Numerical Study of Natural Convection in an Open-Ended Channel: Comparison of Characteristic Quantities Between Air and Water

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Abstract—The present study deals with natural convection flow inside vertical channel with one wall heated at uniform heat flux and the other wall isolated. The purpose of this numerical investigation is to study the differences on flow configurations in pure natural convection between air and water in vertical channels. A comparison in terms of Nusselt number and mass flow rate is then discussed. The numerical code is developed using Finite Differences scheme to solve the Navier-Stokes equations under the Boussinesq assumption in two dimension. The comparison in terms of characteristic quantities shows that Nusselt number are slightly different between air and water, flow patterns are completely different and adimensionless velocity and flow rates are more important in water than in air. Physical understanding of these differences is discussed. Using water to model natural convection in an air vertical channel gives reasonable Nusselt numbers but only for small modified Rayleigh numbers. Discrepancies increases with increasing the modified Rayleigh number.

Index Term-- Natural convection; Open-Ended Channel.

I. INTRODUCTION

Cooling by natural convection in a vertical channel has been used in several industrial applications. In fact, the channel is representative of problems such as chimney, solar wall, Trombe wall, electronic components of computers, or double-skin façade. Two approaches emerge from the studies carried out in this context. A macroscopic approach which provides analytical expressions assuming uniformity of global quantities such as velocity profiles and temperatures. For example, Brinkworth [1] determined and analytical expression for the mass flow for different heating power and gave head losses between the inlet and the outlet of the channel.

A second numerical approach based on detailed experimental studies was also apprehended. The first experiment reported in this context was that Elenbaas [2] for an air-filled channel which walls are heated to uniform temperature, he studied various flow regimes according to the modified Rayleigh number ($10^3 \leq Ra \leq 10^5$), he has also provided correlations in terms of dissipation rate and Nusselt number for the same range of $Ra$. Bodoia and Osterle [3] presented the first numerical results in agreement with the results of Elenbaas [2] only in the case of small numbers $Ra$. For the case of large $Ra$, the results showed more disparity (11% of difference). This discrepancy can be explained by the fact that in the study of Bodoia a uniform velocity was used as a boundary condition at the entrance of the channel, which is not sufficient to consider the case where the flow regime reaches the boundary layer regime (large $Ra$).

Other numerical and experimental studies were presented later, Aung [4], Aung et al. [5], Kihm et al. [6], Sparrow and Bahrami [7], Sparrow et al. [8], Webb and Hill [9] for both heating temperature and uniform flow imposed. Some authors have considered adiabatic extensions and artificial volumes at the input and output of the channel Kettleborough [10], Naylor et al. [11], Morrone et al. [12], Campo et al. [13], Bianco and Nardini [14], Giroux-Julien et al. [15]. They showed interest to add extensions to a parallel channel whose walls are heated at uniform temperature or heat flux. They give the results in terms of maximum temperatures on the walls, mass flow rate, and pressure profiles. They also obtain correlations of mass flow rate and maximum wall temperature as a function of the modified Rayleigh number and the extension spect ratio. Later, other correlations in terms of Nusselt number and optimum spacing for different modified Rayleigh numbers were given by Bar-Cohen and Rohsenow [16]. These correlations were improved and compared by Olsson [17]. He also refers to an optimum spacing between the walls of the channel for a better heat transfer. These previous investigations have focused only on the study of pure convection and neglected heat exchange due to surface radiation. They also overlook or minimize conduction in heated walls. Most of these studies have been done using air as working fluid and the radiation. They also overlook or minimize conduction in heated walls. Most of these studies have been done using air as working fluid and the radiation part of heat transfer is still not completely known. In order to contribute in the understanding of the physical mechanisms of pure natural convection, some authors used water as a working fluid in order to get rid of radiation heat transfer. Miyatake et al. [18] performed both analytical study and experiment for natural convection between vertical parallel plates heated with uniform heat flux. Experiments were performed with water as the working fluid, and results were compared to the theoretical predictions with good agreement. Sparrow et al. [8] investigated the flow reversal by visualizing water flows in a vertical channel heated on one side. They also made a numerical study using a parabolic model to solve the Navier-Stokes equations. They underlined that Nu $(Ra)$ correlations are slightly different between air and water and difference increases with $Ra$. Daverat [19] studied a pure convective flow between two parallel plates placed in a water box. Their velocity profiles and temperature averages reveal the presence of a change in flow regime inside the channel for an identified modified Rayleigh number. They also study the effect of thermal stratification on the flow and temperature. Few experimental and numerical
studies on pure natural convection between parallel plates have been carried out with water. Mitra et al. [20] made a review of Nu (Ra') correlations of only five studies. This study shows how few researches exist on the subject. In addition, most of the investigations were limited to thermal measurements and heat transfer analysis, no studies showed the differences in flow patterns between air and water in vertical channels. In order to contribute in the understanding of the physical mechanisms of pure natural convection, we present a comparison study in terms of thermal and kinematic quantities for pure convective flow between using water and air. The numerical study performed here, used an elliptic model to solve of Navier-Stokes equations.

In the following, the model and numerical methods that address the problem of natural convection of fluid (air or water) between two vertical walls, are detailed. Finally, a comparison of characteristic quantities between the two fluids is discussed.

II. PHYSICAL MODEL AND NUMERICAL METHODS
II.1. Numerical Modeling

The geometrical configuration subject to investigation is shown in Fig. 1. It concerns the simplified (2D) configuration of the experimental apparatus. The wall heating produces a temperature gradient in the fluid leading to small density differences which gives rise to the buoyancy force. Thus, a natural convective flow occurs with the fluid entering at the bottom and leaving at the top. Throughout the analysis the following assumptions are made: fluid properties, except density, are independent of temperature; density variations are significant only in the buoyancy force; and the flow is two-dimensional, laminar and incompressible with negligible viscous dissipation and surface radiation. Thus, the well-known unsteady Boussinesq equations in their elliptic form, governing the flow are:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= 0, \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -1 \frac{\partial p}{\partial y} + \beta \frac{\partial T}{\partial x} - \frac{1}{\rho_0} \frac{\partial^2 u}{\partial x^2} - \frac{1}{\rho_0} \frac{\partial^2 v}{\partial y^2} + \frac{1}{\rho_0} \frac{\partial v}{\partial y} \frac{\partial T}{\partial x}, \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{1}{\rho_0} \frac{\partial^2 T}{\partial x^2} \frac{\partial^2 T}{\partial y^2},
\end{align*}
\]

(1)

Where \( u, v, p \) are respectively, horizontal velocity, vertical velocity, and pressure. \( p_0, v_0, \beta_0, \alpha_0 \) are respectively, mass density, kinematic viscosity, thermal expansion coefficient and thermal diffusivity calculated at some reference temperature. By introducing the following adimensional quantities with reference variables (\( L_r, U_r, t_r \)), Eq. 2.

\[
\begin{align*}
L_r &= b; X = \frac{x}{L_r}; && Y = \frac{y}{L_r}, \\
U_r &= \frac{U_0}{L_r}; \quad \frac{1}{L_r} = \frac{T_r}{U_r}, \\
U_r &= \frac{\beta_0 v^4}{\rho_0 L_r^2} \frac{1}{\rho_0} \frac{\partial^2 T}{\partial x^2}, \quad \frac{1}{L_r} = \frac{T_r}{U_r}, \\
\theta &= \frac{T - T_0}{\Delta T_r}, \quad P = \frac{P - P_0}{\rho U_r^2}, \quad t_r = \frac{b}{U_r},
\end{align*}
\]

(2)

The governing Eq.(3) are transformed as follows

\[
\begin{align*}
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} &= 0, \\
\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} &= -1 \frac{\partial P}{\partial y} + Pr \frac{\partial^2 U}{\partial x^2} + Pr \frac{\partial^2 U}{\partial y^2} + \frac{1}{Pr} \frac{\partial v}{\partial y} \frac{\partial T}{\partial x} + \frac{1}{Pr} \frac{\partial v}{\partial y} + \frac{1}{Pr} \frac{\partial v}{\partial y} \frac{\partial T}{\partial x} + \frac{1}{Pr} \frac{\partial v}{\partial y} \frac{\partial T}{\partial x}, \\
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} &= \frac{1}{Pr} \frac{\partial^2 T}{\partial x^2} + \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2}.
\end{align*}
\]

(3)

Where \( \Delta T \) used to define the adimensional temperature and the reference velocity is: \( \frac{q_u}{\lambda_0} \), with \( q_u \) is the uniform heat flux fixed at the wall. The Prandt number \( Pr \) is defined by \( Pr = \frac{v_0}{\lambda_0} \).

Furthermore, the Rayleigh number, expressing the strength of buoyancy, is constructed with the plate spacing as follows:

\[
Ra_b = \frac{\beta_0 q_0 b^4}{\nu_0 T^2}
\]

(4)

This finally leads to, the modified Rayleigh number (or Elenbaas number):

\[
Ra_b^{*} = \frac{Ra_b}{A_h}
\]

(5)

Where \( \frac{b}{h} \) is the aspect ratio of the heated part of the channel (see fig. 1).
The selected boundary conditions for the thermally driven open channel are shown in system of Eq. (6). When the boundary conditions are expressed in pressure at the inlet, the generalized Bernoulli theorem is used to take into account the pressure loss between the upstream and the entry of the channel (upstream velocity assumed to be zero and the pressure is equal to \( p(x; y) = \text{patm} \)). Indeed, Morrone et al. [12], Marcondes and Maliska [21] found that the driving pressure at the entrance of the channel, due to the velocity of the incoming fluid, cannot be neglected, the influence is even more important as the Rayleigh number increases. Local Bernoulli pressure is applied at the channel entrance. Moreover, pressure at the outlet is taken equal to exterior pressure assumed to be zero (atmospheric pressure in dimensional form). When boundary conditions are written on velocity, zero Neumann boundary condition is applied on streamwise velocity \( V \) and homogeneous Dirichlet boundary condition is applied on transverse velocity \( U \). Actually, we assume that the incoming flow at the inlet is parallel to the channel. Concerning the thermal boundary conditions, we neglect the longitudinal conductive heat flux at the channel outlet, knowing that the transverse thermal gradients are more important. If in some case, an incoming flow can appear at the channel outlet, we assume it has the outside temperature. In this approximation, we neglect the fact that the incoming fluid may be slightly heated in the external surroundings.

### II.2. Numerical Methods

Equations of conservation of mass, momentum, and energy, are discretized with finite differences schemes in space and time. The temporal scheme adopted to discretize the system of equations 2 is based on the three-level second-order Euler backward scheme with Adams-Bashforth extrapolation as proposed in Vanel et al. [22]. In this scheme, we treat implicitly the diffusion terms and explicitly the convective ones. The pressure velocity decoupling is achieved by adopting an iterative solution based on a projection algorithm Chorin [23] Chorin [24]. This algorithm consists of finding first a velocity prediction based on pressure field approximation. Then an equation for pressure correction is derived from the continuity equation, it’s then solved to obtain the pressure correction to update the velocity fields and pressure. Concerning the spatial discretization, the computational domain is discretized on a non-uniform Cartesian grid. Use is made of a staggered variable arrangement. A second order centered difference schemes are used for all variables. A second order forward and backward difference schemes are used to treat the Neumann boundary condition on vertical velocity respectively at the inlet and at the outlet. After discretization, we obtain a Poisson equation on pressure correction and Helmholtz equations on velocity and temperature. We solve the Poisson and Helmholtz equations respectively using partial diagonalization method and tridiagonal matrix algorithm. For more details see Zoubir et al [25].

### III. VALIDATION OF NUMERICAL CODE

The thermally driven vertical channel problem has served as a benchmark problem. The GDR AMETH laboratories G. Desrayaud and Mojtabi [26] have worked on natural convection of air between two vertical walls, one heated by a uniform heat flux and the other insulated. This kind of problems leads to a so-called flow reversal when channel walls are subjected to asymmetric heating Sparrow et al. [8] Ingham et al. [27] Khim et al. [6]. In this benchmark four dynamic boundary conditions are tested in order to approach the Webb and Hill experimentation Webb and Hill [9]. A first validation of numerical code was done in previous works with those of GDR-AMETH laboratories Zoubir et al. [25] and Zoubir et al. [28].The comparison was made for modified Rayleigh number of \( Ra^* = 10^5 \) and an aspect ratio of 10 and different grid resolutions. A spatial grid analysis was also done and shows that our numerical code is globally of second-order in space Zoubir et al. [25]. Furthermore, a good agreement was found with other works on characteristic quantities (mass flow rate and Nusselt number).

### IV. RESULTS AND DISCUSSION

In the present study, the Rayleigh number based on the channel width is varied from \( 10^5 \) to \( 10^6 \) and the aspect ratio is taken equal to 5 and 15. Then, the corresponding modified Rayleigh numbers are \( Ra^* = 10^5, 2.10^5, 10^6, 2.10^5, 10^5 \). The Prandtl numbers used are 0.71 and 7.0, which correspond to air and water respectively. The details of the fluid flow and the temperature fields are presented below, with emphasis on comparison on natural convection results of air and water. Fig. 2 and Fig. 3 present fluid flow configurations of air and water in the form of isotherms and streamlines. In the fig. 3, a flow reversal occurs for all configurations. Indeed, the air heated at the wall rises by natural convection and creates a dynamic boundary layer along the heated wall. To aliment this boundary layer, the air enters from the top when the bottom alimentation is not sufficient. However, the penetration depth of fluid at the channel top is different for different Rayleigh numbers for air and water. The fluid reversal penetration depth increases with Rayleigh number.

In addition, the penetration is more important in the case of air than that in water. There is also a significant difference in the growth and thickness of the dynamical boundary layer for water and air. As expected with the increase on Rayleigh number, the boundary layer at the heated wall decreases, for both water and air. Furthermore, the dynamical boundary layer is smaller in air. In the fig. 2, on the contrary to dynamical boundary layer, the thermal boundary layer is larger in air then in that of water.

The difference in the boundary layer thickness is due to Prandtl number effects and to the presence of recirculation of air at the channel exit near the adiabatic wall. We can also see that for the first modified Rayleigh number, because of flow reversal, the adiabatic wall is slightly heated in the air case, this is subjected by the two isotherms reaching the adiabatic walls.

Fig. 4 and Fig. 5 present the adimensional temperature for \( A = 5 \) and velocity profiles for \( A = 10 \) at different channel height and different modified Rayleigh numbers for both fluids.
Temperatures are greater in air fluid than that in water. The wall temperature differences between the two fluids vary as a function of modified Rayleigh number from 17 to 19% for $A = 5$ and from 9 to 10% for $A = 10$. This is due to an important heat exchange between the heated wall and fluid in water case. It is observed that an increase in the Prandtl number to $Pr = 7$ results in a decrease of the thermal boundary layer thickness and in lower average temperature within the boundary layer. The reason is that smaller value of Prandtl number is equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher value of Prandtl number. Hence there is a reduction in temperature with increase in the Prandtl number. We can also see that discrepancies are more important for the lowest aspect ratio and increase with increasing the modified Rayleigh number.

Let us now analyze the adimensional vertical velocity (see fig. 5, Table. I and Table. II. Because of viscosity and flow reversals, velocity profiles are more homogenized for water than for air and the adimensional mean velocity and mass flow rate is larger in water (see also Table III). We can conclude that the flow characteristics in terms of velocity are different between air and water.

Table III shows the results of average Nusselt number $< Nu >$. Results show that Nusselt number is greater in water than in air. The differences for Nusselt number varied as a function of modified Rayleigh number from 9,3% to 22,75% for $A = 5$ and from 8% to 23,87 for $A = 10$. Discrepancies on Nusselt number are seen to increase with increasing the modified Rayleigh number.
Fig. 4. Results in terms of temperature profiles at different heights (a), (b), (c); and wall temperatures for water and air (d), with $Ra_d = 10^5$, $10^6$, $10^7$ and $A = 5$.

Fig. 5. Results in terms of velocity profiles for different heights for water and air (a,b,c), with $Ra_d = 10^5$, $10^6$, $10^7$ and $A = 10$. 
We have presented a numerical code for studying natural convection in an open-ended channel. Finite difference scheme of second order in time and space was adopted to discretize the Navier-Stokes equations under the Boussinesq assumption. A comparison of characteristic quantities (Nusselt number, flow rate) between water and air has been done and discrepancies are discussed. Traditionally used water models to predict the natural convection of air give reasonable heat transfer results only for small modified Rayleigh numbers. The fluid flow characteristics for air are different from those of water. The maximum velocity and exchange flow rates reflect this point. Further studies need to be carried out in order to improve both experimental and numerical approaches and understand the physical phenomena.

### NOMENCLATURE

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<td>A</td>
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<tr>
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<td>g</td>
<td>gravity acceleration (9.81 m/s²)</td>
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## TABLE I

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## REFERENCES


