Low REYNOLDS Number Effect on Steady Laminar Flow Around Two Circular Cylinders in In-Line Arrangement

Olanrewaju M. Oyewola\(^1\)\(^2\)
\(^1\)School of Mechanical Engineering, Fiji National University, Suva, Fiji
\(^2\)Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria
*Corresponding author email address: ooyoewola001@gmail.com; Olanrewaju.Oyewola@fnu.ac.fj

Abstract-- In this study, steady laminar flow past two identical circular cylinders in in-line arrangement is investigated numerically with aim of examine the effects of low Reynolds number (\(1 \leq Re \leq 40\)) and centre-to-centre distance between the cylinders on the flow field around them. For a given Reynolds number, it is observed that the mean drag coefficients of both cylinders are lower than that of an isolated single cylinder. In addition, the mean drag coefficient of the downstream cylinder is significantly less than that of the upstream cylinder. For a fixed small centre-to-centre distance of two diameters, a pair of symmetric vortices is formed within the gap between the cylinders and no vortices in the rear end of the downstream cylinder are observed for \(Re \leq 10\). However, for \(Re > 10\), additional pair of symmetric vortices is observed behind the downstream cylinder. Also, for a fixed Reynolds number, the flow characteristics pattern in the gap is strongly influenced by centre-to-centre distance.

Index Term-- Laminar flow, circular cylinder, drag coefficient, vortices

1. INTRODUCTION

The wake of a two-dimensional circular cylinder of diameter, \(D\), is mainly characterized by the Kármán vortex street over a wide range of Reynolds number [see e.g.1,2]. At very low Reynolds numbers (\(Re < 1\)), inertial forces are negligibly small compared with viscous forces, the boundary layer separates from the surface at the rear stagnation point and no vortex shedding occurs from the cylinder. In addition, the flow is symmetrical around the cylinder and the flow downstream is almost a mirror image of the upstream flow. But, as the Reynolds number increases, the flow downstream no longer mirrors the flow upstream and the complexity of the flow increases. For Reynolds numbers in the range of 5 to 40, the flow starts to separate from the surface of the cylinder and a pair of symmetrical attached vortices is formed at the rear stagnation point of the cylinder. With further increase of the flow Reynolds number, the vortices become unsteady and oscillate and eventually shed in an alternating manner. Complete detailed of various flow regimes and information about their characteristics can be found in, for examples in [1,2,3]. Given the above, it is not surprising that, the mean drag coefficient of the cylinder varies with the Reynolds number.

The flow velocity field and mean forces acting on two circular cylinders depend on number of parameters which includes: the flow Reynolds number, distance between their centres \(S\), pattern of arrangement (either side-by-side, tandem or staggered) and the ratio of diameter of the cylinders. Despite the practical significance of the flow around two circular cylinders at low Reynolds number, it is much less well understood than when the flow regime is within subcritical Reynolds number flow regime (\(Re > 350\)). There have been relatively few study of this flow reported in the literature. For instance, [3] showed that at a fixed Reynolds number of 23500 and Prandtl number of 0.702, the dynamics of the flow is altered by the spacing ratios and the temperature between the cylinders drop significantly as the spacing is increased. It should be noted that the Reynolds number considered is extremely high and fixed which in turn didn’t give room to assess the Reynolds number influence as well as the behavior of the interaction at extremely low values of Reynolds numbers.

Meanwhile two studies that have focused on low Reynolds number flow around two cylinders are Cheung et al. [4] and Juncu [5]. These studies were based on Reynolds number of 1 and 2 respectively, and reported that the mean drag coefficient of the two cylinders are influenced by the wake interference effect and are considerably different from that of an isolated cylinder. The present study extended their studies to more Reynolds number within the laminar steady flow regime and with objective of improving the present level of understanding of this type of flow. Therefore, the aim of this study is to investigate the laminar flow past two circular cylinders of the same diameter in in-line arrangement especially at very low Reynolds numbers. The results of this study can help to understanding the mechanism of flow interaction between wind turbines in wind farm set-up. For efficiency use of land and reduction in wind farm mini-grid system, it is desirable to locate wind turbines as close as possible. However, as result of wake effects, the power output from the wind farm can significantly reduce when the turbines are packaged together. In addition, there is directly relationship between the drag coefficient of a wind turbine...
and its power output. Furthermore, the findings from this work can also help in understanding of the flow around multi-tube heat exchangers. The effects of low Reynolds number and centre-to-centre distance between the cylinders on the flow field around them will be presented. For this study, the flow Reynolds number is varied from 1 to 40 which comprises the creeping flow regime \((Re \leq 5)\) and steady twin vortex flow regime \((5 < Re \leq 40)\).

2. NUMERICAL SOLUTION

2.1 Governing Equations

For an incompressible two-dimensional and steady flow, the non-dimensional forms of the continuity and Navier-Stokes equation can be expressed as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  

\[
\frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial y} + \frac{\partial uv}{\partial x} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

where \(u\) and \(v\) are velocities in \(x\)- and \(y\)-directions, respectively; \(p\) is pressure and \(t\) is time. The flow Reynolds number \(Re\) is defined as:

\[
Re = \frac{\rho UD}{\mu}
\]  

where \(\rho\) is the medium density, \(U\) is the freestream velocity, and \(\mu\) is the medium dynamic viscosity.

2.2 Computational Configuration and Boundary Conditions

The computational domain for a single circular cylinder of diameter \(D\) is shown in Figure 1a. The domain is \(20D\) wide and \(50D\) long. The cylinder is located at centre of the computation domain so that steady and symmetrical flow can be achieved and the cylinder is positioned at \(5D\) from the inlet flow boundary. The blockage ratio, defined as the ratio of the cylinder diameter to the domain width, is 5%. The boundary conditions were chosen as \(u = U, v = 0\) at the inlet and along the top and bottom boundaries. The value \(U\) depends on the flow Reynolds number to be simulated. At the surface of the cylinder, the non-slip boundary condition is applied, that is \(u = 0\) and \(v = 0\). The zero pressure \((p = 0)\) is applied to the outlet boundary. The same computational domain size and boundary conditions were used for the two cylinders’ study. The centre-to-centre distance \(S/D\) between the cylinders was varied from 2 to 12 (Figure 1b). The non-slip boundary condition is also applied on the second cylinder surface.
2.3 Solution Techniques
For each of set of flow conditions and cylinder arrangement, the continuity and momentum equations were solved using a CFD code, COMSOL Multiphysics software which is based on finite element methods. The complete model (from geometry to post processing analysis) is implemented using this software. The Navier-stokes application mode use Lagrange P2-P1 elements to stabilize the pressure. The second-order Lagrange elements model the velocity components while linear elements model the pressure. A homogeneous unstructured grid was used to mesh the solution domain and with predefined mesh sizes set to extremely fine and the refinement method set to regular. Detailed description of calculation settings is available in software documentation [6]. The viscous forces on the cylinder surface were estimated by carried out boundary integration on the cylinder surface. Since the flow is steady and symmetrical, only the non-dimensional form of the drag force is presented here and it is defined as:

$$C_D = \frac{2F_D}{\rho U_D^2D} \tag{5}$$

where $F_D$ is the mean drag force. Due to the limited information about this type of flow in open literature, the numerical solution was first computed for a single circular cylinder and both the mean drag coefficient and recirculation length were compared with reported values.

3. RESULTS AND DISCUSSION
3.1 Single circular cylinder
The mean drag coefficient for a single cylinder as a function of the Reynolds number is shown in Figure 2. As expected the mean drag coefficient is strongly depend on the flow Reynolds number. Also shows on this figure are the mean drag coefficient from other previous studies. The mean drag coefficient determined in this study is found to agree well with the results from other studies for all the Reynolds numbers investigated. Within the creeping flow regime ($Re<5$), it is observed that the mean drag coefficient decreases rapidly with increasing Reynolds number. However, this reduction is more gradual in the case of steady twin vortex flow regime ($Re>5$). The relatively lower value of mean drag coefficients in the steady vortex regime when compared with creeping flow regime is due to the flow separation from the surface of the cylinder. The flow separation leads to a pair of symmetrical attached vortices to be formed at the rear end of the cylinder. The size and length of this pair of symmetric vortices increases with the flow Reynolds number.

The distance from the rear end of the cylinder to the point where the separated flow is reattached to the line of symmetry is usually referred to as the recirculation length. The non-dimensional recirculation length ($L_r/D$) determined in this study is in good agreement with that of other studies as shown in Figure 3. As previous observed by other studies, there is a roughly linear relationship between the recirculation length and the flow Reynolds number. In this present study, this relationship is given as: $L_r/D = a + b \times Re$ (where $a = -0.414$ and $b = 0.0692$). Similar expression can be derived from other similar studies with slightly variations in the values of $a$ and $b$. For instance, from the results of Dennis and Chang [7], the values of $a$ and $b$ are estimated respectively, as -0.410 and 0.0686. It should be noted that for a given Reynolds number, the recirculation length is sensitive to the flow blockage ratio and it increases with decreasing blockage ratio (see e.g., [8, 9, 10]). Therefore, the slight variations in recirculation length between different studies may be related to different in blockage ratios among these studies.

Fig. 1. (a) Computational domain for a single circular cylinder, and (b) Configuration for two circular cylinders.
3.2 Two circular cylinders
3.2.1 Mean Drag Coefficient
For two cylinders in tandem arrangement, the wake of the upstream cylinder could be significantly affected the velocity field around another cylinder placed in its wake. Likewise, the presence of the downstream cylinder hinders the free development of upstream cylinder wake and the downstream cylinder is exposed to relatively lower velocity field when compared with the velocity field in front of the upstream cylinder. Due to these factors, the mean drag coefficient of both cylinders would be considerably different from that of a single circular cylinder exposed to the same freestream velocity or flow Reynolds number (assumed the same diameter).

The ratio of the mean drag coefficient of the upstream cylinder \( (C_{D1}) \) to the mean drag coefficient of an isolated single cylinder \( (C_D) \), symbolized as \( \lambda_1 \left( = \frac{C_{D1}}{C_D} \right) \) is presented in Figure 4. It is observed that at \( S/D = 2 \) and \( Re = 1 \), the value of \( \lambda_1 \) is about 0.79 but increases with increasing Reynolds number. This is similar to the observation of Cheung et al. [3] for \( Re = 1 \). For \( Re < 5 \), the value of \( \lambda_1 \) increases gradually with increasing value of \( S/D \). For \( Re > 20 \), the value of \( \lambda_1 \) drop initially and thereafter increases almost linear with \( S/D \). The wake interference effect is still
noticeable at $S/D = 12$ for the all Reynolds number considered. The characteristic of the mean drag coefficient is related and depend to the velocity field within the gap between the two cylinders as will be shown in Section 3.2.2.

![Graph](image)

Fig. 4. The ratio of the mean drag coefficient of the upstream cylinder to that of isolated single cylinder.

The ratio of the mean drag coefficient of the downstream cylinder ($C_{D2}$) to mean drag coefficient of the upstream cylinder ($C_{D1}$), symbolized as $\lambda_2 \equiv \frac{C_{D2}}{C_{D1}}$, is presented in Figure 5. It should be noted that the coefficient for the downstream cylinder was estimated using the freestream velocity upstream of the first turbine since this is constant. It is observed that the mean drag coefficient of the downstream cylinder is significantly lower than that of the upstream cylinder and thus, lower than single cylinder that is exposed to freestream velocity. This reduction is due to the fact that the downstream cylinder is exposed to a considerably lower velocity than the upstream cylinder. It can further be observed that the wake effect is more pronounced with increasing Reynolds number. However, at a fixed Reynolds number the interference effect reduces with increasing $S/D$. This is because with increasing distance between the cylinders, the velocity in the wake of the upstream cylinder slightly recovers before encounter the downstream cylinder. This leads to relatively higher velocity field in front of the downstream cylinder with compared with lower values of $S/D$ and hence, leads to observed increase in the mean drag coefficient of the downstream cylinder.
3.2.2 Flow Pattern

The complexity of flow velocity around the cylinders depends on \( Re \) and centre-to-centre distance between the cylinders. The streamlines plot for selected Reynolds numbers at \( S/D = 2 \) are shown in Figure 6. For this fixed value of \( S/D \), the flow structure around the cylinders can be classified into two regimes according to \( Re \). When \( Re \leq 10 \), a pair of symmetric vortices is formed within the gap between the cylinders and no symmetric vortices in the wake of the downstream cylinder (Figure 6a,b). This flow regime can be referred to as symmetric vortices in gap. The flow separation from the upstream turbine surface is observed to be delayed with decreasing Reynolds number. The gap between the cylinders is dominated by negative streamwise velocity along the wake centre line. However, for \( Re > 10 \) (Figure 6c,d), additional pair of symmetric vortices is observed behind the downstream cylinder. The size of this vortices and its length increases with \( Re \). In addition, the angle of flow separation on the rear side of the upstream cylinder increases with the Reynolds number.

The streamlines plots for \( Re = 10 \) at selected \( S/D \) are shown in Figure 7. With increasing \( S/D \), the flow characteristics pattern in the gap between the cylinders change from completely recirculation flow (Figure 7a) to a pair of symmetric vortices behind the upstream cylinder and the flow reattached to the line of symmetric in front of the downstream cylinder (Figure 7b). The size of this pair of symmetrical vortices reduces with increasing \( S/D \). The critical distance at which this change occurs is between \( S/D = 2 \) and 2.5 for this Reynolds number (Figure 7a,b). In addition, the flow around the downstream cylinder steadily becoming symmetrical and the flow in front flow is gradually resemble the rear flow. This is similar to creeping flow around a single circular cylinder. Similar flow pattern and development were observed in the gap for other higher Reynolds numbers but the range of the critical distance where the changes in flow structure occurs increases with the flow Reynolds number.

The length of the recirculation region behind the upstream cylinder relative to the cylinder case for \( Re = 10 \), symbolized as \( \beta \), is presented in Figure 8. At \( S/D = 2.5 \), the recirculation length behind the upstream cylinder is about 2.3 times of a single cylinder. However, as the gap between the cylinders increases, the value of \( \beta \) reduces asymptotes gradually towards unity. It is expected that at sufficiently large value of \( S/D \), the value will be equal to unity. At this distance, the mean drag coefficient of the upstream cylinder is expected to be the same as that of the single cylinder.

4. CONCLUSIONS

In this study, steady laminar flow past two identical circular cylinders in in-line arrangement is investigated numerically at low Reynolds numbers. The effects of low Reynolds number and separation distance between the centres of the cylinders on the flow field around them were studied. For a given Reynolds number, it is observed that the mean drag coefficients of the upstream cylinders are lower than that of an isolated single cylinder. However, the drag coefficient increases with increasing \( S/D \). In addition, the mean drag coefficient of the downstream cylinder is considerably less than that of the upstream cylinder.

The flow velocity field around the cylinders was shown to be complex and the complexity of the flow depends on \( Re \) and \( S/D \). For small value of \( S/D \) (e.g. 2), the flow
structure around the cylinders can be classified into two regimes according to the Reynolds number. When $Re \leq 10$, a pair of symmetric vortices is formed within the gap between the cylinders and no vortices in the wake of the downstream cylinder. However, for $Re > 10$, additional pair of symmetric vortices is observed behind the downstream cylinder. The size of these vortices depends on Reynolds number.

For a fixed Reynolds number and depending on the value of $S/D$, the flow characteristics pattern in the gap between the cylinders could change from completely recirculation flow to a pair of symmetric vortices in rear end of the upstream cylinder and the flow reattached to the line of symmetric in the front of the downstream cylinder. The critical distance of separation where this change occurs is observed to increases with the flow Reynolds number.

REFERENCES
Fig. 6. The streamlines of the velocity field at $S/D = 2$ for (a) $Re = 5$; (b) $Re = 10$, (c) $Re = 20$ and (d) $Re = 30$.

Fig. 7. The streamlines of the velocity field for $Re = 10$ at (a) $S/D = 2$; (b) $S/D = 2.5$; and (c) $S/D = 3$. 
Fig. 8. The development of the recirculation length behind the upstream cylinder normalized by that of a single cylinder for $Re = 10$. 