A New Backstepping Sliding Mode Control System to prevent Roll Instability of a Four-Wheeled Vehicle

M. A. Saeedi1,*, M. Mirzaee2
1Assistant Professor, Department of Mechanical Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.
2Department of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran.
*Corresponding Author: Department of Mechanical Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran.
*E-mail: amin_saeedi@sru.ac.ir

Abstract—In this paper, in order to prevent the rollover instability of a four-wheeled vehicle, a new roll control system is designed. In this control system an active anti-roll bar is employed as an actuator to generate the roll moment. First, in order to achieve an accurate model a through nonlinear dynamic model of a four-wheeled vehicle is developed. Then, dynamic model validation is done using ADAMS CAR software during a transient maneuver. Next, in order to design the roll control system backstepping sliding mode control method is used. In this study, lateral load transfer ratio is considered as an important factor to investigate the vehicle rollover. The dynamic performance of the vehicle is evaluated in standard maneuvers at different conditions. The simulation results show that the proposed active roll control system is able to reduce the lateral load transfer ratio, especially during severe lane change maneuver in which intense instability occurs. Also, the robustness of the control system is verified during steady state and transient maneuvers at different road conditions.

Index Term—Robust Control System, Rollover, Lateral Load Transfer Ratio, Stability Analysis.

I. INTRODUCTION

The rollover of the vehicles is an important factor for road safety. A vehicle reaches roll instability, whenever the overturning moment caused by the centrifugal forces associated with the directional moment of the vehicle, exceeds the stabilizing moment. High center of gravity heavy vehicles, especially when laden, yields lower roll stability limit, which is further related to the dynamic characteristics of the articulated vehicle and the nonlinearities associated with the suspension and tires. The aim of the rollover prevention is to provide the vehicle with the ability to resist overturning moments generated during maneuvers. The problem with heavy vehicles is a relatively high mass center and narrow track width [1]-[7].

Rollover accidents affect strongly highway safety, especially rollover of heavy-duty tankers carrying fuels and chemicals, which might result in explosions and catastrophic chemical spills. This fact made the study of rollover stability of various vehicles, a major concern for many vehicle manufacturers, organizations, and researchers. Ervin et al. [1] reported rollovers in nearly 75 percent of the total numbers of tank trucks highway accidents, while the rollover rate for conventional solid cargo vehicles was 54 percent.. Strandberg et al. [8] studied the overturning risk of heavy-duty truck-trailer combinations both analytically and experimentally. Gaspar et al. [9] proposes a control system increase the rollover stability of a vehicle. The control system is designed based on a three-degrees-of-freedom linear dynamic model and the basis of adaptive control method. They evaluated the performance of the control system during transient and steady state maneuvers at different velocities. Chen et al. [10] designed an active suspension control system to improve the rollover stability of the vehicle based on LQR control method. The dynamic performance of the vehicle is investigated at different road conditions.

Winkler et al. [11], [12] reviewed the US accident statistics and reported a strongly negative correlation between steady-state roll stability and the average frequency of rollover accidents. He performed more analysis on rollover stability of an articulated vehicle carrying liquid using Strandberg results. Sampson [13] investigated the use of active roll control to enhance the roll stability of a tractor semitrailer. Talebi et al. [14], [15] optimized the tanker rollover threshold for the articulated vehicle carrying liquid. Azadi et al. [16] investigated the effect of tank shape on the lateral dynamic of the vehicle. They found that the filled level and maneuver type are two effective parameters on the dynamic interaction.

In this paper, in order to improve the lateral dynamic of a vehicle a new robust active roll control system is used. So, this paper is organized as follows. At first, the combined yaw-roll model in which the forward velocity changes in time is constructed. The backstepping control method solves both the problems of stabilization and trajectory tracking of nonlinear systems. Also, backstepping technique guarantees global asymptotic stability. In section 3 the backstepping method and the sliding mode control method are combined and backstepping sliding mode control as a new robust control is used. The controller is designed based on the nonlinear dynamic model and the roll angle of the vehicle is considered as the state variable which is targeted to track its desired value. Section 4 demonstrates the results of the control system performance during standard maneuvers at different velocities. Finally, the conclusions are provided.

II. DYNAMIC MODELING

In this paper to simulate the directional characteristics of a vehicle a fourteen-degrees-of-freedom nonlinear dynamic model is proposed. As depicted in Fig. 1, the body has six degrees of freedom for a rigid body motion in a space, and each wheel which is connected to the body via a suspension system, can go through a translational motion along the z-axis and a rotational motion about the y-axis. The front wheels can steer about the z-axis.
Where $v_x$ is the vehicle’s longitudinal velocity, $v_y$ is the vehicle’s lateral velocity, $v_z$ is the vertical velocity, $\dot{\psi}$ is the yaw rate, $\dot{\theta}$ is the pitch rate and $\phi$ is the roll rate. The roll, pitch and yaw motions are defined as

$$
M_z = I_{zz} \ddot{\psi} - (I_{yy} - I_{zz}) \dot{\theta} \dot{\psi} = (F_{zr} - F_{fr}) T_r/2 + (F_{zrr} - F_{zrr}) T_r/2 - (F_{yfr} + F_{yfr} + F_{yr} + F_{yr})
$$

(6)

$$
M_y = I_{yy} \ddot{\theta} - (I_{zz} - I_{xx}) \dot{\phi} \dot{\psi} = (F_{zx} + F_{zr}) L_r - (F_{zfr} + F_{zfr}) L_f + (F_{xfr} + F_{xfr} + F_{xtr} + F_{xtr})(h_{cg} - h_{cr})
$$

(7)

$$
M_z = I_{zz} \ddot{\psi} - (I_{xx} - I_{yy}) \ddot{\phi} = (F_{xf} - F_{xfr}) T_r/2 + (F_{xr} - F_{xrr}) T_r/2 + (F_{yfr} + F_{yfr} + F_{yr} + F_{yr}) L_r - (F_{xfr} + F_{xfr} + F_{xfr} + F_{xfr} + F_{xfr} + F_{xfr})(L_r + \sum_{i=1}^{n} M_{di})
$$

(8)

### B. Tire dynamics

Apart from aerodynamic forces, all of the forces influencing the vehicle are created on the contact surface between the tire and the road. Hence, in order to investigate the vehicle dynamic performance, the nonlinear tire model is considered. In this model, the combined slip situation was modeled from a physical viewpoint. Tires generate lateral and longitudinal forces in a non-linear manner. In this article, the Magic Formula model is used to simulate the tire forces [17].

$$
F_x = F_{x0} \cos \left( \left( R_{cx1} \times \tan (Bx0. alfa) \right) - (Rex1 + Rex2. dfz) \right) (Bx0. alfa - \tan (Bx0. alfa)) / \left( \cos \left( (Rcx1. \tan (Bx0. Rhx1) - (Rex1 + Rex2. dfz) \right) (Bx0. Rhx1 - \tan (Bx0. Rhx1)) \right)
$$

(9)

$$
F_y = F_{y0} \cos \left( \left( R_{cy1} \times \tan (fu. S) \right) - (Ry1 + Rey2. dfz) \right) (fu. S - \tan (fu. S)) / \left( \cos \left( (Rcy1. \tan (fu. (Rhy1 + Rhy2. dfz)) - (Ry1 + Rey2. dfz) - \tan (fu. (Rhy1 + Rhy2. dfz))) \right)
$$

$$
M_{dr} = Dr. \cos \left( (Br. Alpha_r_{eq}) \right);
$$

$$
F_{prim_y} = F_y - SVy_k;
$$

$$
t = Dt. \cos \left( Ct. \tan \left( Bt. Alpha_{eq} \right) - \left( Et. \left( \left( Bt. Alpha_{eq} \right) - \atan \left( Bt. Alpha_{eq} \right) \right) \right) \right) . u(6);
$$

$$
M_{prim_z} = -t. F_{prim_y};
$$

As can be seen from Figure 2, the total longitudinal force and lateral force acting on the model in the coordinate system are given

$$
FX_i = Fx_i \cos(\delta) - Fy_{ik} \sin(\delta),
$$

$$
FY_i = Fy_i \cos(\delta) + Fx_i \sin(\delta),
$$

(1)

for $i = 1, ..., 4$

Where

$$
F_x = \sum_{i=1}^{4} F_{xi}
$$

(2)

$$
F_y = \sum_{i=1}^{4} F_{yi}
$$

### A. The equations of motion of the vehicle

The longitudinal, lateral and vertical motions are defined by

$$
M_i (v_x + \theta v_y - v_y \dot{\theta}) = FX_{fi} + FX_{fr} + FX_{rt} + FX_{rr}
$$

(3)

$$
M_i (v_y + \psi v_x - \dot{\theta} v_y) = FY_{fi} + FY_{fr} + FY_{rt} + FY_{rr}
$$

(4)

$$
M_i (v_z + \dot{\theta} v_x - \dot{\psi} v_y) = FZ_{fi} + FZ_{fr} + FZ_{rt} + FZ_{rr}
$$

(5)

where

- $F_{x0}$ is the initial roll force
- $F_{y0}$ is the initial pitch force
- $Dr$ is the longitudinal stiffness
- $S$ is the yaw stiffness
- $Et$ is the roll stiffness
- $Bx0$ is the front tire radius
- $Rex1$ and $Rex2$ are the center of gravity coordinates
- $Rhy1$ and $Rhy2$ are the roll angles
- $fu$ and $S$ are the static friction coefficients
- $Bt$ is the total tire radius
- $Alpha_{eq}$ is the equivalent wheel angle
- $SVy_k$ is the vertical velocity
- $Dt$ is the total time
- $Ct$ is the total yaw rate
- $Bt$ is the total roll rate
- $Et$ is the total pitch rate
- $u(6)$ is the unit impulse
- $M_z$ is the moment of inertia
- $M_{prim_z}$ is the primary moment
- $F_{prim_y}$ is the primary force
- $v_x$, $v_y$, $v_z$ are the velocities
- $\theta$, $\psi$, $\dot{\theta}$, $\dot{\phi}$, $\dot{\psi}$ are the angular velocities
- $\delta$, $\alpha$, $\beta$, $\gamma$ are the angles
\[ M_z = (M_{prim, z} + M_{zr} + (s \cdot F_x)); \]

C. Wheel Dynamics

The following equation can be written for traction from Fig. 3:

\[ I_w \omega = -R_w F_{xi} + T_i \]  \hspace{1cm} (10)

Fig. 3. wheel diagram [17]

The longitudinal slip, the tire side slip angles and the tire normal forces related equations can be found in our previous paper [18]-[20].

III. Dynamic Model Validation

In order to validate the nonlinear dynamic model, ADAMS CAR software is used [21]. So, the considered developed model parameters are the same as the ones implemented in ADAMS CAR modeled vehicle. The simulation results of the most important roll dynamic responses for standard maneuvers are shown in Figs. 4 to 7.

In this analysis, the vehicle is considered as initial velocity of 70 km/h on a dry road with road friction coefficient of 0.7. As can be seen from Fig. 4, the lateral acceleration of the vehicle matches acceptably with the ones of the real test data results. Moreover, according to Fig. 5, an appropriate coincidence is observed between the vehicle’s yaw rate and ADAMS CAR model. Also, the performance of the dynamic model is evaluated during a transient maneuver. In this maneuver the vehicle moves at initial velocity of 50 km/h on a dry road. The steering angle is illustrated in Fig. 6. As can be seen from Fig. 7, although the yaw rate’s curve shows an appropriate agreement with the ADAMS CAR model results, some deviations between these two are observed during \( t = 2.1 - 3 \text{ s} \).

Fig. 4. Lateral acceleration.

Fig. 5. Yaw rate.

Fig. 6. Steering angle.
IV. CONTROL SYSTEM DESIGN

In this section, in order to prevent the vehicle’s rollover an active roll control system is proposed. Therefore, the backstepping method and the sliding mode control method are combined to produce the new robust control system. This control system is used to eliminate the steady state error. In the proposed control system, the control state variable of the vehicle is the roll angle which is assumed to track its desired value. The desired value of the roll angle of the vehicle is considered zero.

To design the controller, the simplified dynamic model Eq. (11) is used. A second-order system is considered as follows:

\[ l_{xx} \ddot{\phi} = f_1 + M_A \]  

Where

\[ f_1 = (Fz_2 - Fz_1)T_f/2 + (Fz_4 - Fz_3)T_r/2 - (FY_1 + FY_2 + FY_3 + FY_4)(h_{cg} - h_{cr}) + (l_{yy} - l_{zz})\dot{\theta}\dot{\psi} \]  

Now, state variables are considered as follows:

\[
\begin{align*}
\dot{x}_1 &= \phi \\
\dot{x}_2 &= \dot{\phi} \\
\dot{\phi} &= \frac{f_1}{l_{xx}} + \frac{M_A}{l_{xx}}
\end{align*}
\]  

The tracking error and its derivative value is

\[
\begin{align*}
e_1 &= x_1 - \dot{x}_d \\
\dot{e}_1 &= x_2 - \ddot{x}_d
\end{align*}
\]  

Now, Lyapunov function is given as follows:

\[ V_1 = \frac{1}{2} e_1^2 \]  

\[ \dot{V}_1 = e_1 \dot{e}_1 = e_1 (x_2 - \ddot{x}_d) \]  

For \( \dot{V}_1 \leq 0 \), we have

\[ x_2 = s - c_1 e_1 + \ddot{x}_d \]  

\[ s = x_2 + c_1 e_1 - \ddot{x}_d \]  

Based on mentioned Eqs. (14) and (16) we have

\[ s = c_1 e_1 + \dot{e}_1 \]  

Substituting the Eq. (17) in Eq. (16) we have

\[ \dot{V}_1 = e_1 s - c_1 e_1^2 \]  

Then

\[ s = 0 \rightarrow \dot{V}_1 \leq 0 \]  

The Lyapunov function can be defined as

\[ V_2 = V_1 + \frac{1}{2} s^2 \]  

By derivation from the above equation

\[ \dot{V}_2 = \dot{V}_1 + s \dot{s} \]  

\[ \dot{V}_2 = e_1 s - c_1 e_1^2 + s \left( \frac{f_1}{l_{xx}} + \frac{M_A}{l_{xx}} + c_1 \dot{e}_1 - \ddot{x}_d \right) \]  

In order to realize \( \dot{V}_2 \leq 0 \), the control input is

\[ M_A = I_{xx} \left( -\frac{f_1}{l_{xx}} - c_2 s - e_1 - c_1 \dot{e}_1 + \ddot{x}_d - \eta_1 \text{sign}(s) \right) \]  

Where \( c_2 > 0 \), therefore

\[ \dot{V}_2 = -c_1 e_1^2 - c_2 s^2 - \eta_1 |s| \leq 0 \]
V. SIMULATION RESULTS

In this section, the performance of roll control system to increase the rollover stability of the vehicle is investigated in standard maneuvers. In this section in order to investigate the performance of the BSMC a linear control system is designed.

A. Fishhook Maneuver

In this study, the vehicle at 80 km/h initial velocity on a dry road with 0.7 friction coefficient moves and steering input is as Fig. 10:

Fig. 10. Steering angle.

Figs. 11 to 13 show the simulation results for with and without control conditions. As can be seen from Fig. 11(a), for uncontrolled vehicle, the roll angle increases considerably. The amount of this increase is considerable at the 3.6 second for the four-wheeled vehicle. It can be seen from Fig. 11(b) both controllers are able to reduce the roll angle considerably in comparison with uncontrolled conditions. The robust control system improves the peak value and the settling time significantly in comparison with the linear control system. The lateral acceleration of the vehicle is illustrated in Fig. 12 for with and without control conditions. As can be seen from Fig. 12(b), the first peak value and the second peak value are reduced using the control system. In this state, the BSMC has an appropriate performance to reduce the peak value and settling time in comparison with linear controller. The control effort is shown in Fig. 13(a) for the vehicle.

Lateral dynamic behavior of the vehicle is investigated using lateral load transfer ratio (LTR). The LTR is defined as below [22]:

\[
LTR = \sum_{j=1}^{N} \frac{|F_{xxj} - F_{xtj}|}{F_{xxj} + F_{xtj}}
\]  

(24)

The LTR starts from zero and becomes unit when there is no contact between the wheels and the road. The Fig. 13(b) shows that the maximum value of lateral load transfer ratio has been decreased considerably in comparison with uncontrolled state. The maximum value of LTR decreases from 0.6 to 0.4.
Fig. 11. The roll angle: (a) without control, (b) controlled.

Fig. 12. Lateral acceleration, (a) without control, (b) controlled.

Fig. 13. (a) The control effort, (b) Lateral load transfer ratio.
B. Lane change Maneuver

In this maneuver, the vehicles run on a level icy road with a friction coefficient of 0.3 at the constant speed of 90 km/h and the steering angle input shown in Fig. 14.

![Steering Angle of Front Wheels](image1)

Fig. 14. Steering angle.

The most important roll dynamic responses of the vehicle are shown in Figs. 15 to 18. According to the Fig. 15(a), the roll angle of the uncontrolled vehicle increases considerably. The amount of this increase for the second peak is more. In this condition, the maximum value of roll angle is 5.9 degree. As can be seen from Fig. 15(b), for controlled conditions, the vehicle’s roll angle becomes zero after 6 seconds. According to Fig. 16, the active roll control system is able to reduce the peak value of the lateral acceleration in comparison with uncontrolled state. The way that control signals vary in order to ensure that the system outputs follow their desired values is shown in Fig. 17. The peak of the roll moment in response to a critical maneuver is 40 kN.m. Fig. 18 shows that a considerable reduction on the LTR is observed for transient condition.

![Fig. 15. The roll angle: (a) without control, (b) controlled.](image2)

![Fig. 16. Lateral acceleration.](image3)

![Fig. 17. The control effort.](image4)
VI. CONCLUSION

In this paper, in order to reduce the rollover probability of the vehicle a robust active roll control system is proposed. At first, a thorough nonlinear dynamic model of a four-wheeled vehicle is developed. Then, a new active roll control system is designed based on the nonlinear dynamic model using backstepping sliding mode control method. Afterward, the controller performance is evaluated in standard maneuvers for different velocities for which the following conclusions are derived.

- The uncontrolled vehicle is severely subjected to the rollover instability, as the driving condition becomes critical.
- Clearly, the control system significantly improves the rollover stability of the vehicle in response to steering inputs.
- The vehicle with active roll control system could remain stable even if the steering input is changed.
- Using the controller, a considerable reduction is observed in LTR during Lane Change maneuver.
- The more investigations demonstrate that the active roll control system is able to increase the rollover stability of the vehicle during critical maneuvers.
- The controller is robust to the height of center of gravity, road friction of coefficient variation as well as vehicle’s velocity.

APPENDIX

Table I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{sf}$</td>
<td>Front suspension damping constant</td>
<td>3000(N. s/m)</td>
</tr>
<tr>
<td>$C_{sr}$</td>
<td>Rear suspension damping constant</td>
<td>4000(N. s/m)</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Height of the sprung mass center of gravity</td>
<td>0.5(m)</td>
</tr>
<tr>
<td>$I_w$</td>
<td>Wheel moment of inertia</td>
<td>1.1(Kg. m$^2$)</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>Roll moment of inertia</td>
<td>496(Kg.m$^2$)</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>Pitch moment of inertia</td>
<td>2212(Kg.m$^2$)</td>
</tr>
<tr>
<td>$I_{zz}$</td>
<td>Yaw moment of inertia</td>
<td>2249(Kg. m$^2$)</td>
</tr>
<tr>
<td>$K_{sf}$</td>
<td>Front suspension stiffness constant</td>
<td>46.8(KN. m/rad)</td>
</tr>
<tr>
<td>$K_{sr}$</td>
<td>Rear suspension stiffness constant</td>
<td>50(KN. m/rad)</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Distance of the center of gravity from the front axle</td>
<td>1.053(m)</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Distance of the center of gravity from the rear axle</td>
<td>1.559(m)</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Vehicle sprung mass</td>
<td>1176(Kg)</td>
</tr>
<tr>
<td>$M_t$</td>
<td>Vehicle total mass</td>
<td>1349(Kg)</td>
</tr>
<tr>
<td>$M_{uf}$</td>
<td>Front unsprung mass</td>
<td>24.15(kg)</td>
</tr>
<tr>
<td>$M_{ur}$</td>
<td>Rear unsprung mass</td>
<td>27.2(kg)</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Front track width</td>
<td>1.483(m)</td>
</tr>
<tr>
<td>$T_r$</td>
<td>Rear track width</td>
<td>1.483(m)</td>
</tr>
</tbody>
</table>

NOMENCLATURE

- $C_{si}$ front/ rear suspension damping constant
- $C_{st}$ Front/ rear tire damping constant
- $h_s$ height of sprung mass
- $I_w$ Wheel moment of inertia
- $I_{xx}$ roll moment of inertia
- $I_{yy}$ pitch moment of inertia
- $I_{zz}$ yaw moment of inertia
- $L_f$ distance of the center of gravity from the front axle
- $L_r$ distance of the center of gravity from the rear axle
- $M_s$ Vehicle sprung mass
- $M_{uf}$ front unsprung mass
- $M_{ur}$ rear unsprung mass
- $M_t$ Vehicle total mass
- $T_f$ front track width
- $T_r$ rear track width
- $\varphi$ roll angle
\[ \theta \quad \text{pitch angle} \]
\[ \psi \quad \text{yaw angle} \]
\[ \delta \quad \text{steer angle} \]
\[ v_x \quad \text{longitudinal velocity} \]
\[ v_y \quad \text{lateral velocity} \]
\[ v_z \quad \text{vertical velocity} \]
\[ F_x \quad \text{tire’s longitudinal force} \]
\[ F_y \quad \text{tire’s lateral force} \]
\[ r \quad \text{yaw rate} \]
\[ c_{dl} \quad \text{Cornering stiffness} \]

REFERENCES


