CFD Simulation with Analytical and Theoretical Validation of Different Flow Parameters for the Wedge at Supersonic Mach Number

Sher Afghan Khan¹, Abdul Aabid¹, Ahamed Saleel C.²

¹Department of Mechanical Engineering, Faculty of Engineering, International Islamic University Malaysia, Kuala Lumpur, Malaysia
²Mechanical Engineering Dept., College of Engineering, King Khalid University, Kingdom of Saudi Arabia

Abstract-- In this paper analytical and numerical methods are used to evaluate flow over a wedge at supersonic Mach numbers. Closed form solutions are obtained for various semi-vertex angle of the wedge and the Mach numbers. Supersonic similarity parameter has been used to obtain the pressure distribution over wedge at different angle of attack with attached shock wave case. Results are in good agreement with the theory. For the analysis a strip theory is used which are independent in the direction of the flow. To simulate the results, the finite element (FE) method has been used. The validation has been done using the second order shock-expansion theory and the analytical solution obtained by Ghosh’s unified supersonic/hypersonic theory.

Index Term-- CFD, Wedge, FEM, Mach number, surface pressure.

I. INTRODUCTION

A Similarity law for the unsteady hypersonic flow was first invented by Tsien, which is applicable for a wide range of Mach numbers. The results obtained by him are in good agreement with the experimental results. Many attempts have been made for theoretical analysis for an oscillating wedge [1]. Theory of an oscillating airfoil in pitch at high Mach number and unsteady piston theory developed by Lighthill which was later named it ‘piston analogy’. For a pitching symmetric airfoil, Lighthill found the analytical solution assuming the air to be perfect gas [2]. Hayes studied unsteady flow around thin airfoils at high Mach number using the piston theory [3]. Unsteady hypersonic flow has been studied by Zartarian et al., using the tangent-wedge approximation method and shock-expansion theory [4]. Carrier Found an exact solution for an oscillating wedge in 2-D flow, which is valid for supersonic flow Mach numbers for an arbitrary angle of attack, provided the shockwave is attached [5].

Monis et al., [6] examined the variation of pitch damping gradient with incidence and Mach number by varying the pivot position and found that the damping derivative decreases with Mach number and increases with the angle of incidence. Crasta et al., [7] used unified supersonic theory to derive the expressions for stiffness derivative of a wedge in hypersonic flow and demonstrated the effects of the pivot position on the pitching derivative with angle of incidence as well as the pitch rate for inertia level. Shabana et al., [8] obtained analytical results for static pressure and its corresponding derivative in pitching motion as a function of piston Mach number for an oscillating cone. The effects of Mach number and its geometry on the static pressure and its variation with the pitch rate with inertia level and angle of incidence were obtained.

Crasta et al., [9] reported the effect of aspect ratio of the wing on rolling moment derivatives at a various angle of attack and Mach numbers for Supersonic flow. Monis et al., [10] attempted to derive the expressions for stiffness derivative for a wing with forty five degrees sweep angle whose leading edges are curved for an unsteady wing at hypersonic flow. Shabana et al., [11] developed analytical relations derivatives in pitch due to change in angle of attack and pitch rate with the assumptions that the gas is non-viscous and perfect, the motion is quasi-steady, and the semi-vertex angle of the cone is such a way so that the Mach M₂ after the shock is larger than or equal to 2.5.

Crasta et al., [12]–[17] identified the influence of angle of attack as well angle due to the sweep the wing on roll damping derivative of a triangular wing whose angle is forty five degrees along with the edges which are not straight but have been give some curvature with the conditions that the shock wave is attached. A similarity parameter has been obtained for a pitching oscillating Non-planar two-dimensional wedge provided the shock wave is attached at high angle of attack in hypersonic flow. Also, they presented the effectiveness of angle of incidence on pitching derivatives due angle of incidence and pitch rate for a delta wing for the attached shock case. Strip theory was used in where the strips at different locations along the span are independent and this clubbed with the supersonic similarity parameter to give a piston theory. A similitude and the piston theory have been extended to a flat wing with straight leading edges. A similitude was obtained for a pitching oscillating non-planar wedge provided the shock at the nose of the wedge is attached at high angle of attack in supersonic flow and this combines with the similitude to lead to a one-dimensional piston theory. Analytical method used to predict the aerodynamic pitching derivatives of oscillating delta wings whose leading edge is straight. It uses the Ghosh...
similitude and the strip theory to obtain the expressions for stability derivatives in pitch and roll in the Newtonian limit.

Crasta et al., [18]–[20] investigated the influence of the flow deflection angle and sweep angle on damping derivative due to the change in the roll angle as well as roll rate of a delta wing whose edges are not straight but curved for attached shock case in supersonic flow.

Pavitra et al., [21] implemented two-dimensional slender body theory in hypersonic flow at high angle of attack for various Mach number by utilizing the concept of piston theory. The expressions for stability derivatives were obtained for a wedge, which depends on the flight Mach number and semi vertex angle. They found that the pitching moment derivatives behaves linearly with the progressive increment in the inertia level for the pivot positions of $h = 0, 0.2, 0.4$ but for Mach number greater than 10 the steady state is achieved, and stability derivative achieves the steady state and does not change with the Mach number. The present theory has been applied to a sharp wedge with attached shock. Using the theory and the piston theory, the moment coefficient in pitch for stiffness damping are obtained for zero incidences of the wedge and they are dependent on flight Mach number and wedge semi vertex angle.

The strip theory is used where the flow in spanwise location are independent of each other. For the two-dimensional flow, a normal shock wave is formed directly in front of the body and becomes a curved oblique shock as it extends around the body [22]–[30]. Sahana et al., analyses the box wing configuration using CFD method [31]. Singh et al., optimized the flow parameters as pressure, temperature and density through a planar wedge using CFD [32]. The pressure and velocity effects from the nozzle to evaluate the effect of microjets to enhance the base pressure for suddenly expanded ducts using FE method was identified [33]–[35].

The objective of this paper to use finite element method to validate the theoretical and analytical results by computing the different flow parameters for the planar wedge. The CFD analysis has been carried out along with the parametric study using ANSYS. The Mach number considered was in the range from 2 to 4.5 and the angle of half wedge in the range from 5 to 20 degree.

II. DEFINITION OF THE PROBLEM

The modeling of the wedge was done in ANSYS with the assumption of the free stream flow passed through the two-dimensional wedge. When the flow is ahead of the wedge called upstream (or inlet) flow and when the flow after the wedge is called downstream (or outlet) flow. In the present solution, two-dimensional geometry modeling has been done as shown in figure 1. The dimension of wedge considered horizontal distance H7 and H8 of 1.5 m, and 0.5 m respectively. The vertical distance V9 is 1.259 m and V 10 is depending on the half-wedge angle in degrees (figure 2).

III. FINITE ELEMENT METHOD AND ANALYSIS

The finite element method is next to the numerical method to evaluate the approximate solution. In this method, all the problems, such as structural, fluent, and thermal solutions can be obtained approximately. This method is used extensively for the analysis and design of ships, aircraft, space crafts, electric motors and heat engines [36]. In this problem, commercially available ANSYS code has been used to simplify the solution. ANSYS Fluent runs comprehensive modeling capabilities for a wide range of compressible and incompressible, laminar and turbulent fluid flow problems. Another very useful group of models in ANSYS Fluent is the set of free surface and multiphase flow models. The packages are continuously being updated by incorporating more and more elements and adding new modules like non-linear analysis, dynamic analysis, and optimization techniques [37]. The solution runs in this method with the three basic steps: Pre-processing, Processing/Solution and Post-processing/Results.

The various input required to define a problem may be grouped into the following:

1. Geometric Data is the specification of a given problem.
2. Load Data consists of total types of loads and for each load its magnitude, point of application etc.
3. Material Properties to be supplied consists of total number of materials used, for each material required material property.
4. Next input required is about total number of boundary conditions and for each boundary condition specified displacements.
5. Number of Gaussian points to be used and for each Gaussian point weight function and coordinates in local system should be supplied

In this case the inputs are:

- Solver: steady, absolute, 2D planar pressure-based
- Model: Inviscid, Energy equation
- Fluid: air, ideal gas
- Boundary conditions: far field, pressure far field (pa); symmetry, symmetry of wall; wedge, wall
- Solution method: Pressure (standard); density, momentum, turbulence kinetic energy, turbulence dissipation rate, energy (second-order upwind)

Solution steps are:
• Solution initialization: standard, from pressure-far-field
• Reference value: pressure-far-field (solid surface body)
• Solution run: Up to solution converged

A. Geometry and Modelling

The modelling of the wedge has been done in ANSYS workbench with the specified dimension and there is an external surface has been created to model fluid area ahead of the wedge to show the fluid process from the wedge. The model is symmetry, therefore, considered only half of the wedge shown in figure 2. Figure 2(a) represents the designed FE model.

B. Meshing and Analysis

The meshing has been done for the wedge using ANSYS Meshing. To obtain a good solution very fine mesh has been created with a structural mesh option. Figure 2(b) shows the type of mesh in a two-dimensional shape. 73745 total binary nodes have been created and element size 0.01 m was used. Suitable boundary conditions were applied to solve the problem.

IV. Analytical Approach

Light hill [38] established an idea for oscillating wedge at high Mach number. In order to validate finite element solution of surface pressure distribution considered the piston theory for two dimensional wedge at supersonic Mach number [39]. Ghosh theory is valid only when the shockwave is attached to the wedge. Ghosh piston theory is given below

\[
P_2 \over P_1 = 1 + AM_2^4 + AM_1^4(\sqrt{B + M_2^2} - 1)
\]

Where,

\[
A = \frac{\gamma(y+1)}{4} \quad \text{and} \quad B = \left(\frac{4}{y+1}\right)^{\frac{4}{y+1}}
\]

P1 and P2 is the free stream (inlet) pressure and surface pressure (outlet) respectively. Mp is the piston Mach number perpendicular to the wedge surface. Hence, the Ghosh piston theory is only validated for surface pressure distribution on the wedge at supersonic Mach numbers. Therefore, to validate the FE results for different Mach number, density and temperature are considered.

V. THEORETICAL VALIDATION

The shockwave created at the nose of the wedge when the flow passes the wedge. It is an extremely thin region, across the wedge where the flow properties can change abruptly. There are two types of shockwaves one is oblique shock and the other is normal shock [40]. The properties change at downstream of the oblique shock. Oblique shock in terms of the normal component of the upstream Mach number \(M_{n1}\)

\[
M_{n1} = M_1 \sin \beta
\]

Thermodynamic properties of density, pressure and temperature across a normal shockwave are [40].

\[
M_{n2}^2 = \frac{1 + \frac{\gamma+1}{2}M_{n1}^2}{\frac{\gamma}{\gamma - 1} - \frac{\gamma+1}{2}}
\]

\[
\rho_2 \over \rho_1 = \frac{(y+1)M_{n1}^2}{2 + (y-1)M_{n1}^2}
\]

\[
P_2 \over P_1 = 1 + \frac{2y}{y+1} \left(M_{n2}^2 - 1\right)
\]

\[
\frac{T_2}{T_1} = \frac{P_2 \over P_1 \rho_1 \over \rho_2}{1 + \frac{2y}{y+1} \left(M_{n2}^2 - 1\right) \left(\frac{2(y-1)M_{n1}^2}{y+1} + \frac{(y+1)M_{n1}^2}{2}\right)^{\frac{4}{y+1}}}
\]

\(M_{n2}\) is the normal Mach number behind the shockwave. The downstream Mach number \(M_2\) can be found from \(M_{n2}\) which is illustrate as,

\[
M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}
\]

The changes across an oblique shock depend on two parameters, \(M_2\) and \(\beta\).

\[
\tan \theta = 2 \cot \beta \frac{M_2^2 \sin^2 \beta - 1}{M_2^2 (\gamma + \cos 2\beta) + 2}
\]

This is called \(\theta - \beta - M\) relation and it specifies deflection angle \(\theta\) as a unique function of Mach number \(M_1\) and shock wave angle \(\beta\) [32].

For a given upstream Mach number \(M_1\), there is a maximum deflection angle \(\theta_{max}\). If the physical geometry is such that \(\theta > \theta_{max}\), then no solution exists for a straight oblique shock. The value of \(\theta_{max}\) increases with increasing \(M_1\), hence, at higher Mach numbers, the straight oblique shock solution can occur at higher deflection angles. Nevertheless, there is a limit;
a $M_1$ approaches infinity, $\theta_{max}$ approaches $45.5^0$ (for $\gamma = 1.4$). In this case, the maximum angle of half wedge considered $20^0$ therefore, the problem assumed only attached shockwaves.

VI. RESULTS AND DISCUSSION

A. Surface Pressure Distribution

Ghosh piston theory is used in relation with the static pressure on the surface of the wedge with the increment in the Mach number and the results are compared with FEM. Figure 3(a) illustrates the surface pressure distribution on the edge for various Mach numbers for a different half wedge angle $\theta = 5, 10, 15,$ and $20$ degrees using analytical and FEM. It is observed from the figure that there is an increase in the magnitude of surface pressure from $1.8$ to $5.1$ with the increase in Mach numbers from $M = 2$ to $4.5$ for various flow deflection angles. This trend was expected as the inertia level increases the surface pressure will increase, however, for Mach number five and above this trend will get reversed and with further enhancement in the inertia level of the flow will achieve the steady state and later with any increment in inertia level of the flow parameters will be unchanged, which implied that this is Mach number beyond which the Mach number independence principle as achieved.

Figure 3(b) represents the variation of surface pressure in supersonic stream for Mach numbers $M = 2, 2.5, 3, 3.5,$ and $4$. Pressure distribution on the surface of a wedge whose leading edge is curved, which results in expansion with the flow deflection angle which is nothing but the semi-vertex angle in this case for various Mach numbers. If the half wedge deflection angle is small then the shock wave angle will also be small. Here, while considering the half wedge, this can be considered as a flat plate at an angle of attack. Here the semi-vertex angle is the same as the angle of incidence. Once the analysis is done for half wedge angle the results are multiplied by a factor $2$ to account for the entire wedge. In fact, similar type of study was also discussed in Ref. [8] and Ref. [22].

B. Effect of Mach number

Figure 4 (a) illustrates that due the increment in the upstream value of inertia as Mach number the downstream Mach number also increases but the magnitude is lower than the upstream Mach number (i.e. $M_1 > M_2$). On the other hand, it is seen that the increase of flow deflection angle will result in decrease of the Mach number figure 4 (b). Therefore, when there is increment in the flow deflection angle of the wedge the shock wave angle $\beta$ will increase and beyond a certain value, the shock wave will be detached sing [32].
C. Effect of Density

In order to investigate the flow around the wedge the density approach is considered. The results of the density ratio of the fluid across the wedge are shown in figure 5. From the results, it is found that the density increases up to a certain flow deflection angle of the wedge. It also is seen that the density will increase with the increment in the free stream Mach number (figure 6a) and the flow deflection angle of the wedge (figure 5b).

The behavior is similar to that of the pressure, however, the magnitude of the density is large as compared to the temperature. Therefore, the density is more effective for high Mach number and cone (half wedge) angle.

D. Effect of Temperature

Figure 6 presents the results of the effect of temperature for various flow parameters of the wedge. From the results, it is seen that there is a progressive increase in the temperature with the increase of the Mach number (figure 6a) as well as the flow deflection angle of the wedge (figure 6b). The phenomenon of temperature variation is not like that of the pressure except the inclination, however, the value of temperature does not influence unlike the pressure which means that the temperature effects are negligible in view of the flow bearing low Mach numbers and can be considered that the flow is isothermal and the values are very close to the normal temperature.
E. Flow Over a Wedge

In addition, consider the contour and plots results for a single case of Mach number = 2, Half wedge Angle = 10. The position considered at x-axis in all plots is the symmetry of the wedge. Therefore, the value of all parameter is constant until it reaches 0.5 m due to the upstream position of the wedge. Pressure variation are shown in figure 7. However, for the present case the Mach number refer Fig. 8, the value is high at the upstream of the attached shock and suddenly and later it becomes low and remains constant throughout the length of the wedge. Moreover, for density and temperature (Figs. 9 and 10), the phenomenon is similar to the pressure.
VII. CONCLUSIONS:

This paper explains the dependency of the different flow variables over the wedge. The results presented in this paper are valid only for the attached shock case. The effect of the increase of flow deflection angle of the wedge for a given upstream Mach number and the effect of the Mach number at various flow deflection angle of the wedge has been analyzed using finite element method. It is seen that due to viscous-inviscid and the boundary layer interaction the flow behaves differently at high supersonic Mach numbers. The results have been compared with theoretical study and they are found to be in good agreement. The flow parameters considered here are the pressure, the density, the temperature, and the Mach number. To obtain the pressure distribution and flow parameter, Ghosh piston theory was used for analytical approach then the results compared with finite element solutions and it is found a good agreement with less than 10% relative error in all the cases of this paper. The changes in flow properties across the shockwaves take place within a small distance as the thickness of the shock wave is very small of the order of micrometer. The simulation results show that the temperature values are close to the normal temperature in comparison to the other flow variables.

REFERENCES


