The Response of a Particles-Reinforced Composite with a Plastic Matrix Phase

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***Abstract*--** **This paper describe a new computational homogenization methodology of the prediction the effective elastic-plastic response of random N-phase two-dimensional heterogeneous material. A numerical homogenization technique, based on finite element method, with representative volume element, was used with various boundary conditions. It is based on finite element simulations using 2D cells of different size, smaller than the deterministic representative volume element (DRVE) of the microstructure. A 3-phases materials and 4-phases one, are selected to illustrate this approach; it consists of a random dispersion of elastic circular inclusions in an elastic-plastic matrix. It is found that the effective elastic-plastic response of this composite can be precisely determined by computing a sufficient number of small subvolumes of fixed size smaller than the DRVE and containing different realizations of the random microstructure.**

***Index Term*--** **RVE, Representative volume element, Computational homogenization, Finite element modeling, N-phase elastic-plastic composites.**

INTRODUCTION

The estimation of the effective mechanical response of random heterogeneous material is a very important active research area. Many analytical methods using homogenization technics have been developped, to bound or estimate their effective material properties [Nemat-Nasser and Hori 1993.]. These methods which assume that the effective material properties can be defined as relations among the volume averages of stress and strain fields were primarily used for the linear elastic domain. In the literature, we can found a lot of model, Voigt-Reuss and Hashin-Shtrikman [Hashin and Shtrikman 1963] bounds, Mori-Tanaka model [Mori and Tanaka 1973] and the self-consistent scheme [Hill 1965]. For a comparison of these models, see the papers of Anoukou et al. [Anoukou et al 2011]. Although the large use of these analytical approaches, it remains quite complex to transpose all these model from the linear elastic regime to the plastic regime. For this raison, tangent and secant formulations were developed.

In tangent formulations, the effective elastic-plastic response is computed incrementally by integrating along the loading path the effective stiffness tensor obtained from the tangent stiffness tensor of each phase [Hutchinson 1970, Ju and Sun 2001, Doghri and Friebel 2005, Zaïri et al 2011]. For the secant formulations, the effective elastic-plastic response is computed from the secant stiffness tensor of each phase within the nonlinear elastic framework [Ponte Castaneda and Suquet 1998, Tandon and Weng 1988]. In [Zaïri et al 2011, Shen and Catherine 2006] a numerical investigation of the effect of boundary conditions and representative volume element sizefor porous titanium. Alternatively, the numerical simulations directly performed on the microstructure can give solution to problems such as the plasticity in N-phases composite. The notion of representative volume element (RVE) is used to represent the microstructure. The RVE must be chosen sufficiently large compared to heterogeneities to contain sufficient information about the microstructure in order to be representative, but it must remain small enough, much smaller than the macroscopic body, in order to be considered as a material volume element. Drugan and Willis [Drugan and Willis 1996] proposed to define the RVE as follows: "It is the smallest material volume element of the composite for which the usual spatially constant (overall modulus) macroscopic constitutive representation is a sufficiently accurate model to represent the mean constitutive response".

This definition of the "deterministic" representative volume element (DRVE) ought to be verified in the context of elastic-plastic composites. The effective stress-strain response, defined from spatial averages of stress and strain fields over the volume element, must be obtained with a given precision. For large-scale computations the computational cost is a principal issue and it is appealing to work on volumes smaller than the DRVE. The use of smaller volumes induces fluctuations of the estimated responses which must be compensated by averaging over several realizations of the microstructure in order to get the same estimation as that obtained for the entire volume, [Khdir 2013]. The methodology based on the (DRVE) was proposed in [Hazanov and Huet 1994, Kanit 2003 and 2006], to estimate the linear elastic response of heterogeneous materials and it was extended by [Khdir 2013] to the 2-phase elastic-plastic composites. The purpose of the present work is to extend the computational homogenization strategy proposed by khdir et al. [Khdir 2013] to estimate the effective elastic-plastic response of N-phases random composites. The methodology is applied to a 3-phase and 4-phases composites, and extended to a N-phases random composites. The numerical estimates of the stress-strain response, and their scatters, obtained on volumes of fixed size but containing different realizations of a given volume of the microstructure are investigated.

The present paper is organized as follows. In Section 2, we present the investigated microstructure and the computational method. The results are discussed in Section 3. Some concluding remarks are given in Section 4.

MATERIALS AND METHODS

The finite element method is used for the computations presented in this work. This requires the definition of some elements.

Microstructure and mechanical properties of material

The examples of materials chosen in this investigation to illustrate the methodology is a 3-phases and a 4-phases composites. The examples are constituted by a disordered distribution of elastic inclusions in a stiff matrix. The number of inclusion of each phase is identified by the letter M. The mechanical properties are known for every constituents. The inclusions are assumed linear elastic while the matrix is elastic-plastic. The Young's moduli of the matrix m is 1550 MPa, the Poisson's ratios is 0.4. For the inclusions phase the contrast in the mechanical properties of all the inclusions is 100. The inelastic properties of the matrix were taken from the experimental data employed by Zaïri in [Zaïri 2011], see Table I.

Table I
Elastic properties of considered inclusions phases.

Material			E[MPa]	ν	Particles shape		Volume fraction
Case 1	Three-phase material	Inclusion 1	100	0.3	Circulars inclusions	$D_1 = D_2$ $M_1 = M_2$	$f_1 = f_2 = 15\%$
		Inclusion 2	1				
Case 2	Four-phase material	Inclusion 1	100	0.3	Circulars inclusions	$D_1 = D_2 = D_3$ $M_1 = M_2 = M_3$	$f_1 = f_2 = f_3 = 10\%$
		Inclusion 2	1				
		Inclusion 3	0.01				

MESHING MICROSTRUCTURES

The finite element (FE) method was chosen for the computations presented in this work. Free meshing technique is used to mesh the 2D images of the microstructures. The used elements are two dimensional Delaunay triangles, 3 nodes, with complete integration. In all the simulations, a good mesh density has been used to compute the macroscopic properties with a good precision.

BOUNDARY CONDITIONS

Another important element for the computations tests concerns the boundary conditions [Kanit 2013, Li and Ostoja-Starzewski 2006] which for a uni-axial tensile loading in the x-direction for example, are prescribed as follows, see figure 2:

- u (point O(0,0))= 0
- v (point O(0,0))= 0
- u (LEFT $(x = 0, y)$)= 0
- v (LEFT $(x = 0, y)$)= 0
- u (RIGHT $(x = l, y)$)= d

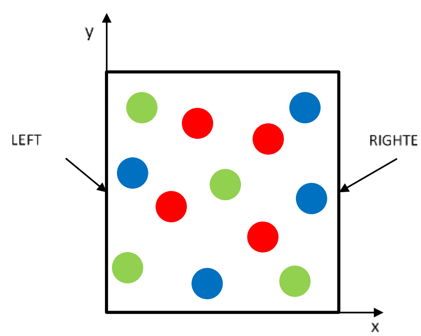

Fig. 2. Description of the boundary conditions.

in which u and v are the applied displacements in the x and y directions, l is the volume length and d is the prescribed displacement.

NUMBER OF REALIZATIONS AND RVE SIZES

Finit element computations on subvolumes of different sizes smaller than the RVE and extracted from the entire volume V were performed. Different sizes of material are used in this investigation; from 3000x3000 pixels, 1000x1000 pixels and 500 x500 pixels. The main advantage of this strategy is that it allows us to work on a sufficiently large volume for a low computational cost. Figure 1 shows examples of 3-phases material and 4-phases one with differents volumes sizes used in this paper. The term n denotes the number of inclusions in each volume and p denotes the number of realizations. Different configurations of material with increasing sizes are summarized in table 2 and 3. Note that decreasing p means an increasing number of inclusions n in a subvolume.

$$n = \sum_{i=1}^{N} M_i \quad (1)$$

That leads to a total of 58 different realizations which includfor all the volumes the same number of inclsuions M. The apparent strains and stresses computed for each subvolume are used to calculate the average strain $\overline{E}^{app}$ and the average stress $\overline{\Sigma}^{app}$ given at each increment as follows:

$$\overline{E}^{app} = \frac{1}{p}\sum_{i=1}^{p} E_i^{app} \quad (2)$$

$$\overline{\Sigma}^{app} = \frac{1}{p}\sum_{i=1}^{p} \Sigma_i^{app} \quad (3)$$

in which Σ_i^{app} is the stress for a given strain E_i^{app} of the realization i.

RESULTS AND DISCUSSION

In all the simulations, only one volume fraction $f = 0.7$ of matrix phase is considered. As displayed in figure. 3a identical effective stress-strain responses in the two orthogonal directions were obtained for the VER$(p = 1, n = 200)$ in the case of 3-phases composite and the VER $(p = 1, n = 300)$ in the case of 4-phases one, see figure. 4a. For the cases of averaging on subvolumes, numerical calculations of the apparent stress-strain response of subdomains smaller than the RVE show a differences in the two directions x and y for the two type of materials considered here. For only one subvolume, the mechanical response is anisotropic see figures. 3d and 4d . If the average process is undertaken for all subvolumes in each direction (figure. 3b and 3c), we note that the averages of apparent responses are nearly identical in the two directions (figure. 3d). Even this recovered isotropy is shown for a specific case, the same trends were observed for all arrangements. One may conclude that only one subvolume could not be used as RVE to describe the mechanical response since the observed anisotropy is not in agreement with the isotropic character of the random microstructure at the macroscopic scale. The mechanical responses obtained for other realizations of the 4-phases composite are presented in figure. 5. As expected, for a given subdivision, the dispersion from one subvolume to another decreases when the size of subvolumes increases. It turns out that a sufficiently large subvolume must be selected inside the whole volume to avoid fluctuations and to reach a good estimation of the average.

Table II
Characteristics of all considered realizations for 3-phases material.

p	36	9	4	4	4	1
n	2	24	50	76	100	200

Table III
Characteristics of all considered realizations for 4-phases material.

p	36	9	4	4	4	1
n	3	33	75	99	150	300

The number of subvolumes provide, the same number of apparent stress-strain curves (figure. 5a and b) for the 3-phases material, (figure. 6a and b) for the 4-phases one with a significant dispersion for all the case of material. This significant dispersion can be understood as a quantity of the loss of representativity due to a high variability of properties over limited domains. It can be appreciated that the error decreases with the domain size. For characterizing the dispersion in the apparent stress-strain response, the quadratic error x_l is used here. This parameter is defined as:

$$x_l = \frac{1}{m}\sum_{j=1}^{m}\left(\frac{\Sigma_j^{app}}{\overline{\Sigma}_j^{app}} - 1\right)^2, 1 \le l \le p \quad (4)$$

where Σ_j^{app} represent the axial stress value for a given axial strain E_j^{app} $1 \le j \le m$ (m is the total number of increments) and $\overline{\Sigma}_j^{app}$ denote the corresponding average axial stress computed on the p curves. The average quadratic error $\overline{x}$ of p realizations is given by equation (5). The value of the quadratic error $\overline{x}$ are reported for the 3-phases material and the 4-phases one in the table 4 and 5.

$$\overline{x} = \frac{1}{p}\sum_{l=1}^{p} x_l \quad (5)$$

Table IV
Error for each group of realizations for 3-phases material.

n	2	24	50	76	100	200
$\overline{x}$ Error	2.12	1.7	0.9	0.07	0.01	0

Table V
Error for each group of realizations for 4-phases material.

n	3	33	75	99	150	300
$\overline{x}$ Error	2.34	1.5	1.02	0.5	0.05	0

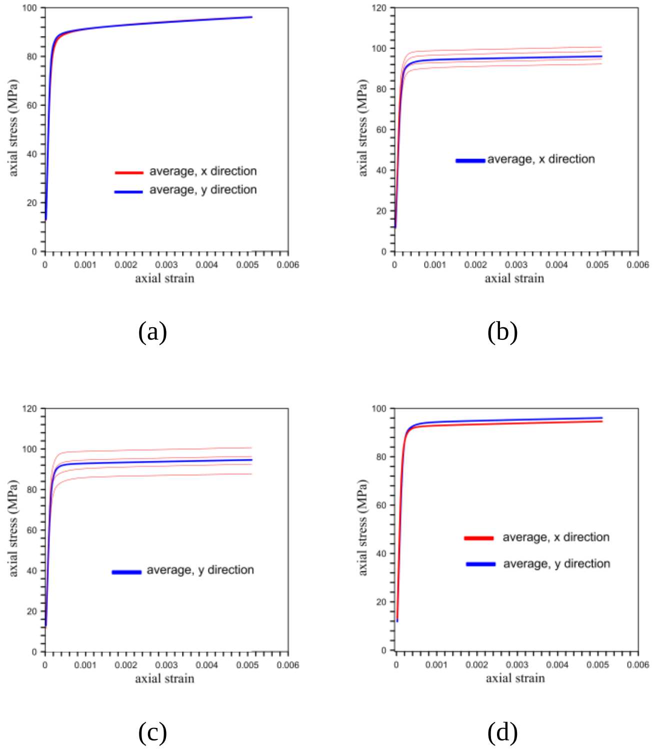

Fig. 3. Apparent stress–strain curves for the 3-phases composite: (a) for the VER (p = 1, n = 200) with $f = 0.7$, (b) for only one subvolume (p = 4, n = 100) stretched in the x direction, (c) y direction, and (d) comparison of average curves in the two directions ($f = 0.7$).

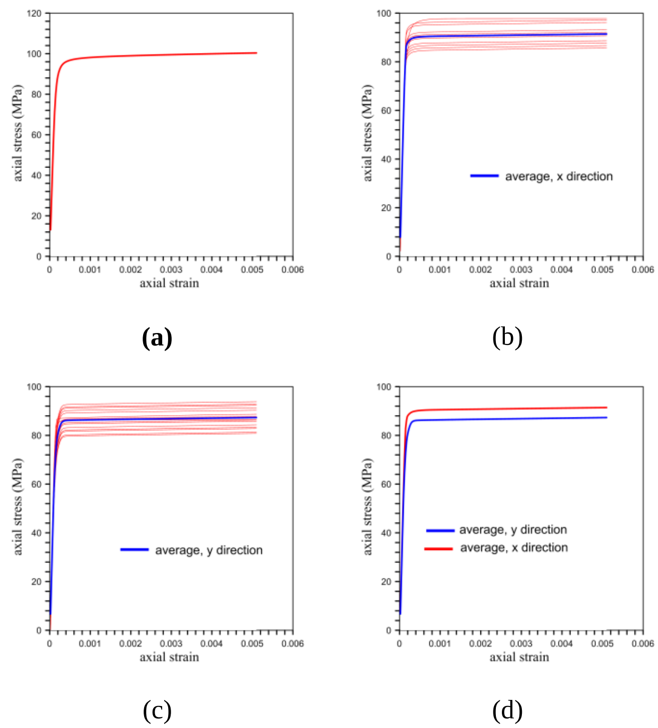

Fig. 4. Apparent stress–strain curves for the 4-phases composite: (a) for the VER (p = 1, n = 300) with $f = 0.7$, (b) for only one subvolume (p = 36, n = 3) stretched in the x direction, (c) y direction, and (d) comparison of average curves in the two directions ($f = 0.7$).

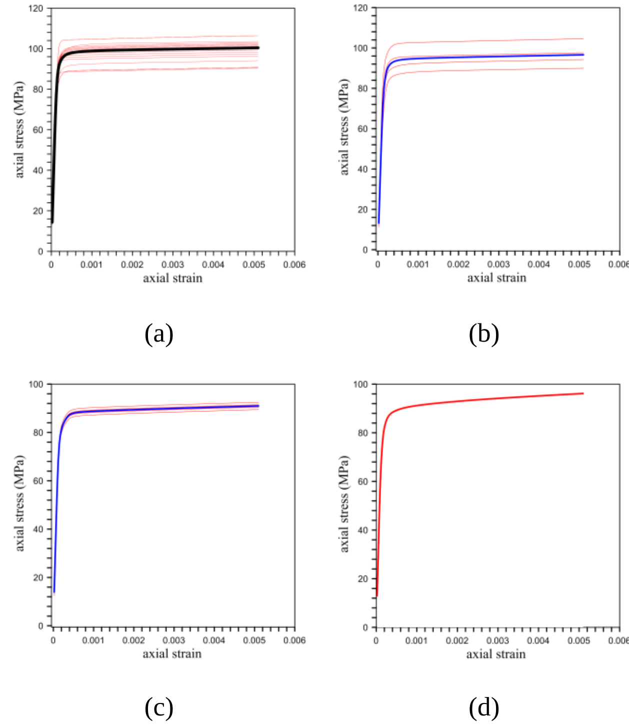

Fig. 5. Examples of apparent stress–strain curves (thin lines) and comparison with the average curve (thick line) for the different configurations of 3-phases material with $f = 0.7$: (a) $n = 24$, $p = 9$, (b) $n = 76$, $p = 4$, (c) $n = 100$, $p = 4$, (d) $n = 200$ and $p = 1$.

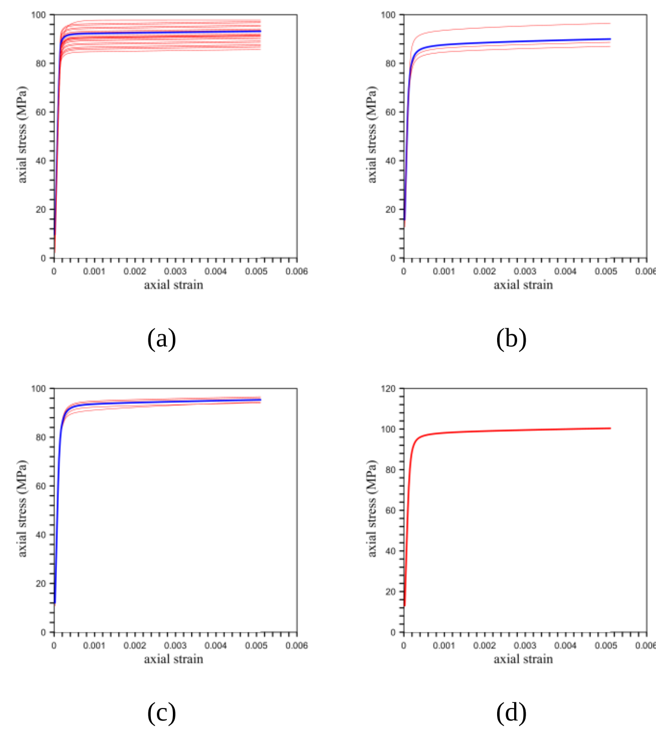

Fig. 6. Examples of apparent stress–strain curves (thin lines) and comparison with the average curve (thick line) for the different configurations of 4-phases material with $f = 0.7$: (a) $n = 3$, $p = 36$, (b) $n = 99$, $p = 4$, (c) $n = 150$, $p = 4$, (d) $n = 300$ and $p = 1$.

CONCLUSION

A novel computational homogenization technique was proposed to evaluate the effective elastic-plastic response of N-phases composites. The technique is based on the computations of small limited volumes of fixed size and containing different realizations of the random particles. The small subvolume did not obligatory display an isotropic response; we verified that the average response of a appropriate number of different realizations is isotropic. A significant scatter in the plastic regime of the apparent stress-strain curves was observed for too small subvolumes. The dispersion of the results decreases when the volume size increases. Finally, we have shown that the methodology used in [Khdir 2013] for the two phases elastic-plastic composite is applicable in the case of 3-phases material, 4-phases one and can be extended to the N-phases elastic-plastic heterogeneous composites

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