An Anisotropic Deformation Analysis of Orthotropic Materials Subjected to High Velocity Impacts

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Abstract—A finite strain constitutive model to predict a complex elastoplastic deformation behaviour involves very high pressures and shockwaves in orthotropic materials is developed in this work. The important feature of the proposed hyperelastic-plastic constitutive model is a Mandel stress tensor combined with the new generalised orthotropic pressure. The formulation is developed in the isoclinic configuration and allows for a unique treatment for elastic and plastic orthotropy. The elastic orthotropy is taken into account through a stress tensor decomposition combined with the new pressure. A yield surface of Hill’s yield criterion aligned uniquely within the principal stress space is adopted to characterise plastic orthotropy by means of the evolving structural tensors. An isotropic hardening is adopted to define the evolution of plastic orthotropy. The formulation is further combined with a shock equation of state (EOS) and Grady spall failure model to predict shockwave propagation and spall failure in the materials, respectively. The proposed constitutive model is implemented as a new material model in the Lawrence Livermore National Laboratory (LLNL)-DYNA3D code of UTHM’s version. The ability of the newly constitutive model to describe finite strain deformation and shock propagation in orthotropic materials is first investigated against plate impact data of aluminium alloy in the longitudinal and transverse directions before a comparison against plate impact test data of carbon fibre reinforced epoxy composites along the through-thickness direction is finally conducted. A good agreement is obtained in each test.

Index Term--Elastoplastic Deformation; Shockwave Propagation; Orthotropic Materials; Aluminium Alloy; Carbon Fibre Reinforced Epoxy Composites

1. INTRODUCTION

In practice, most of the engineering materials such as composites and sheet metal components, manufactured using sheet metal forming processes, are orthotropic. Composite materials in particular are one of the important types of materials in the construction of modern aerospace structures due to their mechanical properties. The strain rate dependent behaviour of composite materials is important for applications involving impact and dynamic loading. It can be observed that such materials exhibit orthotropic behaviour under large elastoplastic deformation at unit-cell level due to the preferred orientation.

Modelling composites behaviour undergoing finite strain deformation including shock wave propagation and spall failure has attracted attention due to its broad engineering applications. Much research has been carried out to capture the complex materials behaviour of this orthotropic materials under dynamic loading conditions, leading to results in various technologies involving analytical, experimental and computational methods. Despite of this current status, it is generally agreed that there is a need for improved constitutive formulation as well as the corresponding procedures to identify the required input parameters. Modelling finite strain deformation and failure in such materials requires an appropriate mathematical description, which can be very complex specifically to deal with the orientations of materials orthotropy (Sitnikova et al., 2014).

In general, the formulation to describe complex materials responses at high strain rate and pressures should be one of integration between elasticity, plasticity and equation of state (EOS) including spall failure. It must first be emphasised that the shock response of an orthotropic materials cannot be accurately predicted using the conventional decomposition of the stress tensor into isotropic and deviatoric parts, (Anderson, 1994). The developed constitutive models for modelling shock wave propagation in solids comprise two parts, an equation of state (EOS) and a strength model which define the response of the material to uniform compression (change of volume) and the response of the material to shear deformation (change of shape) respectively.

This separation of material response into volumetric and deviatoric strain components is matched for isotropic materials which have an isotropic elastic stiffness tensor \( c_{ijkl} \). As a consequence the spherical part of the stress tensor \(-P\delta_{ij} = c_{ijkl}\delta_{kl}\varepsilon_{pp}/3\), being a product of two isotropic tensors, is isotropic itself. Furthermore, the isotropy of \( c_{ijkl} \) results in the co-linearity of the principal axis of the stress and strain tensors. In other words, components of stress and strain are proportional to each other and orthogonality between the volumetric and deviatoric components of strain is reflected in orthogonality between the volumetric and deviatoric components of stress. This is successfully done for isotropic materials through the conventional decomposition of the stress tensor into the spherical and deviatoric parts.
However, in the case of orthotropic materials this co-linearity is not in place. Hence the equivalent relationship cannot be defined for orthotropic materials. If one maintains the assumption that pressure is the state of stress induced by an isotropic state of strain (uniform compression or expansion) then a more general definition of pressure is required, (Vignjevic et al., 2007). This leads to a number of possible definitions of pressure as a vector in the principal stress space which does not co-linear with the conventional hydrostatic alignment for orthotropic materials. To explore this statement further Vignjevic has proposed a new expression for generalized pressure or stress related to uniform compression.

To derive the formulation of this generalized pressure, let first write the stress due to the isotropic component of strain (isotropic strain pressure) as

\[-P \psi_{ij} = c_{ijkl} \delta_{kl} \varepsilon_{ss}/3 = c_{ijkl} \varepsilon_v\]  \hspace{1cm} (1)

where \(\psi_{ij} = 0 \forall \ i \neq j\), \(\psi_{ij} = 0 \forall \ i = j\), and \(\varepsilon_v = \varepsilon_{ss}/3\). In above equation, \(\bar{P}\) and \(\psi_{ij}\) can be defined as

\[-\bar{P} = \varepsilon_v \frac{1}{\psi_{st} \psi_{st}} c_{ijkl} c_{ijkl}\]

and

\[\psi_{ij} = -c_{ijkl} \varepsilon_v / \bar{P} = c_{ijkl} / \left(\frac{1}{\psi_{st} \psi_{st}} c_{ijkl} c_{ijkl}\right)\]  \hspace{1cm} (3)

The double contraction tensor \(\psi_{ijkl}\) must be defined to uniquely define \(\bar{P}\) and tensor \(\psi_{ij}\). One possible assumption is to set \(\psi_{st} \psi_{st} = 3\). Note that the tensor \(\psi_{ij}\) is fully defined by the material elastic stiffness properties. Further, Equations (1)-(3) can be expressed in Voigt notation as shown in Equations (4)-(6) respectively:

\[
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
0 \\
0 \\
0
\end{pmatrix}
= -
\begin{pmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\
c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_v \\
\varepsilon_v \\
\varepsilon_v
\end{pmatrix}
\]  \hspace{1cm} (4)

\[P = \frac{\left(c_{11} + c_{12} + c_{13}\right)^2 + \left(c_{12} + c_{22} + c_{23}\right)^2 + \left(c_{13} + c_{23} + c_{33}\right)^2}{3} - \varepsilon_v = -3 K_p \varepsilon_v\]  \hspace{1cm} (5)

The parameter \(K_p\) reduces to the conventional bulk modulus in the limit of material isotropy. \(\psi_{ij}\) which is used defines the direction of the new volumetric axis in stress space then can be defined as

\[\psi_{(iii)} = \frac{c_{11} + c_{12} + c_{13}}{\sqrt{\left(c_{11} + c_{12} + c_{13}\right)^2 + \left(c_{12} + c_{22} + c_{23}\right)^2 + \left(c_{13} + c_{23} + c_{33}\right)^2}}\]  \hspace{1cm} (6)

Repeated indices in brackets in the above equation indicate no summation. The alternative formulation of generalized pressure for orthotropic materials finally can be expressed as follows:

\[P = \frac{\sigma_{kl} \psi_{kl}}{\psi_{st} \psi_{st}}\]  \hspace{1cm} (7)

It should be noted \(\psi_{ij}\) becomes \(\delta_{ij}\) when dealing with isotropic materials. Hence the new decomposition reduces to the conventional decomposition developed for isotropic materials. The capability of the above formulation to describe shock propagation in orthotropic materials was investigated with experimental plate impact data and showed a good agreement with the physical behaviour of carbon fibre reinforced epoxy, (Vignjevic et al., 2007). Further, this new stress tensor decomposition has been used to develop a new yield criterion for orthotropic sheet metals under plane-stress conditions by assuming the yield surface to be circular in the new deviatoric plane (Mohd Nor et al., 2013b). The predictions of the new effective stress expression showed good agreement with respect to the experimental data for 6000 series aluminium alloy sheet (A6XXX-T4) and Al-killed cold-rolled steel sheet SPCE (Banabic et al., 2003).

2. CONSTITUTIVE FORMULATION

In general, the formulation to describe complex materials responses at high strain rate and pressures should be one of integration between elasticity, plasticity and equation of state (EOS) including spall failure. The behaviour of orthotropic materials at quasi-static strain rates has been studied extensively by many researchers, see for examples (Vignjevic et al., 2002; Sinha and Gosh, 2006; Mohd Nor et al., 2013b, Mohd Nor and Mohamad Suhaimi, 2014). The investigation on the behaviour of such materials that impacted with dynamic shock loading are due to Furnish and Chhabildas (1998), Minich et al. (2004), and Colvin et al. (2009), Kanel et al. (2009), Khan and Meredith (2010), Zaretsky and Kanel (2011) and Meredith and Khan (2012). It can be observed that many have contributed to the study of anisotropic influence on material behaviour undergoing finite strain deformation, including shock wave propagation, (Smallman, 1985; Vignjevic et al., 2002; Gray et al., 2003; Khan et al., 2007a,b;
In respect to shock response, a significant investigation was first made by Rosenberg et al. (1983). It is shown that in differently heat treated states, the Hugoniot Elastic Limit (HEL) and spall strengths for AA2024 followed the identical trends as the quasi-statically measured properties. In recent decades, the topic related to shock wave propagation in anisotropic materials has received considerable attention in the isotropic solid-state physics and mechanics literature (Wackerle, 1962; Zel'dovich and Raizer, 1966; Davison and Graham, 1979; Eliezer et al., 1986; Assay and Shahinpoor, 1993; Meyers, 1994; Drumheller, 1998). The shock response of the aluminium alloy 7010-T6 in plate impact test was investigated in Vignjevic et al., (2002). As emphasized in Mohd Nor (2016a), an appropriate strength model and equation of state (EOS) must be adopted in addition to the conservation laws to accurately describe the material’s nonlinear behaviour and shockwave propagation in solids due to shock loading.

Earlier work by Butcher predicted the spall strength of AA6061-T6 should vary in accordance with the one-dimensional stress yield strength according to orientation, (Butcher, 1968). Further it was concluded that there is no significant effect on crack formation. Other works have also conducted to investigate the behaviour of spall response of orthotropic materials. Stevens and Tuler for instance concluded that the degree of pre compression, that is the shock amplitude, had no effect on the spall strength of AA6061-T6, (Stevens and Tuler, 1971). In addition, it is shown that the spall strength of AA2024-T86 decreased with increasing temperature, (Schmidt et al., 1978). However, both works concerned on geometrical effects. During shock loading, the compressive input stress is details on the rising part of the shock defined by the HEL of the material. It is observed that the shock is reflected back as a release wave when it reaches a free surface, which consequently takes the material back to ambient stress conditions. Releases from the rear of flyer and target can be arranged by controlling the thickness of the specimen and flyer plate to meet in the middle of the target itself, and where they do so, a zone of net tension will result. If that tension exceeds the tensile strength of the material, the net result is failure (spall), which can be detected by appropriate measurement techniques, (Vignjevic et al., 2002).

In many hydrocodes, the common approach to model spall failure using pressure cut-off and maximum principal stress. Pressure cut-off and maximum principal stress can be regarded the simplest models. Generally speaking, the pressure cut-off model compares the pressure with a user defined pressure cut-off value. The material is assumed to have spall when the pressure is less than this pressure cut-off value. The deviatoric stress tensor and the pressure are set to zero, and no hydrostatic tension is subsequently allowed. The pressure is set to zero again with no hydrostatic tension is subsequently allowed and the maximum principal stress criterion is check whether the detected the deviatoric stress tensor.

Grady spall model is an energy-based failure model. It assumes the material spall when the strain energy reaches a certain level. The model provides a rigorous mathematical modelling of fragmentation. Grady reports on results from dynamic compression and dynamic tension (spall) tests (Grady, 1988). Grady failure model was first developed to predict spall in ductile metals in 1997 as performed in Grady and Kipp (1997). This model has been used to analyse fracture and fragmentation of naturally fragmenting munitions of different materials and geometries, (Wilson et al., 2001). The model has shown great ability to predict the test results in terms of fragment mass distributions and proved to be excellent in modelling the breakup of 4140 steel, 70% tungsten, and thick-walled Aermet 100, and reasonable in matching the data from 8-inch Aermet 100 tests. Fountzoulas et al. (2007) have proposed a constitutive model to predict impact and penetration into tungsten carbide sphere into high-strength-low-alloy (hsla)-100 steel targets. Generally, the Grady failure model produced a good agreement with respect to the experimental data of the crater diameter and dept. Further, De Vuyst proposed a constitutive model for impact on water using spall model pressure cut-off, maximum principal stress and Grady spall criterion, (De Vuyst, 2003). The results obtained from the pressure-based spall criterion and the maximum principal stress spall criterion are almost identical. It is observed that these spall models are inadequate to model spall behaviour qualitatively, while a better agreement is obtained using Grady failure model.

In respect to yield function, the first anisotropic homogeneous yield function of degree two that is used to describe an orthotropic plastic response of rolled sheet is proposed by Hill (1948). This concept can be regarded as a solid foundation for the subject in the case of metals. In the literature, numerous researchers have tried to investigate and examine the validity of this basic framework.). The ability to represent the full behaviour of orthotropic materials is one of the advantages of Hill’s yield criterion compared to others. For example, Hill’s formulation in a plane stress case which refers to the principle orthotropic axes and consist of a shear stress component. When the shear parts disappeared the yield function is limited to planar isotropy and this is important as a yield function (Barlat, 1987, Barlat and Lian, 1989).

The benefit presenting that the formulation of Hill’s effective stress conserve this yield criterion homogeneous characteristic. Thus, by using Hill’s yield criterion the convexity of the yield surface is upheld. The expectations of the Hill’s yield criterion are easy to understand in terms of formulation. Furthermore, the parameters consist in the yield functions expression have a direct physical meaning and this is the main cause for the criterion’s wide use in practice. Moreover, this model provides a simple formulation in a three-dimensional case (Banabic, 2010). The yield function meanwhile can be characterised from a low number of mechanical parameters. For the case of plane stress condition, the yield function requires only three parameters.

Method suitable for implementation into a finite element analysis is a procedure for which no simple, highly accurate method exists from explanation of general anisotropic...
yield surface. One consideration particularly motivates the need for modification of the Hill yield criterion. In addition to Hill’s yield function, various types of yield function have been constructed in the literature. For a comprehensive discussion on yield criteria for orthotropic materials, the interested reader is directed to Mohd Nor (2016a).

2.1 Kinematics for Finite Strain Deformation

The construction of the new hyperelastic-plastic constitutive model for orthotropic materials in this work is formulated based on the multiplicative decomposition of the deformation gradient \( F \):

\[
F = F_e F_p
\]  

(8)

The experimental work in Man (1995) shows a strong correlation between elastic and plastic material symmetries. Therefore, the constitutive model described in this manuscript is developed and integrated in the isoclinic configuration. In other words, the hyperelastic part of the constitutive model is based on the assumption that the principal directions of material elastic and plastic orthotropy coincide and are not influenced by inelastic deformation. As emphasized by Aravas (1994), this definition simplifies the numerical implementation since one can steer clear of the explicit use of any corotational rate as definition simplifies the numerical implementation since one can steer clear of the explicit use of any corotational rate as adopted in recent works of Vignjevic et al. (2012), Mohd Nor (2016a), Mohd Nor (2016c). To avoid confusion, \( \hat{\psi} \) is used in this manuscript upon each of kinematic and kinetic variables defined with respect to the isoclinic configuration.

The structural tensors \( M_i \) are pulled back from the elastically unloaded configuration (having an arbitrary orientation) \( \hat{\Omega}_p \) to the isoclinic configuration \( \hat{\Omega}_t \) by rotating back for plastically induced rigid body rotation due to plastic related deformations using below orthonormal transformation.

\[
\hat{\Omega}_t = \mathbf{Q}_p^T \hat{\Omega}_p \mathbf{Q}_p
\]  

(9)

where \( F_e \) and \( F_p \) represent thermo-elastic part of the deformation and plastic part of the deformation (dislocation mechanics) respectively. This concept distinguishes the proposed constitutive model from hypoelastic-plastic material model (when elastic strains are small compared to the plastic strains). As demonstrated by Itskov (2004), this formulation leads to spurious shear stresses which are independent of the elastic material properties for orthotropic materials. In addition, the evolution of material symmetry in orthotropic materials due to large deformations could not be tracked by the additive strain decomposition based model (Itskov and Aksel, 2004).

Using the definition in Equation (8), the elastic right Cauchy-Green tensor \( C_e \) and the elastic Green-Lagrange strain tensor \( E_e \) can be expressed as

\[
C_e = F_e^T \cdot F_e, \quad E_e = \frac{1}{2} (C_e - I) = \frac{1}{2} (F_e^T \cdot F_e - I)
\]  

(9)
where \(Q_p\) is an orthonormal tensor that defines the rigid rotation due to plastic related deformation.

Referring to Figure 1, a triad of unit vectors which represent the material symmetries is schematically shown by two orthogonal axes with arrows. Both elastic stretching and rotation are contained in the elastic part \(F_e\) of the deformation gradient \(F\). Plastic part of \(F\) with respect to \(\Omega_p\) and \(\hat{\Omega}_p\) is represented by \(\hat{F}_p\) and \(\hat{F}_p\) respectively. The plastic rotation \(R_p\) is assigned to \(F_p\) to ensure there is no rotation to the material principal axes of orthotropy (remains fixed or unaltered by plastic deformation) in the isoclinic configuration \(\hat{\Omega}_i\).

As examined in Schröder and Hackl (2014), the rotation and distortion during elastoplastic deformation are contained in the elastic part of \(F\) as a result of isoclinic configuration. Despite of general incompatibility tensor fields between elastic and plastic deformations, the elastic deformation may incidently compatible to plastic deformation when plastic or elastic deformation is homogeneous, (Schröder and Hackl, 2014). Alternatively, the plastic rotation \(R_p\) can be assigned to the deformation due to damage \(\hat{F}_d\) to define \(\hat{F}_e = \hat{F}_e \hat{F}_d R_p\) if one considers the elastic material parameters evolve due to damage since the changes of material compliance due to damage (Vignjevic et. al., 2012). This assumption however not considered in this work.

The elastic and plastic parts of the deformation gradient \(F\) of the proposed constitutive model then can be defined explicitly in the isoclinic configuration as:

\[
\hat{F}_e = F_e, \quad \hat{F}_p = R_p^T F_p = R_p^T R_p U_p = U_p
\]

(11)

where plastic rotation \(R_p\) is an orthogonal rotation tensor induced by plastic deformation and \(U_p\) refers to plastic right stretch tensor. Subsequently, the total velocity gradient \(L\) can be decomposed additively as follows:

\[
\hat{L} = \hat{F} \cdot \hat{F}^{-1} = \hat{F}_e \cdot \hat{F}_e^{-1} + \hat{F}_p \cdot \hat{F}_p^{-1} \cdot \hat{F}_e^{-1} = L_e + L_p
\]

(12)

An incompressibility constraint is assumed to hold for plastic deformation which therefore gives \(\det(\hat{F}_p) = 1\).

### 2.2 Stress Tensor Decomposition for Composite Materials

In general, the Mandel stress tensor \(\Sigma\) can be defined as follows (Mandel, 1972):

\[
\Sigma = C \cdot S
\]

(13)

where \(C\) and \(S\) refer to Right Cauchy-Green tensor and Second Piola Kirchhoff stress tensor respectively. These tensors can be expressed in the elastically unloaded intermediate configuration \(\Omega_p\) as

\[
\hat{C}_e = F_e^T \cdot F_e, \quad \hat{S} = F_e^{-1} \cdot \hat{\tau} \cdot F_e^-T
\]

(14)

(15)

The Kirchhoff stress tensor \(\tau\) in Equation 15 is a symmetric tensor defined in the current configuration \(\Omega_t\) as

\[
\tau = J \cdot \sigma = \det(F) \cdot \sigma
\]

(16)

where \(J\) is a volume ratio. Substituting Equations 14 and 15 into Equation 13, the Mandel stress tensor in the intermediate configuration \(\Omega_p\) can be defined as

\[
\hat{\Sigma} = \hat{F}_e^T \cdot \hat{\tau} \cdot \hat{F}_e^{-T} = \det(F) \cdot \hat{F}_e^T \cdot \sigma \cdot \hat{F}_e^{-T}
\]

(17)

The Mandel stress tensor defined in Equation 17 is frequently used to describe the behaviour of plastic materials (Holzapfel, 2007). This stress tensor is adopted in this work to formulate the new constitutive model for orthotropic metals.

The formulation of the new generalized pressure for orthotropic metals is introduced in Equation 20:

\[
\hat{\Sigma} = \det(F) \cdot \hat{F}_e^T \cdot \left( S + \frac{\sigma \psi}{\psi \psi} \right) \cdot \hat{F}_e^{-T}
\]

(21)

Equation 21 can be further extended as

\[
\hat{\Sigma} = \hat{\Sigma}_p \cdot \text{deviatoric} + \det(F) \cdot \hat{F}_e^{-T} \cdot \frac{\sigma \psi}{\psi \psi} \cdot \psi \cdot \hat{F}_e^{-T}
\]

(22)

where \(\hat{\Sigma}_p\) denotes the volumetric part (pressure) of the new Mandel stress tensor. Focusing on the deviatoric part instead of a full stress tensor, the deviatoric part of the new Mandel stress tensor is given by

\[
\hat{\Sigma}' = \det(F) \cdot \hat{F}_e^T \left( \sigma - \frac{\sigma \psi}{\psi \psi} \cdot \psi \right) \hat{F}_e^{-T}
\]

(23)
Finally the new deviatoric Mandel stress tensor defined in the isoclinic configuration \( \hat{\Sigma}_i \), can be written as
\[
\hat{\Sigma}' = \det(F) \cdot \hat{\Sigma}_{p} - \det(F) \cdot \hat{\Sigma}_{e} - \psi \cdot \hat{\Sigma}_{e}^{-T}
\]
It can be easily proven the deviatoric component of the new Mandel stress tensor Equation 24 is traceless (deviator tensor). In this work, only the symmetric part of the Mandel stress is considered, (Vladimirov et al., 2008; Reese and Vladimirov, 2008).

2.3 Equation of State for Modelling Shock Waves
Appropriate constitutive equations to describe the strength effect and the equation of state must be investigated to describe the anisotropic material response under shock loading. Therefore, in this work, the formulation is combined with an equation of state (EOS) in addition to the conservation laws to mathematically describe the material’s non-linear behaviour and propagation of strong shock waves in solids due to shock loading.

An EOS represents a closure equation, which completes the relationships between the state variables in front of and behind a shock wave. Theoretically, the relationship described by EOS can be determined from the thermodynamic properties of the material, and require no dynamic data. However, practically, extensive experimental test such as the planar shock wave experiment is required to characterize data on the material’s behaviour at high strain rates. In contemporary hydrocodes available EOS’s are either of an analytical or a tabulated type. In this manuscript a very popular EOS that is extensively used for solid continua is the Mie-Gruneisen EOS (1959), implemented in DYNA3D is used. This an analytic EOS frequently used with solid materials that define the pressure as a function of density \( \rho \) or specific volume and specific internal energy \( e \).

The combination between the proposed stress tensor decomposition and the Mie-Gruneisen EOS requires some modifications to reflect the formulation of the generalized orthotropic pressure. Briefly, \( \psi \) is calculated using the material stiffness matrix \( C \) read from the input file. The increment of deviatoric Mandel stress tensor \( \hat{\Sigma}' \) is then calculated using rate of deformation tensor \( \hat{D} \). By setting pressure equals to \( P_{EOS} \), the stress update at time \( n + 1 \) can be defined as
\[
\sigma^{n+1} = \hat{\Sigma}^{n+1} - \rho^{n+1}_{EOS} \psi
\]

2.4 Elastic Free Energy Function
The behaviour of orthotropic metals within elastic and plastic regimes is formulated as a free strain energy function and a plastic level set function (orthotropic yield criterion) respectively. As mentioned previously, an orthotropic symmetry group \( \vartheta \) is used in this work and assumed to be unchanged during plastic deformation. The Helmholtz free energy is used to formulate the elastic orthotropy as a function of evolving structural tensors. The Helmholtz free energy is additively decomposed into elastic and plastic parts in the isoclinic configuration \( \hat{\Sigma}_i \) as
\[
\varphi = \varphi_e(\hat{\Sigma}_e) + \varphi_p(\hat{\Sigma}_p)(\alpha)
\]
where \( \varphi_e(\hat{\Sigma}_e) \) refers to the energy stored due to elastic deformations. This formulation defined in terms of Elastic Green-Lagrangian strain tensor \( \hat{E}_e \). The energy resulting from isotropic plastic hardening is represented by \( \varphi_p(\hat{\Sigma}_p)(\alpha) \) where \( \alpha \) defines the isotropic hardening variable. The elastic material response is set invariant under transformations of the material symmetry group \( \vartheta \): \( \varphi_e(\hat{\Sigma}_e, \hat{M}_{11}, \hat{M}_{22}) = \varphi_e(\hat{Q}{\hat{E}_e}{\hat{Q}}^T, \hat{Q}M_{11}{\hat{Q}}^T, \hat{Q}M_{22}{\hat{Q}}^T) \), where \( Q \) is orthogonal rotation tensor.

2.5 Orthotropic Yield Criterion
The yield function used to model the dependence on plastic anisotropy is defined using the structural tensors \( \hat{M}_{ii} \). Further, the Hill’s yield criterion is adopted to define the orthotropic yield function (Hill, 1948). For a range of strain rates, it is shown in Maudlin et al., (1999a) and Bronkhorst et al., (2006) that yield surface remained the same shape on BBC tantalum. Using this hypothesis, the hardening is modelled as an isotropic hardening in this constitutive model. It is therefore, the yield surface will always maintain its initial shape (only change in size, no change in shape). Using Equation 24, the yield function can be written in terms of the symmetric Mandel stress tensor as
\[
f = f(\hat{\Sigma}', \alpha)
\]
where \( \alpha \) is an isotropic hardening variable. The structural tensors \( \hat{M}_{ii} \) is introduced to describe the properties of symmetric orthotropy as follows:
\[
f = f(\hat{\Sigma}', \hat{M}_{ii}, \alpha)
\]
Accordingly, the plastic anisotropy of the new constitutive model is characterized by Hill’s anisotropy yield function as follows:
\[
f = \sqrt{\hat{\Sigma}' : \hat{h}}: \hat{\Sigma}' - \hat{f}(\alpha) = 0
\]
where \( \hat{h} \) is a fourth-order tensor defined in the isoclinic configuration \( \hat{\Sigma}_i \). The dependence of the above yield function on Hill’s yield criterion and structural tensors is represented by this tensor. \( \hat{f}(\alpha) \) in the above equation defines the evolving flow stress that is controlled by isotropic hardening. The Hill’s effective stress can be expressed in terms of the deviatoric Mandel stress in the isoclinic configuration \( \hat{\Sigma}_i \) as follows:
\[ \textbf{\Sigma} = \frac{1}{2} \left[ \begin{array}{c} F (\Sigma_{y} - \Sigma_{x})^2 + G (\Sigma_{x} - \Sigma_{y})^2 + H (\Sigma_{r} - \Sigma_{y})^2 + 2L \Sigma_{z}^2 + 2M \Sigma_{x}^2 + 2N \Sigma_{y}^2 \end{array} \right] \]

\[ F + G + H \]

2.6 Grady Failure Model

Generally, Grady model is used to check how much energy is required. Grady (energy based model) stand out due to their large scale used models regarding fragmentation phenomenon (Tranà et al., 2015). In this Grady model, two mechanisms are investigated which are brittle and ductile fracture. In the case of brittle fractured the energy that is required based on the critical fracture toughness. In contrast, For the case of ductile fracture is based on work required to reach failure strain. This procedure then results in the following spall strengths:

\[ p_{s(\text{ductile})} = 2 \rho c_0^2 \sigma_f \varepsilon_{\text{fail}} \]  \text{Ductile Failure} \tag{31}

\[ p_{s(\text{brittle})} = 3 \rho c_0 K_c^2 \varepsilon \]  \text{Brittle Failure} \tag{32}

where:

- \( \rho \) = density
- \( c_0 \) = bulk sound speed
- \( \sigma_f \) = yield stress
- \( \varepsilon_{\text{fail}} \) = Critical strain failure
- \( K_c \) = Fracture toughness
- \( \dot{\varepsilon} \) = Rate of volumetric dilatation

This spall stress is calculated for each cell at each cycle, thus including the local conditions in the cell. The calculated spall stress is used as the local maximum principal stress failure criterion in the cell. For a certain strain rate there is a transition point between ductile and brittle spall. This critical strain rate, \( \varepsilon_{\text{crit}} \) can be calculated from:

\[ \dot{\varepsilon}_{\text{crit}} = \sqrt{\frac{2a_b (\sigma_f \varepsilon_{\text{fail}})^2}{9 \rho k_6}} \] \tag{33}

where:

- \( \rho \) = density
- \( \sigma_f \) = yield stress
- \( \varepsilon_{\text{fail}} \) = Critical strain failure
- \( K_c \) = Fracture toughness
- \( B \) = Isotropic/kinematic hardening

3. Results and Analysis

The proposed formulations in the preceding sections are implemented into the LLNL-DYNA3D of UTHM’s version and named Material Type 92 (Mat92). The work is structured to deeply examine and validate each part of the proposed formulation including the book keeping of the algorithm to deal with multiple elements analysis (complex structure). (Mohd Nor, 2016a), (Mohd Nor and Ma’at, 2016), and Mohd Nor et al., 2016). At the end of this process, a new elastoplastic-spall failure constitutive model for orthotropic materials is considered successfully implemented and validated. A cm – g – \( \mu \)s unit system is adopted in the involved numerical tests.

3.1 Analysis on Aluminium Alloy

The capability of the proposed constitutive model to predict the deformation behaviour of orthotropic materials under high pressure and shockwave including spall failure is first examined using the experimental data of Plate Impact test published in De Vuyst (2003). Figure 2 shows the configuration used to simulate Plate Impact test analysis for aluminium alloy in this work. The model is divided into three parts of rectangular bars with 4x4 solid elements for its cross section (symmetrical XY plane) the PMMA block, test specimen and flyer which is modelled with 100 solid elements (12mm in length), 75 solid elements (10mm in length) and 25 solid elements (2.5mm in length) respectively. As can be observed, the model orientation is parallel to Z axis (impact axis). The mesh applied to the model is set to allow a 1D wave propagation along bars during the impact. A non-reflecting boundary condition is used to define the back of the Poly (methyl methacrylate) (PMMA) block. There is a contact interface defined in between the test specimen and the flyer (impactor). A time history block is prescribed to the elements at the top of PMMA bar to ensure the stress time histories is recorded during the impact.
Fig. 2. Configuration of the Plate Impact test simulation for Aluminium Alloy

The Z stress in the top elements of the PMMA bar is compared against the experimental data in short transverse and the longitudinal (rolling) directions of the specimen, performed at three different impact velocities: 234 ms\(^{-1}\), 450 ms\(^{-1}\) and 895 ms\(^{-1}\). Table I shows the material properties of each part including the Gruneisen EOS and the Grady failure model adopted in this analysis. The flyer is prescribed as Aluminium 6082 – T6. Mat10 is used to describe the behaviour of the flyer and the PMMA blocks.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Al7010</td>
</tr>
<tr>
<td>Young's modulus</td>
<td></td>
</tr>
<tr>
<td>(E_a)</td>
<td>70.6 GPa</td>
</tr>
<tr>
<td>(E_p)</td>
<td>71.1 GPa</td>
</tr>
<tr>
<td>(E_c)</td>
<td>70.6 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td></td>
</tr>
<tr>
<td>(\nu_{ba})</td>
<td>0.342</td>
</tr>
<tr>
<td>(\nu_{ca})</td>
<td>0.342</td>
</tr>
<tr>
<td>(\nu_{ab})</td>
<td>0.342</td>
</tr>
<tr>
<td>Shear modulus</td>
<td></td>
</tr>
<tr>
<td>(G_{bc})</td>
<td>26.31 GPa</td>
</tr>
<tr>
<td>(G_{ab})</td>
<td>26.48 GPa</td>
</tr>
<tr>
<td>(G_{ac})</td>
<td>26.48 GPa</td>
</tr>
<tr>
<td>Yield stress in a – direction</td>
<td>(\sigma_y)</td>
</tr>
<tr>
<td>Tangent plastic modulus in a – direction</td>
<td>H</td>
</tr>
<tr>
<td>Pressure cutoff</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td></td>
</tr>
<tr>
<td>Hill’s parameters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5200 ms(^{-1})</td>
</tr>
<tr>
<td>(s_1)</td>
<td>1.36</td>
</tr>
<tr>
<td>(s_2)</td>
<td>0.00</td>
</tr>
<tr>
<td>(s_3)</td>
<td>0.00</td>
</tr>
<tr>
<td>(\Gamma)</td>
<td>2.2</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.48</td>
</tr>
<tr>
<td>Gruneisen parameters</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(\rho)</td>
<td>2.81 gcm(^{-3})</td>
</tr>
<tr>
<td>(c_0)</td>
<td>0.52</td>
</tr>
<tr>
<td>(\varepsilon_{fail})</td>
<td>0.5</td>
</tr>
<tr>
<td>(K_f)</td>
<td>0.0025</td>
</tr>
<tr>
<td>(B)</td>
<td>(\rho/c_0^2)</td>
</tr>
</tbody>
</table>

The material axes AOPT 2 is set to \(a = 0a_x + 0a_y + 1a_z\) and \(d = 0d_x + 1d_y + 0d_z\) in these analyses. The results obtained for each impact velocity in respect to both longitudinal and transverse directions are shown in Figures 3 until 8.
Fig. 3. Longitudinal Stress at 234ms$^{-1}$ impact in longitudinal direction

Fig. 4. Longitudinal Stress at 234ms$^{-1}$ impact in transverse direction
Fig. 5. Longitudinal Stress at 450m/s$^{-1}$ impact in longitudinal direction

Fig. 6. Longitudinal Stress at 450m/s$^{-1}$ impact in transverse direction
Referring to the results in the above figures, it can be observed that the proposed Mat92 is capable of describing the elastic-plastic loading-unloading behaviours of the Al7010 since a close prediction is obtained in each test. The comparison between Mat92 and the experimental data are summarised in Table II.
In addition, the generated pulse and the Hugoniot accuracy to deal with a complex spall strength to describe the behaviour of orthotropic materials. The similar behaviour is close agreement is observed in respect to Hugoniot stress levels. The longitudinal and transverse directions are slight difference in each direction. Specifically, the value in the longitudinal stress increment. An adequate level of anisotropy of the material is reflected by a different HEL value in each direction. I

In general, the shape of generated pulse shows a good agreement compared to the experimental data. The Hugoniot Elastic Limit (HEL) is described by the initial slope of the longitudinal stress increment. An adequate level of anisotropy of the material is reflected by a different HEL value in each direction. In addition, the generated pulse and the Hugoniot stress levels are closely agreed with the experimental data to confirm the elastic-plastic formulation including the newly implemented orthotropic pressure of the proposed constitutive model is capable to describe the behaviour of orthotropic material undergoing finite strain deformation.

The simulated longitudinal stress at the interface between the target material and PMMA block (recorded in the elements at the back of the specimen), for plate impact at 234 ms\(^{-1}\) is presented in Figure 3 and Figure 4. It can be clearly seen that the tensile wave failure or spall (demonstrated by the reloading of the longitudinal stress after the first loading-unloading pulse) is not generated with a lower impact velocity (234ms\(^{-1}\)). However, a clear spall criterion can be observed when higher impact velocities (450ms\(^{-1}\) and 895ms\(^{-1}\)) are applied. At 450ms\(^{-1}\) impact velocity, a clear break in slope in the start rising part of trace (HEL) can be seen in both traces as shown in Figure 5 and Figure 6. The value of the HEL shows a slight difference in each direction. Specifically, the value in the longitudinal and transverse directions are 0.41 GPa and 0.38 GPa, respectively. The associated value obtained in the experiment are 0.39 GPa and 0.33 GPa, respectively. A very close agreement is observed in respect to Hugoniot stress levels. This confirmed the capability of the implemented orthotropic pressure of Mat92 to capture shockwaves in orthotropic materials.

In addition, a clear pull back signals (spall) is developed in both traces; measured 0.31 GPa in the longitudinal and 0.42 GPa in the transverse direction. The values obtained using the Grady’s spall criterion are 0.21 GPa and 0.10 GPa in the longitudinal and transverse directions, respectively. It can be observed the values are smaller than the values in experiment. The constitutive model however, shows sensitivity to the direction of impact. This behaviour is depicted by the point where the spall starts. The similar behaviour is obtained experimentally.

Figure 7 and Figure 8 show the results for 895ms\(^{-1}\) impact velocity. It is clear that the shape of the pulse including the Hugoniot stresses show a good agreement with the experimental data in both impact directions as summarised in Table 2. As observed in 450ms\(^{-1}\) impact velocity, again the proposed constitutive model predict a smaller pullback signals in both impact directions.

Referring to the results, the model as it stands is capable to simulate elastic-plastic, shockwave propagation and spall failure in orthotropic materials of aluminium alloy AA7010. The general pulse shape, the Hugoniot stress level and the EOS are predicted satisfactorily. A higher HEL is observed in the longitudinal direction compared to the short transverse. The Hugoniot stress levels show a very good prediction capability. The pulse shape including the width are in line with the experimental data. The adopted Grady’s spall failure model is obviously capable to predict spall in the materials. However, it must be emphasised that the criterion is not capable to provide a very good accuracy to deal with a complex spall strength evolution developed at different impact velocity. Further works, therefore, are required in both experimental and constitutive modelling aspects.

<table>
<thead>
<tr>
<th>Impact Velocity/ Direction</th>
<th>Analysis Criteria</th>
<th>Pulse ((\mu)s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>234ms(^{-1}) (Longitudinal):</td>
<td>HEL (GPa)</td>
<td>Hugoniot Stress Level (GPa)</td>
</tr>
<tr>
<td>Simulation</td>
<td>0.40</td>
<td>0.64</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.39</td>
<td>0.65</td>
</tr>
<tr>
<td>234ms(^{-1}) (Transverse):</td>
<td>Simulation</td>
<td>0.40</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.33</td>
<td>0.63</td>
</tr>
<tr>
<td>450ms(^{-1}) (Longitudinal):</td>
<td>Simulation</td>
<td>0.41</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.43</td>
<td>1.31</td>
</tr>
<tr>
<td>450ms(^{-1}) (Transverse):</td>
<td>Simulation</td>
<td>0.38</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.39</td>
<td>1.38</td>
</tr>
<tr>
<td>895ms(^{-1}) (Longitudinal):</td>
<td>Simulation</td>
<td>0.36</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.21</td>
<td>3.25</td>
</tr>
<tr>
<td>895ms(^{-1}) (Transverse):</td>
<td>Simulation</td>
<td>0.32</td>
</tr>
<tr>
<td>Experiment</td>
<td>0.19</td>
<td>2.80</td>
</tr>
</tbody>
</table>
### 3.1 Analysis on Carbon Fibre Reinforced Epoxy Composites

The capability of the proposed constitutive model to predict shockwave of impact on composite target, manufactured from woven carbon fibres-epoxy plies is finally conducted in this work. The impact is set normal to the fibre direction to ensure the shockwave propagated in the through thickness direction of the composite. Figure 9 shows the configuration of the finite element model used in this analysis where the carbon fibre composite target plate is modelled as a quasi-orthotropic material.

#### Figure 9. Configuration of the Plate Impact test simulation for Carbon Fibre/Epoxy Composite

The material properties of the woven composite plate are determined from the layer macro mechanical properties is provided in Table 3 for the lay up $[0/90, \pm 45]$ (Millet et al., 2007). The longitudinal speed of sound for the initial material density $\rho_0 = 1500 \text{kg/m}^3$ is set as $C_0 = 3020 \text{ms}^{-1}$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus in longitudinal direction</td>
<td>$E_a$ 68.5 GPa</td>
</tr>
<tr>
<td></td>
<td>$E_b$ 66.5 GPa</td>
</tr>
<tr>
<td></td>
<td>$E_c$ 10.0 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu_{ba}$ 0.039</td>
</tr>
<tr>
<td></td>
<td>$\nu_{ca}$ 0.0044</td>
</tr>
<tr>
<td></td>
<td>$\nu_{cb}$ 0.045</td>
</tr>
<tr>
<td>Shear modulus</td>
<td>$G_{bc}$ 3.57 GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{ab}$ 4.57 GPa</td>
</tr>
<tr>
<td></td>
<td>$G_{ac}$ 3.57 GPa</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$ 1500 kgm$^{-3}$</td>
</tr>
</tbody>
</table>

Bear in mind that the $\textbf{a}$ and $\textbf{b}$ directions of the material properties defined in Table 3, correspond to the $\textbf{x}$ and $\textbf{y}$ coordinate axes. Compared to the previous analysis on aluminium alloy, this analysis involved metal flyer plates impacting a target with a surface metal cover. The material defined for both the metal cover and the flyer is identical. Again a thick PMMA is used at the back of the setting, and the test is modelled using a uniaxial strain state under the adiabatic assumption for the deformation process. The same boundary conditions setting is adopted. A surface to surface contact algorithm is used at the flyer target interface.

The aluminium alloy flyer plate is assigned with 504 m/s impact velocity, as measured in the experiment (Vignjevic et al. 2005 and Millet et al., 2007). The flyer plate is 5 mm thick and modelled with 30 elements. The test specimen is modelled with 142 elements parallel to the impact axis ($Z$ axis). Referring to the actual experiment, 1 mm aluminium alloy plate was used to cover the front gauge while 12 mm of PMMA was adopted for the rear gauge. The thickness of the composite plate specimen is 3.8 mm. The Isotropic-Elastic-Plastic-Hydrodynamic material model named Material Type 10 (Mat10) that available in the DYNA3D code is used to model the aluminium plate. Table 4 shows the elastic material properties defined for the aluminium. The density of aluminium is 2703 kgm$^{-3}$. The Mie-Grüneisen EOS parameters for both the aluminium and composite materials are given in Table V. The data for PMMA is taken from Table I.
Table IV
Aluminium Alloy (AA7010) material properties

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Aluminium</th>
<th>Composite</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s Modulus</td>
<td>290.00 MPa</td>
<td>27.60 GPa</td>
</tr>
<tr>
<td>Poisson’s Ratio</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Shear Modulus</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table V
Aluminium and material properties

The results obtained (stress along the axis of impact - Z axis) are shown in Figures 10 and 11. In this analysis the Z stress at the front surface of the composite material is compared with the stress history from the front gauge. Meanwhile, the stress in PMMA is compared with the measurements from the rear gauge.

Fig. 10. Comparison between the stress obtained from the front gauge and Mat92
The value of stress between gauge and the corresponding finite element is directly comparable since the shock front is planar and parallel to the composite plies in this analysis. It can be clearly seen that the results obtained using the newly constitutive model show a good agreement of the stress magnitude and the pulse length with corresponding experimental measurements. In addition, the proposed constitutive model agrees with the experiment in terms of separation between the flyer and the cover plate i.e. the flyer plate stays in contact with the cover plate. It should be noticed that the constitutive model also provides a correct prediction in the previous analysis on aluminium alloy i.e. separation of the flyer plate from specimen plate after the event of impact.

4. CONCLUSION
A new hyperelastic-plastic constitutive model for orthotropic materials undergoing shockwave propagation and spall failure is proposed in this manuscript. The proposed formulation is the key novelty of this work – and in fact a new finding in this field where a new Mandel stress tensor that is combined with the new stress tensor decomposition of the generalised pressure is adopted. The formulation is developed in the isoclinic configuration using a multiplicative decomposition of the deformation gradient framework, and further combined with Equation of States (EOS). The expansion of orthotropic yield surface is performed in a unique alignment of deviatoric plane within the principal stress space that is defined uniquely by the elastic properties of orthotropic materials. In order to test their ability to describe the deformation behaviour in orthotropic materials, the proposed algorithms are implemented in a Lawrence Livermore National Laboratory (LLNL)-HYNA3D code of UTHM’s version, named Material Type 92 (Mat92). The results obtained by the newly constitutive model are first compared with the experimental plate impact data of aluminium alloy in the longitudinal and transverse directions, before the prediction against the plate impact test data of carbon fibre reinforced epoxy composites is finally performed. It can be seen that the proposed formulation showed good agreement with the physical behaviour of the considered material in each test. This achievement is a good indication for more appropriate orthotropic constitutive models in future in order to help towards a better understanding of the complexity of material orthotropy undergoing finite deformation.

5. ACKNOWLEDGEMENT
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6. REFERENCES


