Effect of Liquid Flow Rate on Droplet Size Distribution of Induced Atomized Charged Droplets

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Abstract—The droplets size distribution of electrostatic atomization is considered a very important factor for evaluating the atomization effect and its applicability. Different kinds of research like theoretical analysis, experimental research and numerical modeling method are carried out to analyze the effect of liquid flow rate on droplet size distribution of electrostatic induction atomization. The results show that, droplet size will increase obviously as the flow rate is increased and geometric standard deviation (GSD) index of atomizing liquid is dropped because of the decrement of droplets induction time. The mathematical expression of droplet size, geometric standard deviation (GSD) index and flow rate of the liquid medium are established by fitting the research data which are tested experimentally with highest accuracy.

Index Term—Liquid medium flow, Electrostatic atomization, Induction charge, Droplet diameter, GSD.

1 Principle Analysis

The influence of flow rate on droplet size can be analyzed by the Euler method of fluid motion analysis. In fluid mechanics [4], Euler method is mainly studied for continuous variation of time (t) and particle location (x, y, z). At First, time (t) and particle location (x, y, z) are used to characterize each physical quantity. In space, particles will have different velocities at different locations (x, y, z) at the same time. Also, at different times in the same location, particles with the locations (x, y, z) will also have different speeds. So the velocity of fluid particle is different from its location (x, y, z) and time (t). In the equation (1), ux, uy, and uz are used to indicate the velocity of particles in x, y and z directions respectively at time t. \( \vec{u} \) indicates the velocity vector of the particle.

\[
\begin{align*}
ux &= u_x(x, y, z, t) \\
uy &= u_y(x, y, z, t) \\
uz &= u_z(x, y, z, t) \\
\vec{u} &= \vec{u}(x, y, z, t)
\end{align*}
\] (1)

\( \vec{u} \) can be expressed as

\[
\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}
\] (2)

In which, \( \hat{i}, \hat{j}, \hat{k} \) are the unit vector respectively to x, y, z direction. Because of continuous changes of all physical quantities, there is a certain relationship between the velocity and acceleration of each fluid particle which can be expressed as derivative form as:

\[
\begin{align*}
\alpha_x &= \frac{du_x}{dt} \\
\alpha_y &= \frac{du_y}{dt} \\
\alpha_z &= \frac{du_z}{dt}
\end{align*}
\] (3)

Here, multivariable partial derivative operation is carried out for equation (2) and substituted into equation (3), then
is displacement acceleration is the sum of gravity vector speed to
\[ a_x = \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \]
\[ a_y = \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \]
\[ a_z = \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \] (4)

Both sides of the equation (2) are solved and then the acceleration vector \( \vec{a} \) can be obtained from equation (3) and (4).
\[ \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \] (5)

In equation (5), acceleration vector \( \vec{a} \) is composed of time varying acceleration \( \vec{a}_t \) and displacement acceleration \( \vec{a}_p \) i.e. the equivalent flow of jet model is equivalent to a constant flow. In the region where the electric field intensity is zero, the displacement acceleration is the sum of gravity acceleration and buoyancy deceleration. But when the electric field intensity of the particle is not zero, for example \( x \) kv/m, the charged droplet is subjected to electric field force in the direction of electrode’s gravity center. At this point, the displacement acceleration can be expressed as:
\[ \vec{a}_p = \frac{-q}{3\pi\eta A} \left( \frac{u^2}{\sqrt{x^2+y^2+z^2}} + a - \varphi m \right) \vec{k} + \frac{q}{3\pi\eta A} \left( \frac{u}{\sqrt{x^2+y^2+z^2}} \right) \vec{i} + \frac{q}{3\pi\eta A} \left( \frac{y}{\sqrt{x^2+y^2+z^2}} \right) \vec{j} \] (6)

The time-varying acceleration can be expressed as, \( \vec{a}_t = \frac{\partial \vec{u}}{\partial t} \). But at different liquid velocities, initial droplets energy flowing from the nozzle is different which leads to change of \( \vec{a}_t \); and droplets will have the influence area of electrostatic field at different times\(^5\). The charge quantity \( Q \) of charged droplets in the electric field is related to the flight time \( t \). When the flow rate is increased, the flight time of charged droplets in the electrostatic field will be shortened. Besides, it takes time to break up the old ionic atmosphere or create a new ionic atmosphere for the charged droplets that is the relaxation time, which is influenced by electrical conductivity\(^6\).
\[ \sigma \tau = \frac{n \times e^2}{m_e^*} \] (7)

Here, \( \sigma \) is electrical conductivity, \( \tau \) is approximate relaxation time, \( m_e^* \) is electronic mass, \( n \) is ion concentration.

In the electrostatic atomization system, when the liquid medium is determined, \( n \) and \( \frac{1}{m_e^*} \) are constant, so it can be seen that the conductivity \( \sigma \) is inversely proportional to the relaxation time \( \tau \) i.e. the higher the conductivity, the shorter the relaxation time and droplets charge rate will be better in the same flight time and splitting charged droplets will be smaller as well. So, the flow rate of liquid medium will affect the charged quantity of droplets and affect the droplet size distribution of charged droplets.

2 Experiment Research

The experimental system is adopted a linear model with a torque of 1.8 degree and a minimum thread of 5 mm. The PWM control model is used to control the motor speed to change liquid flow in the nozzle, the minimum liquid flow rate is 0.083 ml/s. The liquid medium is made of 3% Nacl solution, ring electrode is used to provide electrostatic field, the potential is set to 6 KV, coomassie brilliant blue (G-250) is used as a dye, which is existed as molecular state in the water and has little effect on conductivity of the solution\(^7\). By sampling the droplets in the section area and capturing photos by electron microscope, pictures of the charged droplets distribution at different flow rate are obtained. By using the Arcgis software, gray scale image is converted into grid pixel and the pixels area on sampling board is calculated. According to the camera parameters of microscope, the picture size is 3024×3024 with 9144576 pixels and the sampling area is 60 mm×70 mm. The conversion formula can be obtained by the equation (8).
\[ D = 15874052 \times \sqrt[6]{ \frac{n \times 6.3863 \times 10^{-4}}{\pi} } \] (8)

Here, \( N \) is the pixel area and \( D \) is the diameter of charged droplets in the sampling board.

After getting the diameter of charged droplet \( D \) of sampling board, it is also necessary to calculate the actual droplet diameter \( D' \) of charged droplets based on \( D \), because charged droplets are hemispherical on the sampling board and
charged droplets are spherical \[8\], as the volume from sphere to hemisphere does not change, so

\[
\frac{4}{3}\pi \left(\frac{D'}{2}\right)^3 = \frac{1}{2}\pi \left(\frac{D^2}{2}\right)
\]

Equation (9) can be simplified into \(D' \approx 0.7937D\). After calculating the droplet diameter of all charged droplets on sampling board, the average droplet size of charged droplets can be calculated. The experiment uses eight different flow rates from 0.083 ml/s to 0.372 ml/s, the results are shown in figure 1-8.

As shown in fig. 1-8, the droplet size distribution is a distribution map of charged droplet pixels area, where X-axis is pixel area and Y-axis is number of droplets. The GSD index of atomized droplets can be calculated by the equation (10) \[9\].

\[
GSD = \exp\left(\sqrt[\ln(n)]{\frac{\ln(n)}{n}}\right) \quad (10)
\]

In the equation (10), \(u_g = \sqrt{d_1d_2d_3\cdots d_n}\). \(D_i\) is the \(i\)th droplet diameter and \(n\) is the number of droplets.

Eight groups of experiments with different flow rates are done under the same condition as figure 1. By calculating
the pixel area in Arcgis, the average droplet diameter and GSD index of charged droplets at different liquid flow rate are obtained. The results are shown in table I.

![Table 1](image)

<table>
<thead>
<tr>
<th>Liquid flow rate (ml/s)</th>
<th>Average droplet size (mm)</th>
<th>GSD index</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.083</td>
<td>0.56678</td>
<td>1.192856</td>
</tr>
<tr>
<td>0.124</td>
<td>0.59524</td>
<td>1.175692</td>
</tr>
<tr>
<td>0.166</td>
<td>0.61010</td>
<td>1.121871</td>
</tr>
<tr>
<td>0.207</td>
<td>0.62354</td>
<td>1.263048</td>
</tr>
<tr>
<td>0.248</td>
<td>0.71805</td>
<td>1.366595</td>
</tr>
<tr>
<td>0.290</td>
<td>0.79374</td>
<td>1.315572</td>
</tr>
<tr>
<td>0.331</td>
<td>0.80021</td>
<td>1.443154</td>
</tr>
<tr>
<td>0.372</td>
<td>0.81378</td>
<td>1.623072</td>
</tr>
</tbody>
</table>

It can be seen from table (I) that the average droplet diameter of charged droplets is increased with the increase of flow rate. The minimum average droplet size of the charged droplet is 0.56678 mm when liquid flow rate is 0.083ml/s and the maximum value is 0.81378 mm when the liquid flow rate is 0.372 ml/s. The GSD index is showed a nonlinear variation with the increase of flow rate. When the liquid flow rate is 0.166ml/s, the GSD index of charged droplet is the least, which is 1.121871. When the liquid flow rate is 0.372ml/s, the GSD index of charged droplet is the largest, which is 1.623072. The analysis results are as follows: the acceleration of charged droplet is composed of the time-varying acceleration, \( \ddot{a}_t \) and the displacement acceleration, \( \ddot{a}_p \).

When the flow rate is increased, the velocity of the solution is also increased and the time-varying acceleration \( \ddot{a}_t \) increased as well. The increasing coefficient is the same as the increasing coefficient of velocity. Due to the increase of \( \ddot{a}_t \), the droplet charge amount Q is decreased according to the equation (6) and acceleration in the direction of \( \vec{k} \) is increased with same gravity acceleration \( \alpha \) as the buoyancy deceleration \( \bar{y}_m \) and acceleration in the direction of \( \vec{f} \) and \( \vec{j} \) is decreased because the jet mode under the experimental condition has a constant flow in the direction of \( \vec{k} \). Therefore, when the flow rate is increased, the droplet charge amount Q is decreased, the acceleration in the direction of \( \vec{k} \) is increased, the flight time (t) of charged droplet in the electric field is decreased and the average droplet size of charged droplet is increased. This is because of the rapid flow of liquid medium that causes liquid ejection from the nozzle and falls on the sampling board[10], when the liquid is not fully charged.

3 DATA PROCESSING AND VERIFICATION

In order to further analyze the relationship among them, the experimental data are fitted [11], the flow rate is set as the independent variable, \( x \). The average droplet diameter and GSD index are set as the dependent variables, \( y \). For a given set of discrete data \( \{x_i, y_i\}_{i=1}^n \), the sum of squared error \( f(x_i) - y_i \) at node \( x_i \) can be calculated as:

\[
F(a_0, a_1 \ldots a_n) = \sum_{i=0}^{n}(a_0 + a_1x_i + \cdots + a_nx_i^n - y_i)^2
\]

(11)

According to the necessary condition of the function, set \( \frac{\partial F}{\partial a_k} = 0 \; \; k = 0,1 \ldots, n \), then:

\[
2\sum_{i=0}^{n} x_i^k (a_0 + a_1x_i + \cdots + a_nx_i^n - y_i)^2 = 0 \; \; k = 0,1 \ldots, n
\]

(12)

Equation (12) can be reduced to:

\[
a_0 \sum_{i=0}^{n} x_i^k + a_1 \sum_{i=0}^{n} x_i^{k+1} + \cdots + a_n \sum_{i=0}^{n} x_i^{k+n} = \sum_{i=0}^{n} x_i^k y_i \; \; k = 0,1 \ldots, n
\]

(13)

\[
\begin{bmatrix}
  n + 1 & \sum_{i=0}^{n} x_i & \cdots & \sum_{i=0}^{n} x_i^n \\
  \sum_{i=0}^{n} x_i & \sum_{i=0}^{n} x_i^2 & \cdots & \sum_{i=0}^{n} x_i^{n+1} \\
  \vdots & \vdots & \ddots & \vdots \\
  \sum_{i=0}^{n} x_i^n & \sum_{i=0}^{n} x_i^{n+1} & \cdots & \sum_{i=0}^{n} x_i^{2n}
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_n
\end{bmatrix}
= \begin{bmatrix}
\sum_{i=0}^{n} x_i y_i \\
\vdots \\
\sum_{i=0}^{n} x_i^n y_i
\end{bmatrix}
\]

(14)
Equation (13) is a group of \( n + 1 \) linear order equations about \( a_0, a_1 \ldots a_n \). The matrix of the system is expressed as equation (14).

The function equation of liquid flow rate with average droplet size and GSD exponent can be obtained by taking the data from table (1) into equation (14). Among them, the relation between the liquid flow rate and average droplet size is:

\[
f(x) = 2.023 - 58.41x + 905.5x^2 - 6931x^3 + 2.785e^{-4}x^4 - 5.583e^{-4}x^5 + 4.391e^{-4}x^6 (3-12)
\]

The relationship between the liquid flow rate and GSD index is:

\[
f(x) = -12.99 + 492.3x - 6636x^2 + 4.464e^{-4}x^3 -1.592e^{-5}x^4 + 2.879e^{-5}x^5 - 2.075e^{-5}x^6
\]

In order to verify the accuracy of fitting results, the liquid flow rate is randomly set to 0.223 ml/s under the same other conditions and atomization experiment is carried out. The experimental results are shown in figure 9.

![Experiment capture](image1) ![Arcgis process](image2) ![Droplet diameter distribution](image3)

Fig. 9. Experimental result with flow rate of 0.223ml/s

The average droplet size and GSD index of charged droplets are calculated and fitted. The results are shown in table II.

<table>
<thead>
<tr>
<th>Table II</th>
<th>Comparison between fitting results and actual results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average particle diameter(mm)</td>
</tr>
<tr>
<td>Fitting results</td>
<td>0.633417</td>
</tr>
<tr>
<td>Experimental results</td>
<td>0.681749</td>
</tr>
</tbody>
</table>

Comparing the two sets of data, it is shown that when the flow rate is 0.223ml/s, the relative error between the average droplet size fitting result and experimental result is only 3.6% and the GSD index error is only 1.2%.

4 CONCLUSION

Based on the theoretical deduction of the relationship between velocity, acceleration, flight time of the droplet in electrostatic field, the relationship between relaxation time and charge droplet, the influence of flow rate on the droplet size of atomized liquid are analyzed. The experimental results are showed that the average droplet size and GSD index of charged droplets are obviously increased with the increase of flow rate. Based on the experimental results, expression of the average droplet size, GSD index and the flow rate is established which is tested by the experimental analysis in a precise manner.

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REFERENCE


