesses are equal as (Fig. 1.), then the in-plane properties are isotropic. This means that the in-plane properties of the honeycomb can be described by only two independent elastic modules, for instance by Young's modulus Fig. 2 shows a hexagonal honeycomb regular structure in which the angles are equal (30^0) with constant wall thickness. This results in using in-plane properties of the honeycomb structures such as modulus of elasticity E and modulus of rigidiy G **[1]** as two independent properties.



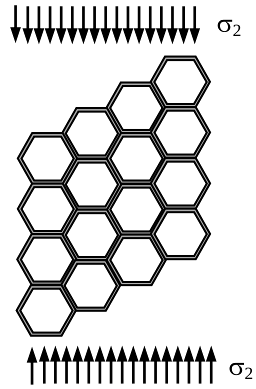


Fig. 1. Honeycomb Structure Loaded in x or y-direction, [1].

If the honeycomb cellular structure is not regular, the in-plane properties are used with four modules (e.g. E_{h1}, E_{h2}, G_{h12}, and ν_{h12}). If the hexagonal honeycomb has a low relative density, ${}^{\rho_h}/_{\rho}$, (${}^{t}/_{l}$ is small) get, **[1]**,

$$\rho_h = \rho.\frac{\frac{t}{l}\left(\left(\frac{h}{l}\right)+2\right)}{2.\cos\theta\left(\left(\frac{h}{l}\right)+\sin\theta\right)} \quad (1)$$

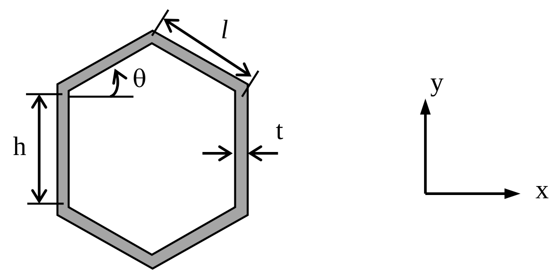


Fig. 2. A Regular Hexagonal Honeycomb, [1].

The deflected shape of the honeycomb when loaded in x- or y-direction (Fig. 1), is in bending direction for all cell walls (see Figure 3.a), and is described by five module: Young's module E_{h1} and E_{h2}, a shear modulus G_{h12} and two Poisson's ratio ν_{h12} and ν_{h21}. Using the reciprocal theorem gives,

$$E_{h1}.\nu_{h21} = E_{h2}.\nu_{h12} \quad (2)$$

Reduce the five independent modules to four independent modules.

When loading in x or y directions, the four independent module described as, **[1]**,

$$E_{h1} = \left(\frac{t}{l}\right)^3.\frac{\cos\theta}{\left(\frac{h}{l}+\sin\theta\right).\sin^2\theta}.E\ ,$$

$$E_{h2} = \left(\frac{t}{l}\right)^3.\frac{\left(\left(\frac{h}{l}\right)+\sin\theta\right)}{\cos^3\theta}.E,$$

$$\nu_{h12} = \frac{\cos^2\theta}{\left(\frac{h}{l}+\sin\theta\right).\sin\theta},$$

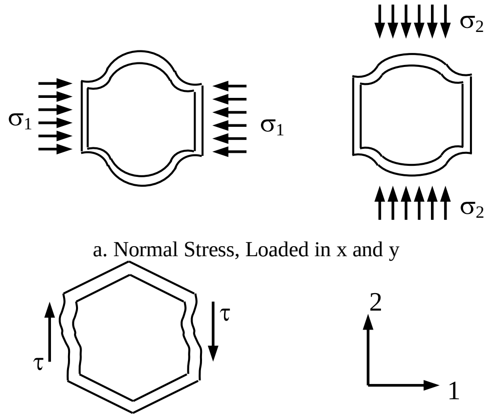
$$\nu_{h21} = \frac{\left(\left(\frac{h}{l}\right)+\sin\theta\right).\sin\theta}{\cos^2\theta} \quad (3)$$

Fig. 3b shows the honeycomb structure under shear loading.

It is possible to show that the shear module G_{h12} to be, **[1]**,

$$G_{h12} = \left(\frac{t}{l}\right)^3 . \frac{\left(\left(\frac{h}{l}\right)+\sin\theta\right)}{\left(\frac{h}{l}\right)^2 . \left(1+2.\left(\frac{h}{l}\right)\right).\cos\theta} . E \qquad (4)$$

Where, E is modulus of elasticity of honeycomb material.



a. Normal Stress, Loaded in x and y

b. Shear Stress

Fig. 3. Honeycomb Deformed, a. Deformed by Normal Stress Loaded in x and y, b. Deformed by Shear Stress, [1].

2.2 Vibration Analysis of Honeycombs Sandwich Plate

The analysis of honeycomb sandwich plate covered the determination of the general equation of motion of honeycomb sandwich plate structure, where, the plate is orthotropic properties in x and y direction. Also, solving the general equation of motion of plate is included, and then, evaluating the natural frequency of honeycomb sandwich plate with various parameters for upper and lower plate parts and honeycomb dimensions and properties part.

To Derive the general equation of motion of honeycomb sandwich plate, using the properties and density of honeycomb sandwich materials, equations (1 to 4), into the general equation of motion of plate, **[9]**,

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \rho h \frac{\partial^2 w}{\partial t^2} = 0 \qquad (10)$$

Where, M_x, M_y, M_{xy} are the bending moments and twisting moment per unit length of the orthotropic plate. The bending and twisting moments per unit length, acting on the plate element, are as follows, **[9]**,

$$M_x = \int_{-(h/2)}^{(h/2)} \sigma_x \, z \, dz$$

$$M_y = \int_{-(h/2)}^{(h/2)} \sigma_y \, z \, dz$$

$$M_{xy} = \int_{-(h/2)}^{(h/2)} \sigma_{xy} \, z \, dz$$

$$\rho h = \int_{-(h/2)}^{(h/2)} \rho \, dz \qquad (5)$$

Where,

$$\sigma_{xx} = -z\left(\frac{E_{xx}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2 w}{\partial x^2} + \frac{\nu_{xy}E_{yy}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{yy} = -z\left(\frac{\nu_{xy}E_{xx}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2 w}{\partial x^2} + \frac{E_{yy}}{1-\nu_{xy}\nu_{yx}}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\tau_{xy} = -2G_{xy}\, z \,\frac{\partial^2 w}{\partial x \partial y} \qquad (6)$$

And, $E_{xx}, E_{yy}, G_{xy}, \nu_{xy}, \nu_{yx}$ are mechanical properties of plate materials parts in x and y directions. and w is the deflection of plate in z-direction.

Then, with subjecting eq. 5 to plate structure as shown in fig. 4, to get the assuming bending moments acting on the honeycomb sandwich plate, as,

$$M_x = \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} (\sigma_x)_{Lp}\, dz + \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} (\sigma_x)_h \, dz + \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} (\sigma_x)_{Up}\, dz$$

$$M_y = \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} (\sigma_y)_{Lp}\, dz + \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} (\sigma_y)_h \, dz + \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} (\sigma_y)_{Up}\, dz$$

$$M_{xy} = \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} (\tau_{xy})_{Lp}\, dz + \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} (\tau_{xy})_h \, dz + \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} (\tau_{xy})_{Up}\, dz$$

And,

$$\rho h = \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} \rho_{Lp}\, dz + \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} \rho_h \, dz + \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} \rho_{Up}\, dz \qquad (7)$$

where, ρ_{Lp} *and* ρ_{Lp} are density of lower and upper plate parts, respectively.

For honeycomb sandwich plate shown in Fig. 4, assuming the structure combine of isotropic materials properties of upper and lower plate parts and orthotropic honeycomb sandwich part, then stresses formulation giving in equation (6), become,

1. Upper plate part, since the plate part is isotropic propeties, then,

$$E_{xx} = E_{yy} = E_{Up},\ G_{xy} = G_{Up} = \frac{E_{Up}}{2(1+\nu_{Up})},\ \nu_{xy}, \nu_{yx} = \nu_{Up} \qquad (8)$$

And,

Where, E_{Up}, G_{Up}, ν_{Up} are mechanical properties of uper plate part.

Then, by subsutution eq. 8 into eq. 6 givs,

$$\sigma_{xx} = -z\frac{E_{Up}}{\left(1-\nu_{Up}^2\right)}\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Up}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{yy} = -z\frac{E_{Up}}{\left(1-\nu_{Up}^2\right)}\left(\nu_{Up}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$\tau_{xy} = -\frac{E_{Up}}{(1+\nu_{Up})}\, z\, \frac{\partial^2 w}{\partial x \partial y} \qquad (9)$$

2. Orthotropic honeycomb sandwich part, since the plate part is orthotropic propeties, then,

$$E_{xx} = E_{h1},\ E_{yy} = E_{h2},$$
$$G_{xy} = G_{h12}$$
$$\nu_{xy} = \nu_{h12},\ \nu_{yx} = \nu_{h21} \qquad (10)$$

Then, by subsutution eq. 10 into eq. 6, gives ,

$$\sigma_{xx} = -z\left(\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{yy} = -z\left(\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\tau_{xy} = -2G_{h12}\, z\, \frac{\partial^2 w}{\partial x \partial y} \qquad (11)$$

3. Lower plate part, since the plate part has isotropic propeties, then,

$$E_{xx} = E_{yy} = E_{Lp}\ ,\ G_{xy} = G_{Lp} = \frac{E_{Lp}}{2(1+\nu_{Lp})}\ ,$$

$$\nu_{xy}, \nu_{yx} = \nu_{Lp} \tag{12}$$

Where, E_{Lp}, G_{Lp}, ν_{Lp} are mechanical properties of uper plate part.

Then, by subsutution eq. 8 in to eq. (6), gives ,

$$\sigma_{xx} = -z\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Lp}\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{yy} = -z\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(\nu_{Lp}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$\tau_{xy} = -\frac{E_{Lp}}{(1+\nu_{Lp})}\, z\, \frac{\partial^2 w}{\partial x \partial y} \tag{13}$$

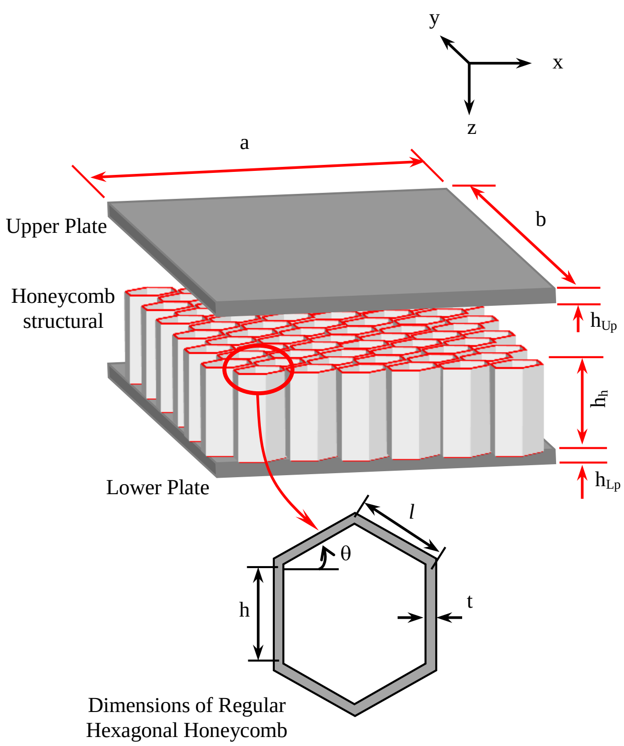


Fig. 4. Dimensions of Honeycomb Sandwich Combined Plate Structure.

Therefore, by substation eqs. 11, 12 and 13 into eq. 7, gives the bending moments acting on the honeycomb sandwich plate, as,

$$M_x = \left(\begin{array}{c} \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} \left[-z\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Lp}\frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz + \\ \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} \left[-z\left(\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz + \\ \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} \left[-z\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Up}\frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz \end{array}\right)$$

$$M_y = \left(\begin{array}{c} \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} \left[-z\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(\nu_{Lp}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz + \\ \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} \left[-z\left(\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz + \\ \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} \left[-z\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(\nu_{Up}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right] z\, dz \end{array}\right)$$

$$M_{xy} = \left(\begin{array}{c} \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} \left[-\frac{E_{Lp}}{(1+\nu_{Lp})}\, z\, \frac{\partial^2 w}{\partial x \partial y}\right] z\, dz + \\ \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} \left[-2G_{h12}\, z\, \frac{\partial^2 w}{\partial x \partial y}\right] z\, dz + \\ \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} \left[-\frac{E_{Up}}{(1+\nu_{Up})}\, z\, \frac{\partial^2 w}{\partial x \partial y}\right] z\, dz \end{array}\right)$$

$$\rho h = \int_{-\left(\frac{h_h}{2}+h_{Lp}\right)}^{-\left(\frac{h_h}{2}\right)} \rho_{Lp}\, dz + \int_{-\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}\right)} \rho_h\, dz + \int_{\left(\frac{h_h}{2}\right)}^{\left(\frac{h_h}{2}+h_{Up}\right)} \rho_{Up}\, dz \tag{14}$$

Then, with integration eq. 14, gives ,

$$M_x = -\left[\begin{array}{c} \frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Lp} + 3\frac{h_h}{2}h_{Lp}^2\right)\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Lp}\frac{\partial^2 w}{\partial y^2}\right) + \\ \frac{2}{3}\left(\frac{h_h}{2}\right)^3\left(\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right) + \\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Up} + 3\frac{h_h}{2}h_{Up}^2\right)\left(\frac{\partial^2 w}{\partial x^2} + \nu_{Up}\frac{\partial^2 w}{\partial y^2}\right) \end{array}\right]$$

$$M_y = -\left[\begin{array}{c} \frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Lp} + 3\frac{h_h}{2}h_{Lp}^2\right)\left(\nu_{Lp}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \\ \frac{2}{3}\left(\frac{h_h}{2}\right)^3\left(\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial x^2} + \frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}\frac{\partial^2 w}{\partial y^2}\right) + \\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Up} + 3\frac{h_h}{2}h_{Up}^2\right)\left(\nu_{Up}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \end{array}\right]$$

$$M_{xy} = -\left[\begin{array}{c} \frac{1}{3}\frac{E_{Lp}}{(1+\nu_{Lp})}\left(h_{Lp}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Lp} + 3\frac{h_h}{2}h_{Lp}^2\right)\frac{\partial^2 w}{\partial x \partial y} + \\ \frac{4}{3}\left(\frac{h_h}{2}\right)^3 G_{h12}\frac{\partial^2 w}{\partial x \partial y} + \\ \frac{1}{3}\frac{E_{Up}}{(1+\nu_{Up})}\left(h_{Up}^3 + 3\left(\frac{h_h}{2}\right)^2 h_{Up} + 3\frac{h_h}{2}h_{Up}^2\right)\frac{\partial^2 w}{\partial x \partial y} \end{array}\right]$$

$$\rho h = \rho_{Lp}h_{Lp} + \rho_h h_h + \rho_{Up}h_{Up} \tag{15}$$

And then, by substitution eq. 15 into the general equation of motion of orthotropic plate, eq. (10), gives, the suggested general equation of motion of honeycomb sandwich plate, as,

$$\left(\begin{matrix}\begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)+\\ \frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\frac{\partial^4 w}{\partial x^4}+\\ \begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)+\\ \frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\frac{\partial^4 w}{\partial y^4}+\\ \begin{bmatrix}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)\frac{2}{3}\frac{E_{Lp}}{(1-\nu_{Lp})}+\\ \frac{4}{3}\left(\frac{h_h}{2}\right)^3\left(\frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}+\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}+2G_{h12}\right)+\\ \left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\frac{2}{3}\frac{E_{Up}}{(1-\nu_{Up})}\end{bmatrix}\frac{\partial^4 w}{\partial x^2\partial y^2}\end{matrix}\right)+$$

$$\begin{pmatrix}\rho_{Lp}h_{Lp}+\\ \rho_h h_h+\\ \rho_{Up}h_{Up}\end{pmatrix}\frac{\partial^2 w}{\partial t^2}=0 \qquad (16)$$

To solve equation (16), separation method can be used with assuming the function of deflection as **[11]** ,

$$\mathrm{w}(x,y,t)=\mathrm{w}(x,y)*w(t) \qquad (17)$$

To evaluate the behavior of deflection plate as a function of x and y directions, the boundary conditions of the plate are required. Then, for the equation of plate as a function of x and y direction, as, **[10]**,

$$\mathrm{w}=\sin\frac{\mathrm{m\pi x}}{\mathrm{a}}\sin\frac{\mathrm{n\pi y}}{\mathrm{b}} \qquad (18)$$

Then, by substation eq. 18 into eq. 16, the suggested general equation of motion for honeycomb sandwich plate structure, is obtained as,

$$\left(\begin{matrix}\begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)\\ +\frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\left(\frac{\pi}{a}\right)^4+\\ \begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)\\ +\frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\left(\frac{\pi}{b}\right)^4+\\ \begin{bmatrix}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)\frac{2}{3}\frac{E_{Lp}}{(1-\nu_{Lp})}+\\ \frac{4}{3}\left(\frac{h_h}{2}\right)^3\left(\frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}+\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}+2G_{h12}\right)+\\ \left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\frac{2}{3}\frac{E_{Up}}{(1-\nu_{Up})}\end{bmatrix}\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\end{matrix}\right)w(t)+$$

$$\begin{pmatrix}\rho_{Lp}h_{Lp}+\\ \rho_h h_h+\\ \rho_{Up}h_{Up}\end{pmatrix}\frac{\partial^2 w(t)}{\partial t^2}=0 \qquad (19)$$

With, comparison eq. (19) with general equation of motion of single degree of freedom for free vibration structure, as follows **[12]**

$$\omega_{mn}^2 w(t)+\frac{\partial^2 w(t)}{\partial t^2}=0 \qquad (20)$$

the suggested general equation of natural frequency for honeycomb sandwich plate structure with various mechanical properties and dimensions of honeycomb sandwich plate combined, as,

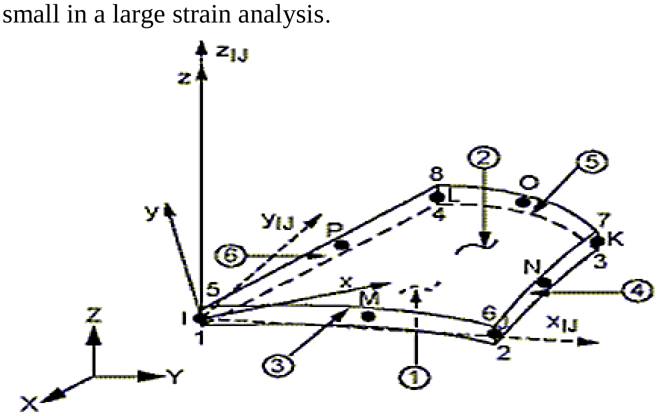
$$\omega_{mn}^2=\frac{\left(\begin{matrix}\begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)+\frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h1}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\left(\frac{\pi}{a}\right)^4+\\ \begin{bmatrix}\frac{1}{3}\frac{E_{Lp}}{(1-\nu_{Lp}^2)}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)+\frac{2}{3}\left(\frac{h_h}{2}\right)^3\frac{E_{h2}}{1-\nu_{h12}\nu_{h21}}+\\ \frac{1}{3}\frac{E_{Up}}{(1-\nu_{Up}^2)}\left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\end{bmatrix}\left(\frac{\pi}{b}\right)^4+\\ \begin{bmatrix}\left(h_{Lp}^3+3\left(\frac{h_h}{2}\right)^2 h_{Lp}+3\frac{h_h}{2}h_{Lp}^2\right)\frac{2}{3}\frac{E_{Lp}}{(1-\nu_{Lp})}+\\ \frac{4}{3}\left(\frac{h_h}{2}\right)^3\left(\frac{\nu_{h12}E_{h2}}{1-\nu_{h12}\nu_{h21}}+\frac{\nu_{h12}E_{h1}}{1-\nu_{h12}\nu_{h21}}+2G_{h12}\right)+\\ \left(h_{Up}^3+3\left(\frac{h_h}{2}\right)^2 h_{Up}+3\frac{h_h}{2}h_{Up}^2\right)\frac{2}{3}\frac{E_{Up}}{(1-\nu_{Up})}\end{bmatrix}\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2\end{matrix}\right)}{(\rho_{Lp}h_{Lp}+\rho_h h_h+\rho_{Up}h_{Up})} \qquad (21)$$

By, building a computer program, with using Fortran Power Station 4.0, can be evaluated the natural frequency of honeycomb sandwich plate with various parameters effect, as,

1. Thickness of upper and lower plates and honeycomb parte.
2. Hexagonal honeycomb.
3. t, h, and *l* of honeycomb parte.
4. Mechanical properties and density of honeycomb sandwich parte.

3. NUMERICAL INVESTIGATION

The numerical investigation is carried out to calculate the natural frequency of honeycomb sandwich plate structure by using the finite elements method ANSYS program (Ver.14). Where, the shell element (SHELL93) is used to build the model of honeycomb sandwich plate structure, figure 5.It is assumed that the shell element is employed with zero thickness or element tapering down to a zero thickness at any corner is not allowed Also the shear deflections are included in this element. The out-of plane stress for this element varies linearly through the thickness..The transverse shear stresses (SYZ and SXZ) are assumed to be constant through the thickness, and, the transverse shear strains are assumed to be small in a large strain analysis.



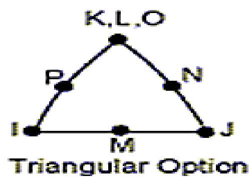


Fig. 5. Shell93-8 Node Element Geometry.

4. RESULTS AND DISCUSSIONS

The results evaluated are included the natural frequency of honeycomb sandwich simply supported plate with various angle and thickness of honeycomb part and different upper and lower plates parts thicknesses. The materials are assumed to be distributed to upper and lower plate parts and honeycomb part are aluminium materials with mechanical properties as,

$E = 75\ Gpa$

$Density = 2800\ kg/m^3$

And the dimensions of plates are,

$a = b = 0.5\ m$

$h_{Lp} = h_{Up} = 1, 1.5, and\ 2\ mm$

$h_h = 0.5, 1, 1.5,\ and\ 2\ cm$

$t = 0.025, 0.05, 0.075, 0.1\ mm$

$h = l = 5\ mm$ (22)

the results of natural frequency evaluated are shown in figures 6 to 9 using various parameters.

Figs. 6 and 7 show the natural frequency of honeycomb plate structure with different Hexagonal Honeycomb thickness , angle , various honeycomb parts thickness with upper and lower plates parts are $h_{Lp} = h_{Up} = 1\ and\ 2\ mm$, respectively. From figures , it is seen that the natural frequency increases with increase the Hexagonal Honeycomb thickness and angle.

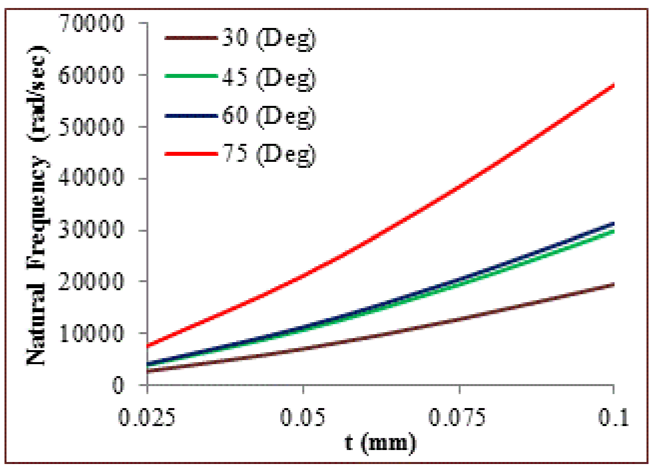
Figs. 8 and 9 show the natural frequency of honeycomb plate structure with different upper and lower plates parts and various of Hexagonal Honeycomb angle and various honeycomb part thickness, for Hexagonal Honeycomb thickness $t = 0.025\ mm$. From figures , it is clear that the natural frequency is increasing with increase the Hexagonal Honeycomb angle and honeycomb part thickness. The natural frequency decreases with increasing the upper and lower plates parts thickness.

The results indicate that the increasing of Hexagonal Honeycomb thickness and angle and increasing the honeycomb plate part thickness causes an increase in the strength to weight ratio, therefore, an increase in the natural frequency of plate. The increase of upper and lower plate parts causes a decrease in the strength to weight ratio, therefore, decreasing the natural frequency of honeycomb sandwich plate.

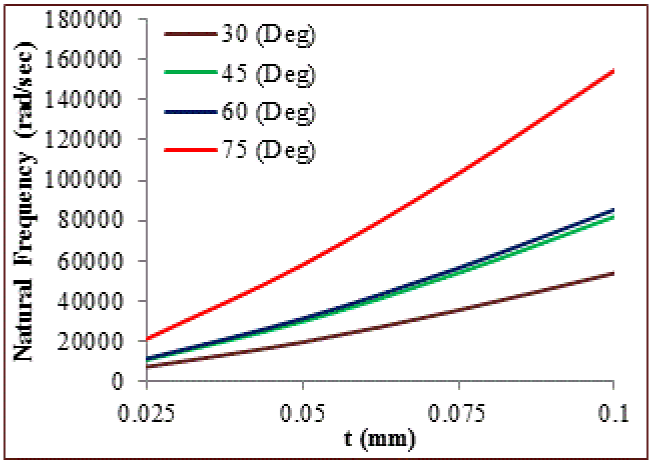
Table I shows the comparison between theoretical and numerical results of fundamental natural frequency of honeycomb plate structure with different honeycomb parts and upper and lower plate parts thickness. It is clear that the suggested analytical solution of honeycomb plate gives a good agreement compared with numerical results evaluated by using finite element technique, ANSYS program ver. 14. The maximum percentage discrepancy between the analytical and numerical results is about (2%).

Table I
Comparison between analytical and numerical natural frequency results for various honeycomb and upper and lower plate thickness.

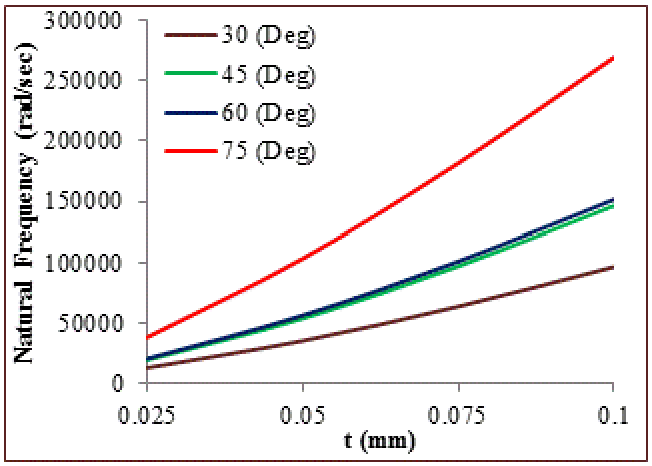
Case	θ (Deg)	Natural Frequency (rad/sec)		Discrepancy %
		Analytical	Numerical	
h_h=1 cm, h_{Lp}=h_{Up}= 1 mm, t=0.05 mm	30	19706.66	19400.25	1.55
	45	29897.46	29443.25	1.52
	60	31374.13	30944.25	1.37
	75	58075.07	56963.5	1.91
h_h=2 cm, h_{Lp}=h_{Up}= 1 mm, t=0.05 mm	30	54046.09	52974.25	1.98
	45	82041.66	81065.3	1.19
	60	85471.03	84268.35	1.41
	75	154559.2	151928.2	1.70
h_h=0.5 cm, h_{Lp}=h_{Up}= 1 mm, t=0.05 mm	30	7140.829	7068.25	1.02
	45	10778.76	10586.35	1.79
	60	11352.08	11243.69	0.95
	75	21250.12	20987.35	1.24
h_h=0.5 cm, h_{Lp}=h_{Up}= 2 mm, t=0.05 mm	30	5247.032	5139.365	2.05
	45	7787.841	7642.58	1.87
	60	8207.793	8068.35	1.70
	75	15344.21	15025.38	2.08



a. h_h=0.5 cm

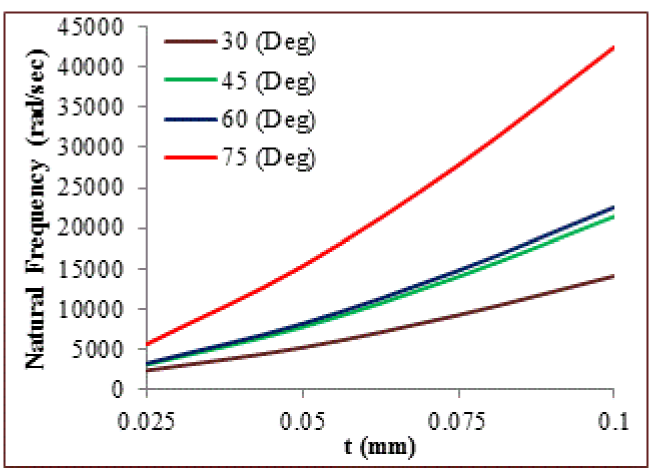


b. h_h=1 cm

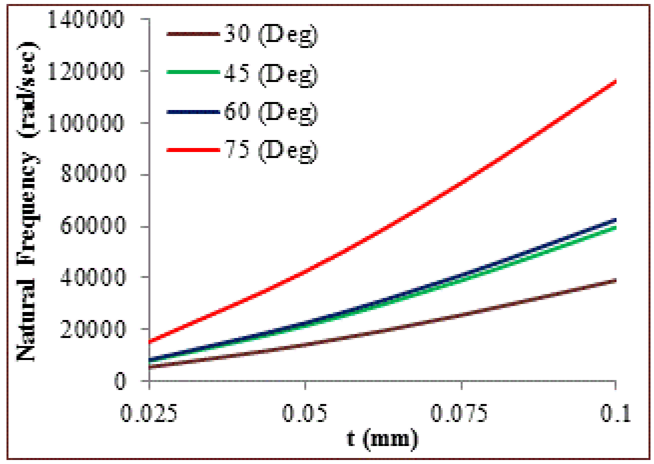


c. h_h=1.5 cm

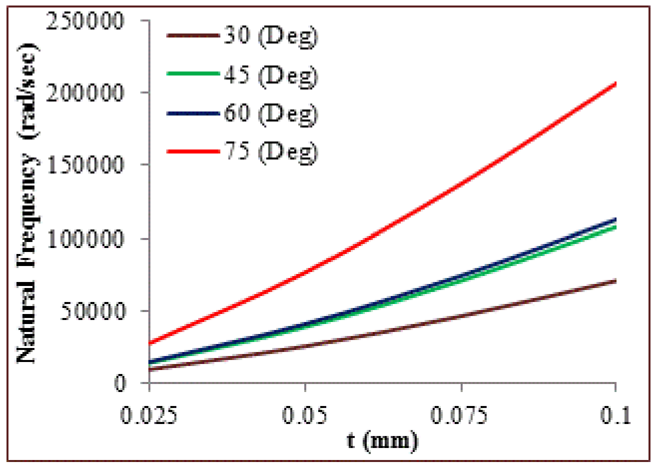
Fig. 6. Natural frequency of honeycomb plate with different dimensions of Regular Hexagonal Honeycomb and various honeycomb thickness, for thickness of upper and lower plate hLp=hup=1 mm.



a. h_h=0.5 cm

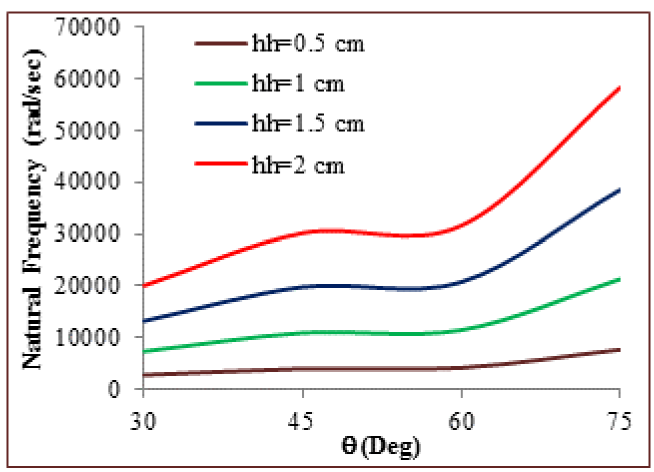


b. h_h=1 cm

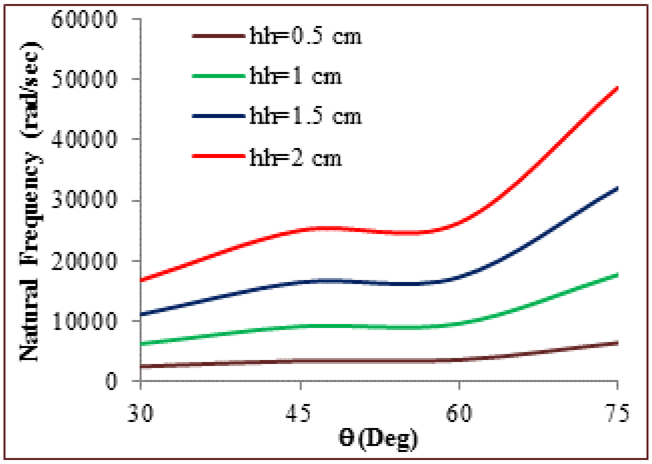


c. h_h=1.5 cm

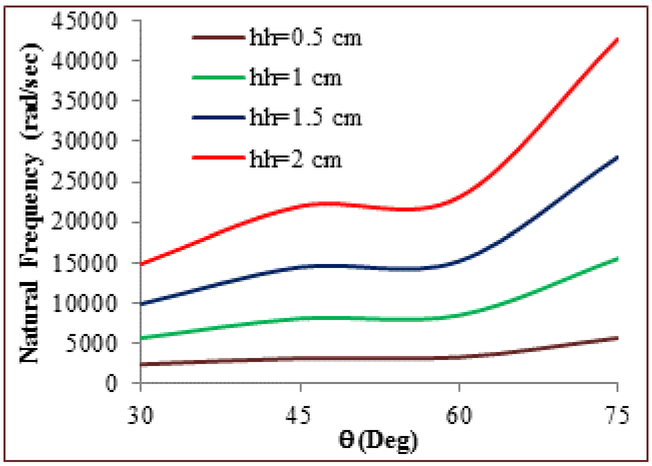
Fig. 7. Natural frequency of honeycomb plate with different dimensions of Regular Hexagonal Honeycomb and various honeycomb thickness, for thickness of upper and lower plate hLp=hup=2 mm.



a. h_{Lp}=h_{up}=1 mm

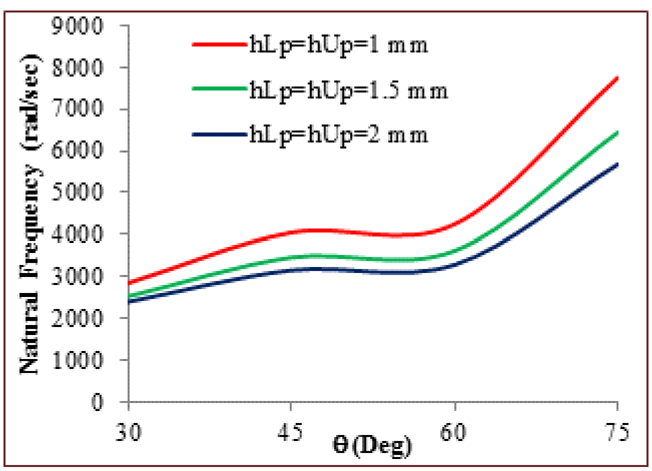


b. $h_{Lp}=h_{up}=1.5$ mm

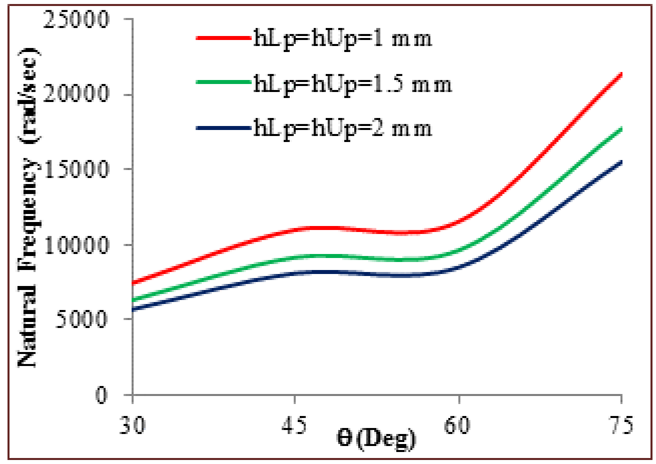


c. $h_{Lp}=h_{up}=2$ mm

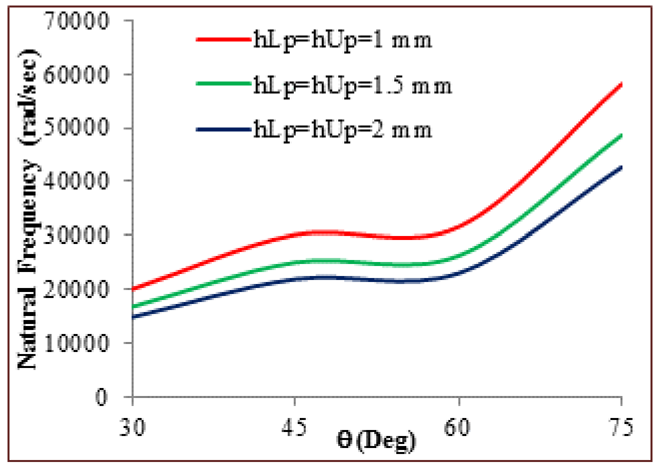
Fig. 8. Natural frequency of honeycomb plate with different angle of Regular Hexagonal Honeycomb and various honeycomb thickness and thickness of upper and lower plate, for t=0.025 mm and .



a. $h_h=0.5$ cm



b. $h_h=1$ cm



c. $h_h=2$ cm

Fig. 9. Natural frequency of honeycomb plate with different angle of Regular Hexagonal Honeycomb and various thicknesses of upper and lower plate and honeycomb thickness, for t=0.025 mm and .

5. CONCLUSIONS

Some concluding observations from the investigation are given below:

1. The increase of Hexagonal Honeycomb thickness causes an increase in the strength of honeycomb structure ,then, increasing the natural frequency of honeycomb plate.
2. With increasing the Hexagonal Honeycomb angle increases the strength and stiffness of Honeycomb structure, therefore, with increase the Hexagonal Honeycomb angle increases the natural frequency of honeycomb sandwich plate structure.
3. The increase of honeycomb thickness causes an increase in the strength of honeycomb structure then, increasing the natural frequency of honeycomb plate.
4. The increasing of upper and lower plate parts thickness of honeycomb sandwich plate causes a decrease in the strength to weight ratio of honeycomb plate structure. Therefore, with increasing the upper and lower plate parts thickness decreases the natural frequency of honeycomb sandwich plate structure.

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