A Comparison Between HPM and ADM for the Nonlinear Benjamin–Bona–Mahony Equation

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***Abstract*--** **In this paper, we studied whether we could obtain numerical solution of the nonlinear Benjamin–Bona–Mahony Equation (BBME) by Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM) and then formed charts which is include numerical conclusion for this equation by drawing graphics of the this equation by means of programming language Mathematica. Finaly, we have done a comparison between HPM and ADM for BBME.**

***Index Term*--** **Nonlinear Benjamin–Bona–Mahony Equation; Homotopy Perturbation Method; Adomian Decomposition Method;**

1. INTRODUCTİON

In this paper, we use the HPM [4,5,9,11,12] in order to find the analytic solution of nonlinear BBME [1–3]. The method in applied mathematics can be an effective procedure to obtain the analytic and approximate solutions. It is too important to find analytic solutions of nonlinear BBME. BBME is mathematical models of complex physical occurrences that arise in engineering, chemistry, biology, mechanics and Physics [1]. A novel approach to linear or nonlinear problems is particularly valuable as a tool for Scientists and applied Mathemati-cians.

The technique has many advantages over the classical techniques [6–7] because the method does not need linearization or weak nonlinearity assumptions. It is providing an efficient explicit solution with high accuracy, minimal calculation. HPM [8] is the most effective and convenient on effor both linear and nonlinear equations. HPM does not depend on a small parameter. Using homotopy technique in topology, a homotopy is constructed with an embedding parameter $p\in[0,1]$, which is considered as a “small parameter”. HPM has been shown to effectively, easily and accurately solve a large class of linear and nonlinear problems [10] with components converging rapidly to accurate solution. HPM was first proposed by He and was succesfully applied to various engineering problems.

2. AN ANALYSİS OF THE METHODS AND THEİR APPLİCATİONS

2.1 Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following nonlinear differential equation,

$$A(u)-f(r)=0,\ \ r\in\Omega, \tag{1}$$

with the boundary conditions

$$B(u,\partial u/\partial n)=0,\ \ r\in\Gamma, \tag{2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω.

Generally speaking, the operator A can be divided into two parts which are L and N, where L is linear, while N is nonlinear. Eq.(1) can be rewritten as follows:

$$L(u)+N(u)-f(r)=0. \tag{3}$$

By the homotopy technique, we construct a homotopy $V(r,p):\Omega\times[0,1]\to R$ which satisfies:

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)]=0 \tag{4}$$

or

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p\left[N(v)-f(r)\right]=0, \tag{5}$$

where $p \in [0,1]$ is an embedding parameter, u_0 is an initial approximation of Eq.(1). Obviously, from Eq.(4) and Eq.(5) we will have

$$H(v,0) = L(v) - L(u_0) = 0 \tag{6}$$

and

$$H(v,1) = A(v) - f(r) = 0 \tag{7}$$

the changing process of p from zero to unity is just that of $V(r,p)$ from $u_0(r)$ to $u(r)$. In topology, this is called deformation, and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopy. According to the HPM, we can first use the embedding parameter *p* as a "small parameter", and assume that the solution of Eq.(4) and Eq.(5) can be written as a power series in p:

$$V = V_0 + pV_1 + p^2V_2 + p^3V_3 + \cdots. \tag{8}$$

Setting $p = 1$ results in the approximate solution of Eq.(1) and Eq.(2):

$$u = \lim_{p \to 1} V = V_0 + V_1 + V_2 + V_3 + \cdots. \tag{9}$$

The convergence of series Eq.(9) has been proved by He in his paper [12]. This technique can have full advantage of the traditional perturbation techniques. The series Eq.(9) is convergent rate depends on the non-linear operator $A(v)$ (the following opinions are suggested by He [12]):

(1) The second derivative of $N(v)$ with respect to v must be small because the parameter may be relatively largee, i.e., $p \to 1$.

(2) The norm of $L^{-1}(\partial N / \partial v)$ must be smaller than one so that the series converges.

2.2 Adomian Decomposition Method

To demonstrate the basic ideas of this method, we consider the following nonlinear differential equation;

$$L_t(u) - L_x\left(L_{xt}\left(u\right)\right) + L_x\left(u\right) + N\left(u\right) = 0 \tag{10}$$

which is called BBME. Initial condition is

$$u(x,0) = \sec h^2\left(\frac{x}{4}\right). \tag{11}$$

We consider the standard form of problem Eq.(10) in operator form as follows:

$$u_t - u_{xxt} + u_x + uu_x = 0$$

$$L_t = L_{xxt}u - (1-u)L_xu \tag{12}$$

where the differential operator L is

$$L_t = \frac{\partial}{\partial t},\ L_{xxt} = \frac{\partial^3}{\partial x^2 t},\ L_x = \frac{\partial}{\partial x}. \tag{13}$$

L_t^{-1} is regarded as the integration from 0 to t, i.e.

$$L_t^{-1}(.) = \int_0^t (.)\,dt \tag{14}$$

The decomposition method represents the solution of Eq.(12) as a series, i.e.

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t) \tag{15}$$

and the nonlinear operator $F(u) = uu_x$ is decomposed as

$$F(u) = \sum_{n=0}^{\infty} A_n \tag{16}$$

where the components $u_n(x,t)$ will be determined recurrently, and A_n are the so-called Adomian polynomials of $u_0, u_1, u_2, \cdots, u_n$ defined

$$A_n = \frac{1}{n!}.\left[\frac{d^n}{d\lambda^n}\left[F\left(\sum_{k=1}^{\infty} \lambda^i u_i\right)\right]\right]_{\lambda=0},\ n = 0,1,2,\cdots \tag{17}$$

Thus, applying the inverse operator L_t^{-1} on both sides of Eq.(12) and using the initial condition, we find

$$u(x,t) = f(x) - L_t^{-1}\left(L_{xxt} - L_x - uu_x\right)$$
$$u(x,t) = f(x) - L_t^{-1}\left(L_{xxt} - L_x - A_k\right) \tag{18}$$

where is $L_x = \frac{\partial}{\partial x},\ L_{xxt} = \frac{\partial^3}{\partial x^2 t}$ *and* $A_k = (u_k)(u_k)_x$.

Substituting Eq.(15) and Eq.(16) into Eq.(18) gives

$$\sum_{n=o}^{\infty} u_n(x,t) = f(x) - L_t^{-1}\left\{\left(\sum_{n=o}^{\infty} u_n(x,t)\right)_{xtx} - \left(\sum_{n=o}^{\infty} u_n(x,t)\right)_x - \left(\sum_{n=o}^{\infty} A_n(x,t)\right)\right\}. \tag{19}$$

Identifying the zeroth component $u_0(x,t)$ by all terms that arise from the initial condition gives as a result, the remaining components $u_n(x,t), n \geq 1$ which can be determined by using the recurrence relation:

$$u_0(x,t) = f(x,t)$$
$$u_{k+1}(x,t) = -L_t^{-1}\left[(u_k)_{xtx} - (u_k)_x - A_k\right], k \geq 0 \tag{20}$$

where A_n are the Adomian polynomials that represent the nonlinear term uu_x given by

power of the A^k :

$$A^k = (u_0 + u_1 + u_2 + u_3 + \cdots)\left[(u_0) + (u_1) + (u_2) + (u_3) + \cdots\right]_x$$
$$A_0 = u_0 (u_0)_x$$
$$A_1 = u_{0x}(u_1) + u_0(u_{1x})$$
$$A_2 = u_{0x}(u_2) + u_{1x}(u_1) + u_{2x}(u_0)$$
$$A_3 = u_{0x}(u_3) + u_{1x}(u_2) + u_{2x}(u_1) + u_{3x}(u_0)$$
$$\vdots$$

Consequently, one can recursively determine every term of the series $\sum_{n=0}^{\infty} u_n(x,t)$ and hence the solution $u(x,t)$ is readily obtained in a series form. In order to verify numerically whether the proposed methodology lead to higher accuracy, we can evaluate the approximate solution using the n-term approximation to $u(x,t)$ by ψ_n such that

$$u(x,t) = \lim_{n\to\infty} \psi_n$$

where

$$\psi_n = \sum_{k=0}^{n-1} u_k(x,t) \tag{21}$$

can be used.

Example We consider the simplest nonlinear BBME in its simplest form is given by

$$u_t - u_{xtx} + u_x + uu_x = o \tag{22}$$

and initial conditions is

$$u(x,0) = \sec h^2(\frac{1}{4}x). \tag{23}$$

In order to solve Eq.(22) by using HPM, we can construct a homotopy for BBME as follows:

$$(1-p)[Y^{\cdot} - u_0^{\cdot}] + p[Y^{\cdot} - Y^{''\cdot} + Y^{'} + YY^{'}] = 0 \tag{24}$$

$$Y^{\cdot} - u_0^{\cdot} + pu_0^{\cdot} - pY^{''\cdot} + pY^{'} + pYY^{'} = 0 \tag{25}$$

where $Y^{\cdot} = \frac{\partial Y}{\partial t}$, $Y^{''\cdot} = (Y_{xt})_x$ $Y^{'} = \frac{\partial Y}{\partial x}$ and $p \in [0,1]$.

Initial approximation is $Y_0 = u_0(x,0) = \sec h^2\left(\frac{x}{4}\right)$. The approximation solution of Eq.(22) has the form as follows:

$$Y = Y_0 + pY_1 + p^2Y_2 + p^3Y_3 + \cdots = \sum_{n=0}^{\infty} p^n Y_n\left(x,t\right) \quad (26)$$

$$Y^{\cdot} = Y_0^{\cdot} + pY_1^{\cdot} + p^2Y_2^{\cdot} + p^3Y_3^{\cdot} + \cdots \quad (27)$$

$$Y^{''\cdot} = Y_0^{''\cdot} + pY_1^{''\cdot} + p^2Y_2^{''\cdot} + p^3Y_3^{''\cdot} + \cdots \quad (28)$$

$$Y^{'} = Y_0^{'} + pY_1^{'} + p^2Y_2^{'} + p^3Y_3^{'} + \cdots \quad (29)$$

Then, substituting Eq.(26-29) into Eq.(25) and rearranging based on powers of p -terms, we obtain

$$p^0 : Y_0^{\cdot} - u_0^{\cdot} = 0 \quad (30)$$

$$p^1 : Y_1^{\cdot} + u_0^{\cdot} - Y_0^{''\cdot} + Y_0^{'} + Y_0Y_0^{'} = 0 \quad (31)$$

$$p^2 : Y_2^{\cdot} - Y_1^{''\cdot} + Y_1^{'} + Y_0Y_1^{'} + Y_1Y_0^{'} = 0 \quad (32)$$

$$p^3 : Y_3^{\cdot} - Y_2^{''\cdot} + Y_2^{'} + Y_0Y_2^{'} + Y_1Y_1^{'} + Y_2Y_0^{'} = 0 \quad (33)$$

$$\vdots$$

with solving Eq.(30-33)

$$p^0 : Y_0^{\cdot} - u_0^{\cdot} = 0 \Rightarrow Y_0^{\cdot} = u_0^{\cdot} \Rightarrow Y_0 = u_0 = \sec h^2\left(\frac{x}{4}\right)$$

$$\Rightarrow Y_0 = \sec h^2\left(\frac{x}{4}\right), \quad (34)$$

$$p^1 : Y_1^{\cdot} + u_0^{\cdot} - Y_0^{''\cdot} + Y_0^{'} + Y_0Y_0^{'} = 0 \Rightarrow Y_1^{\cdot} = -u_0^{\cdot} + Y_0^{''\cdot} - Y_0^{'} - Y_0Y_0^{'}$$

$$\Rightarrow Y_1 = \int_0^t \left[-u_0^{\cdot} + Y_0^{''\cdot} - Y_0^{'} - Y_0Y_0^{'}\right] dt$$

$$\Rightarrow Y_1 = \int_0^t \left[Y_0^{''\cdot} - Y_0^{'} - Y_0Y_0^{'}\right] dt \quad (35)$$

$$\Rightarrow Y_1 = \frac{1}{8}\text{tsech}^5\left(\frac{x}{4}\right)\left[5\sinh\left(\frac{x}{4}\right) + \sinh\left(\frac{3x}{4}\right)\right],$$

$$p^2 : Y_2^{\cdot} - Y_1^{''\cdot} + Y_1^{'} + Y_0Y_1^{'} + Y_1Y_0^{'} = 0 \Rightarrow Y_2^{\cdot} = \left[Y_1^{''\cdot} - Y_1^{'} - Y_0Y_1^{'} - Y_1Y_0^{'}\right]$$

$$\Rightarrow Y_2 = \int_0^t \left[Y_1^{''\cdot} - Y_1^{'} - Y_0Y_1^{'} - Y_1Y_0^{'}\right] dt$$

$$\Rightarrow Y_2 = \frac{1}{256} t\sec h^8\left(\frac{x}{4}\right)\left[-104t + 23t\cosh\left(\frac{x}{2}\right)\right] \quad (36)$$

$$+\frac{1}{256} t\sec h^8\left(\frac{x}{4}\right)\left[16t\cosh\left(x\right) + t\cosh\left(\frac{3x}{2}\right)\right]$$

$$+\frac{1}{256} t\sec h^8\left(\frac{x}{4}\right)\left[-107 sinh\left(\frac{x}{2}\right) + 8 sinh(x) + sinh\left(\frac{3x}{2}\right)\right],$$

$$p^3: Y_3^{\cdot} - Y_2^{'''} + Y_2^{'} + Y_0 Y_2^{'} + Y_1 Y_1^{'} + Y_2 Y_0^{'} = 0 \Rightarrow Y_3^{\cdot} = \left[Y_2^{'''} - Y_2^{'} - Y_0 Y_2^{'} - Y_1 Y_1^{'} - Y_2 Y_0^{'} \right]$$

$$\Rightarrow Y_3 = \int_0^t \left[Y_2^{'''} - Y_2^{'} - Y_0 Y_2^{'} - Y_1 Y_1^{'} - Y_2 Y_0^{'} \right] dt$$

$$\begin{aligned}\Rightarrow Y_3 = \frac{1}{12288}(t\sec h^9(\frac{x}{2})(6t(3775 - 2726\cosh(\frac{x}{2}) \\ - 2280\cosh(x) + 30\cosh(\frac{3}{2}x) + \cosh(2x) \\ + \sec h(\frac{x}{4}) + 6(5533 sinh(\frac{1}{4}x) - 1179 sinh(\frac{3}{4}x) \\ + 19 sinh(\frac{5}{4}x) + sinh(\frac{7}{4}x)) \\ + t^2 \sec h^2(\frac{x}{4})(-7086 sinh(\frac{x}{4}) - 1096 sinh(\frac{3}{4}x) \\ + 272 sinh(\frac{5x}{4}) + 43 sinh(\frac{7x}{4}) + sinh(\frac{9x}{4})))) \ ,\end{aligned} \tag{37}$$

$\vdots$

the above terms of the series Eq.(26) could be easily calculated. When we consider the series Eq.(26) with the terms Eq.(34–37) and suppose $p = 1$, we obtain approximation solution of Eq.(22) as follows:

$$u(x,t) = Y_0 + pY_1 + p^2 Y_2 + p^3 Y_3 + \cdots$$

$$u(x,t) = \lim_{p \to 1} \left(Y_0 + pY_1 + p^2 Y_2 + p^3 Y_3 + \cdots \right)$$

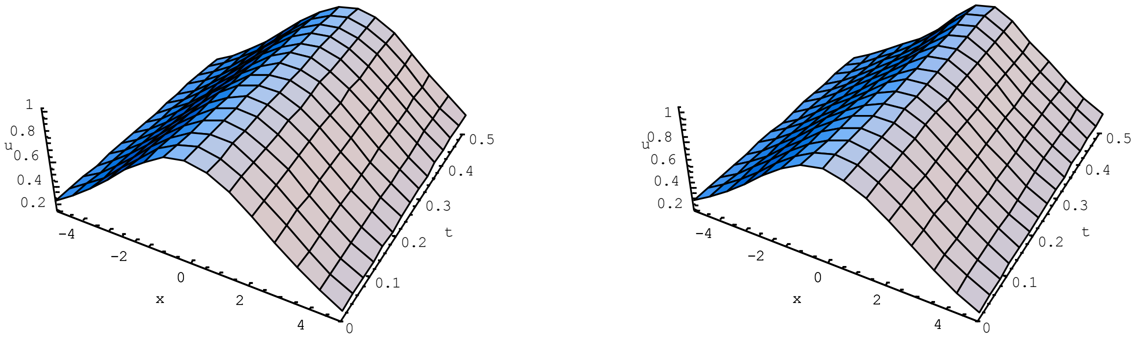
$$u(x,t) = Y_0 + Y_1 + Y_2 + Y_3 + \cdots$$

$$\begin{aligned}u(x,t) = \sec h^2(\frac{x}{4}) + \frac{1}{8}\text{tsech}^5(\frac{x}{4})\ (5\sinh(\frac{x}{4}) + \sinh(\frac{3}{4}x)) + \frac{1}{256}t\sec h^8(\frac{x}{4})(-104t \\ + 23t\cosh(\frac{1}{2}x) + 16t\cosh(x) + t\cosh(\frac{3}{2}x) - 107 sinh(\frac{1}{2}x) + 8 sinh(x) \\ + sinh(\frac{3}{2}x)) + \frac{1}{12288}(t\sec h^9(\frac{x}{2})(6t(3775 - 2726\cosh(\frac{x}{2}) - 2280\cosh(x) \\ + 30\cosh(\frac{3}{2}x) + \cosh(2x) + \sec h(\frac{x}{4}) + 6(5533 sinh(\frac{1}{4}x) - 1179 sinh(\frac{3}{4}x) \\ + 19 sinh(\frac{5}{4}x) + sinh(\frac{7}{4}x)) + t^2\sec h^2(\frac{x}{4})(-7086 sinh(\frac{x}{4}) - 1096 sinh(\frac{3}{4}x) \\ + 272 sinh(\frac{5x}{4}) + 43 sinh(\frac{7x}{4}) + sinh(\frac{9x}{4})))).\end{aligned}$$

We obtain the closed form exact solution as follows:

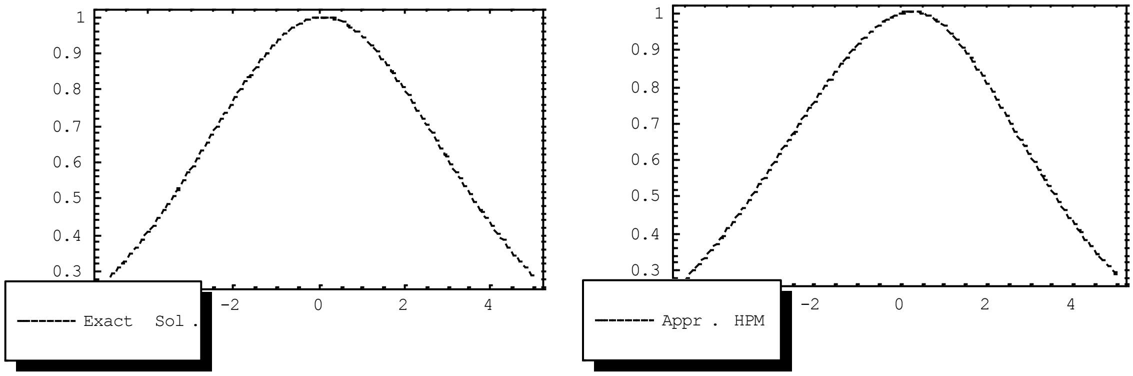
$$u(x,t) = \sec h^2\left(\frac{x}{4} - \frac{t}{3}\right).$$

As a result, the components $Y_0, Y_1, Y_2, Y_3, \cdots$ are identified and the series solution thus entirely determined.



(a) Exact solution (b) Approximation solution HPM

Fig. 1.1 The numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t=0.05$ with initial condition of Eq.(22) by means of HPM



(a) Exact solution (b) Approximation solution HPM

Fig. 1.2 The plots of the numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t=0.05$ with initial condition of Eq.(22) by means of HPM

Table I

The numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t=0.05$ with initial condition of Eq. (22) by means of HPM

t\x	0.03	0.04	0.05
0.01	2.26646E-4	2.77073E-4	3.27453E-4
0.02	6.03525E-4	7.04304E-4	8.04969E-4
0.03	1.13061E-3	1.28165E-3	1.43250E-3
0.04	1.80786E-3	2.00908E-3	2.20999E-3
0.05	2.63524E-3	2.88653E-3	3.13739E-3

In order to solve Eq.(22) by using ADM, we consider the following nonlinear BBME

$$u_t - u_{xxt} + u_x + uu_x = 0 \tag{38}$$

which is called nonlinear BBME. Initial condition is

$$u(x,0)=\sec h^2\left(\frac{x}{4}\right). \tag{39}$$

We consider the standard form of problem Eq.(38) in operator form as follows:

$$u_t - u_{xxt} + u_x + uu_x = 0$$
$$L_t = L_{xxt} - L_x - uu_x \tag{40}$$

where is $L_t = \frac{\partial}{\partial t}$, $L_{xxt} = \frac{\partial^3}{\partial x^2 \partial t}$ *and* $L_x = \frac{\partial}{\partial x}$.

$$u(x,t) = L_t^{-1}\left[L_{xxt} - L_x - uu_x\right]$$
$$u(x,t) = u(x,0) - \int_0^t \left[L_{xxt} - L_x - uu_x\right]dt$$
$$u_{n+1}(x,t) = -\int_0^t \left[(u_n)_{xxt} - (u_n)_x - A_n\right]dt, \tag{41}$$

the Eq.(41) take as the following for different n value; $u_0 = u(x,0) = \sec h^2\left(\frac{x}{4}\right)$. A_n are the Adomian polynomials that represent the nonlinear term uu_x given by power of the A_k :

$$A_k = (u_0 + u_1 + u_2 + u_3 + \cdots)\left[(u_0) + (u_1) + (u_2) + (u_3) + \cdots\right]_x$$
$$A_0 = u_0 (u_0)_x$$
$$A_1 = (u_0)_x (u_1) + u_0 (u_1)_x$$
$$A_2 = u_2 (u_0)_x + u_1 (u_1)_x + u_0 (u_2)_x$$
$$A_3 = u_3 (u_0)_x + u_2 (u_1)_x + u_1 (u_2)_x + u_0 (u_3)_x$$
$$\vdots$$

$$n = 0 \Rightarrow u_1 = -\int_0^t \left[(u_0)_{xxt} - (u_0)_x - A_0\right]dt$$
$$\Rightarrow u_1 = \int_0^t \left[-(u_0)_{xxt} + (u_0)_x + u_0 (u_0)_x\right]dt \tag{42}$$
$$\Rightarrow u_1 = \frac{1}{8}\text{tsech}^5\left(\frac{x}{4}\right)\left[5\sinh\left(\frac{x}{4}\right) + \sinh\left(\frac{3x}{4}\right)\right],$$

$$n=1 \Rightarrow u_2 = -\int_0^t \left[(u_1)_{xtx} - (u_1)_x - A_1 \right] dt$$

$$\Rightarrow u_2 = \int_0^t \left[-(u_1)_{xtx} + (u_1)_x + (u_0)_x (u_1) + u_0 (u_1)_x \right] dt$$

$$\Rightarrow u_2 = \frac{1}{256} t \sec h^8 \left(\frac{x}{4}\right) \left[-104t + 23t \cosh\left(\frac{x}{2}\right) + 16t \cosh(x) + t \cosh\left(\frac{3x}{2}\right) \right] \quad (43)$$

$$+ \frac{1}{256} t \sec h^8 \left(\frac{x}{4}\right) \left[107 sinh\left(\frac{x}{2}\right) - 8 sinh(x) - sinh\left(\frac{3x}{2}\right) \right],$$

$$n=2 \Rightarrow u_3 = -\int_0^t \left[(u_2)_{xtx} - (u_2)_x - A_2 \right] dt$$

$$\Rightarrow u_3 = \int_0^t \left[-(u_2)_{xtx} + (u_2)_x + u_2 (u_0)_x + u_1 (u_1)_x + u_0 (u_2)_x \right] dt$$

$$\Rightarrow u_3 = \frac{1}{12288} (t \sec h^9 (\frac{x}{4}) 6 \left[5533 sinh(\frac{x}{4}) - 1179 sinh(\frac{3x}{4}) + 9 sinh(\frac{5x}{4}) + sinh(\frac{7x}{4}) \right]$$

$$+ t^2 \sec h^2 (\frac{x}{4}) \left[-1096 sinh(\frac{3x}{4}) + 272 \sinh(\frac{5x}{4}) + 43 \sinh(\frac{7x}{4}) + \sinh(\frac{9x}{4}) \right]$$

$$- 6t \sec h(\frac{x}{4})(3775 - 2726 \cosh(\frac{x}{2}) - 280 \cosh(x) + 30 \cos h(\frac{3x}{2}) + \cos h(2x)$$

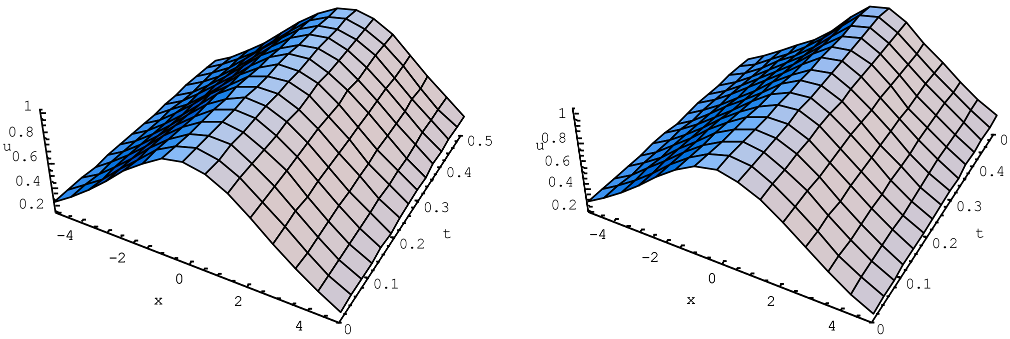
$$+ 1181 t . tanh(\frac{x}{4}))) ,$$

$$\vdots$$

In this manner, three component of the decomposition series were obtained of which u(x,t) was evaluated to have the following expansion.

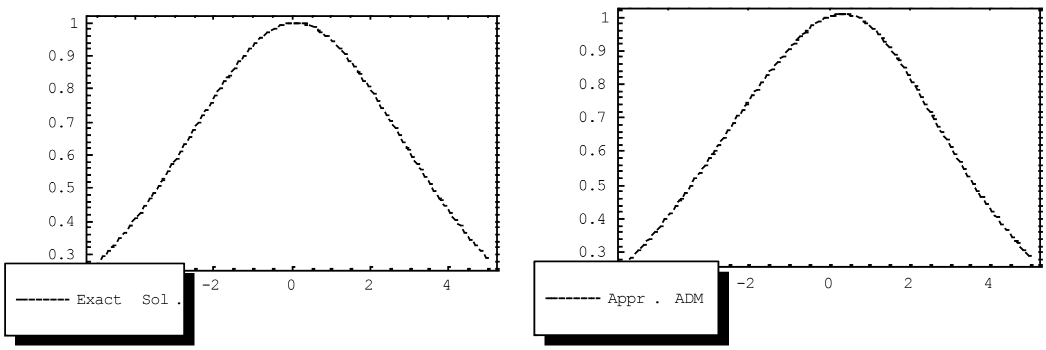
$$
\begin{aligned}
u(x,t) &= \sum_{n=0}^{\infty} u_n(x,t) = u_0 + u_1 + u_2 + u_3 + \cdots \\
&= \sec h^2\left(\frac{x}{4}\right) + \frac{1}{8}\text{tsech}^5\left(\frac{x}{4}\right)\left[5\sinh\left(\frac{x}{4}\right) + \sinh\left(\frac{3x}{4}\right)\right] \\
&+ \frac{1}{256} t \sec h^8\left(\frac{x}{4}\right)\left[-104t + 23t\cosh\left(\frac{x}{2}\right) + 16t\cosh(x) + t\cosh\left(\frac{3x}{2}\right)\right] \\
&+ \frac{1}{256} t \sec h^8\left(\frac{x}{4}\right)\left[107 sinh\left(\frac{x}{2}\right) - 8 sinh(x) - sinh\left(\frac{3x}{2}\right)\right] \\
&+ \frac{1}{12288}(t \sec h^9(\frac{x}{4})6\left[5533 sinh(\frac{x}{4}) - 1179 sinh(\frac{3x}{4}) + 9 sinh(\frac{5x}{4}) + sinh(\frac{7x}{4})\right] \\
&+ t^2 \sec h^2(\frac{x}{4})\left[-1096 sinh(\frac{3x}{4}) + 272\sinh(\frac{5x}{4}) + 43\sinh(\frac{7x}{4}) + \sinh(\frac{9x}{4})\right] \\
&- 6t \sec h(\frac{x}{4})(3775 - 2726\cosh(\frac{x}{2}) - 280\cosh(x) + 30\cos h(\frac{3x}{2}) + \cos h(2x) \\
&+ 1181 t . tanh(\frac{x}{4}))).
\end{aligned}
$$

As a result, the components $Y_0, Y_1, Y_2, Y_3, \cdots$ are identified and the series solution thus entirely determined.



(a) Exact solution (b) Approximation solution ADM

Fig. 2.1 The numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t = 0.05$ with initial condition of Eq.(22) by means of ADM



(a) Exact solution (b) Approximation solution ADM

Fig. 2.2 The plots of the numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t = 0.05$ with initial condition of Eq.(22) by means of ADM

Table II
The numerical results for Y_3 in comparison with the analytic solution $u(x,t)$ when $t = 0.05$ with initial condition of Eq. (22) by means of ADM

t\x	0.03	0.04	0.05
0.01	2.26646E-4	2.77073E-4	3.27453E-4
0.02	6.03525E-4	7.04304E-4	8.04969E-4
0.03	1.13061E-3	1.28165E-3	1.43250E-3
0.04	1.80786E-3	2.00908E-3	2.20999E-3
0.05	2.63524E-3	2.88653E-3	3.13739E-3

3. CONCLUSION

In this work, the HPM and ADM was used nonlinear BBME with initial conditions and obtained their analytic solutions. The analytic solutions of the nonlinear equations have a fundamental importance. Various effective methods have been developed to understand the mechanisms of these physical models, to help physicians and engineers and to ensure knowledge for physical problems and its applications. A clear conclusion can be drawn from the numerical results that the HPM algorithm provides analytic solutions without special discretizations for the nonlinear partial differential equations.

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