Dynamic Prediction of Laminated Glass Plate Based on Higher-Order Plate Finite Element
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Abstract— A drop weight impact on laminated glass plate like automotive windshield can be caused the reduction of the strength of the material of window glass plate. Dynamic predictions on automotive windshield like laminated glass are approached by the use of a refined finite element formulation based on Reddy’s Higher-order Shear Deformation Theory (HSDT) in conjunction with Hertz’s contact law and Dharani’s PVB interlayer model. This higher-order plate theory contains the same dependent unknowns as in Whitney and Pagano’s First-order Shear Deformation Theory (FSDT), and accounts for parabolic distribution of the transverse shear strains through the thickness of the plate. And also, this theory requires no shear correction coefficients and predicts the deflections and stresses more accurately when compared to FSDT. Consequently, dynamic predictions like deflection, kinetic energy, wave fronts and in-plane stress in monolithic glass plate are a few sensitive than those of laminated glass plate and prone to more fracture risk. But we can see that the variation of PVB thickness of laminated glass plate does not affect so much on dynamic responses. These results present a similar trend with those of simulation by using FSDT and very small differences in its magnitudes of every simulation. That is, we can’t see so much difference in application of macroscopic behaviors for laminated glass plate between HSDT and FSDT.

Index Term— Dynamic prediction, Automotive windshield, Higher-order shear deformation theory, PVB interlayer model.

1. INTRODUCTION

Automotive windshield consists of multilayer of glass plies adhered by a polyvinylbutyral (PVB). Two glass plies are divided by a PVB interlayer that prevents the glass plies from failure by foreign mass impact and then, reducing the possibility of passenger’s injury. But laminated glass (LG) plate unlike the mono glass (MG) plate composed of a brittle material can reduce the number of dangerous flying fragments as many small parts because of a PVB interlayer.

The general purposes of the PVB interlayer are to absorb the dynamic energy and adhere the two glass plies. Therefore, the PVB interlayer can be provided as a barrier between the two glasses avoiding penetration and fracture. But despite of their a lot of advantages, the efficient application of LG plate is limited because of the difficulties in their optimal design at the preliminary stage. Therefore, the higher accurate simulation for LG plate is required a thorough study of the dynamic responses of automotive windshield due to impact.

A lot of analytical and numerical works on isotropic and anisotropic materials subjected to static and dynamic loading have been studied by Whitney and Pagano’s First-order Shear Deformation Theory (FSDT) [1]. And many papers about contact law and PVB effect on impact of LG plate for architectural applications have been presented by Hertz [2] and Dharani [3, 4]. Recently, Lee and Ahn etc. [5, 6] has been studied the impact behaviors of LG plate system by Whitney and Pagano’s First-order Shear Deformation Theory (FSDT).

A Higher-order Shear Deformation Theory (HSDT) of laminated composite plates is developed by Reddy [7-10]. This theory contains the same dependent unknowns as in FSDT, and accounts for parabolic distribution of the transverse shear strains through the thickness of the plate. Exact closed-form solutions of symmetric cross-ply laminates are obtained and the results are compared with three-dimensional elasticity solutions and FSDT solutions. The HSDT requires no shear correction coefficients and predicts the deflections and stresses more accurately when compared to FSDT.

In this paper, Reddy’s Higher-order Shear Deformation Theory (HSDT) based on Hertz’s law and Dharani’s PVB interlayer model is used to predict the overall dynamic responses on the LG plates like automotive windshield. The dynamic prediction such as the histories of contact force, deflection, kinetic energy, wavefront and in-plane stress for various PVB interlayer thicknesses during impact is obtained for the LG plates. For the dynamic behaviors of LG plate are compared and studied with those of the LG plates with the same total glass thickness and the different PVB interlayer thickness due to small mass impact.

2. DYNAMIC FINITE ELEMENT SIMULATION

Assume a geometry and a quarter model of target (laminated glass plate) consisting of three layers of total thickness \( h \) (outer ply thickness \( h_0 \), PVB interlayer thickness \( h_p \), inner ply thickness \( h_i \) under drop weight impact of radius \( R \) at the center with initial velocity \( V_0 \) as shown in Figs. 1 and 2.
To study the dynamic prediction between target and impactor, the displacement components of a point at a distance of $z$ from the reference plane according to Reddy’s HSDT\cite{7-10} may be expressed as

$$u(x,y,z) = u_0(x,y) + z[\phi_x - \frac{4z^2}{3h^2}(\phi_x + w(x,y))]$$

$$v(x,y,z) = v_0(x,y) + z[\phi_y - \frac{4z^2}{3h^2}(\phi_y + w(x,y))]$$

$$w(x,y,z) = w(x,y)$$

These formulations are based on the assumptions followed in Reddy’s HSDT. The middle plane of the plate is taken as the reference plane of model.

The stress state in each layer is given by Kooke’s law as

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} e_x \\ e_y \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{bmatrix}$$

(2-1)

where $Q_{ij}$ are the stiffness coefficients, which are defined in terms of engineering constants in the material axes of the layer as

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}^2}; \quad Q_{22} = \frac{E_{22}}{1-\nu_{21}^2}; \quad Q_{12} = \frac{\nu_{12}E_{22}}{1-\nu_{12}^2}; \quad Q_{44} = G_{23}Q_{44} = G_{23} = G_{13}$$

(2-2)

The stress resultants are related to the strains by the relations as

$$\begin{bmatrix} \{N\} \\ \{M\} \end{bmatrix} = \begin{bmatrix} [A] & [B] & [E] \\ [C] & [F] & [H] \end{bmatrix} \begin{bmatrix} \{e_0\} \\ \{e_1\} \end{bmatrix}$$

$$\begin{bmatrix} \{Q\} \\ \{R\} \end{bmatrix} = \begin{bmatrix} [A] & [D] \\ [D] & [F] \end{bmatrix} \begin{bmatrix} \{\gamma_0\} \\ \{\gamma_2\} \end{bmatrix}$$

(3)

$$\begin{aligned}
(A_1, B_1, E_1, F_1, H_1) &= \sum_{i=1}^{N_s} \int_{x_i}^{x_{i+1}} Q_{ij}^1(1,z_1,z_2,z_3,z_i^2) dz \\
(A_2, D_2, F_2) &= \sum_{i=1}^{N_s} \int_{x_i}^{x_{i+1}} Q_{ij}^2(1,z_1^2,z_2^2) dz
\end{aligned}$$

The stiffnesses in Eq. (3) are defined for $i, j = 1, 2, 6$ and those in Eq. (3) are defined for $i, j = 4, 5$.

The laminated glass plate is frequently treated as an equivalent monolithic glass plate using equivalent geometry and material properties. Wei et al.\cite{11} suggested equivalent properties for the whole laminated glass plate. $E_c, G_c, v_c$ and $\rho_c$ are equivalent Young’s modulus, equivalent shear modulus, equivalent Poisson’s ratio and equivalent density of the laminated glass plate, respectively. That is, these relationships are given as

$$E_c = (E_g h_o + E_p h_p + E_s h_i)/(h_o + h_p + h_i)$$

$$G_c = (G_g h_o + G_p h_p + G_s h_i)/(h_o + h_p + h_i)$$

$$v_c = (v_g h_o + v_p h_p + v_s h_i)/(h_o + h_p + h_i)$$

$$\rho_c = (\rho_g h_o + \rho_p h_p + \rho_s h_i)/(h_o + h_p + h_i)$$

(4)

where $E_g, E_p, h_o, h_p$, and $h_i$ are equivalent Young’s modulus of the glass, equivalent Young’s modulus of the PVB, outer ply thickness, PVB interlayer thickness and inner ply thickness, respectively. These relations are called the rule of the mixtures.

The equivalent material properties of laminated glass plate are used to calculate the contact stiffness by Hertz’s contact law as

$$\begin{aligned}
\left(\frac{E_{eq}}{E_g h_o + E_p h_p + E_s h_i} \right)^{\frac{1}{2}} &+ \left(\frac{E_{eq}}{E_g h_o + E_p h_p + E_s h_i} \right)^{\frac{1}{2}} \\
&+ \left(\frac{E_{eq}}{E_g h_o + E_p h_p + E_s h_i} \right)^{\frac{1}{2}} + \left(\frac{E_{eq}}{E_g h_o + E_p h_p + E_s h_i} \right)^{\frac{1}{2}} = \left(\frac{E_{eq}}{E_g h_o + E_p h_p + E_s h_i} \right)^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{G_{eq}}{G_g h_o + G_p h_p + G_s h_i} \right)^{\frac{1}{2}} &+ \left(\frac{G_{eq}}{G_g h_o + G_p h_p + G_s h_i} \right)^{\frac{1}{2}} \\
&+ \left(\frac{G_{eq}}{G_g h_o + G_p h_p + G_s h_i} \right)^{\frac{1}{2}} + \left(\frac{G_{eq}}{G_g h_o + G_p h_p + G_s h_i} \right)^{\frac{1}{2}} = \left(\frac{G_{eq}}{G_g h_o + G_p h_p + G_s h_i} \right)^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{v_{eq}}{v_g h_o + v_p h_p + v_s h_i} \right)^{\frac{1}{2}} &+ \left(\frac{v_{eq}}{v_g h_o + v_p h_p + v_s h_i} \right)^{\frac{1}{2}} \\
&+ \left(\frac{v_{eq}}{v_g h_o + v_p h_p + v_s h_i} \right)^{\frac{1}{2}} + \left(\frac{v_{eq}}{v_g h_o + v_p h_p + v_s h_i} \right)^{\frac{1}{2}} = \left(\frac{v_{eq}}{v_g h_o + v_p h_p + v_s h_i} \right)^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{\rho_{eq}}{\rho_g h_o + \rho_p h_p + \rho_s h_i} \right)^{\frac{1}{2}} &+ \left(\frac{\rho_{eq}}{\rho_g h_o + \rho_p h_p + \rho_s h_i} \right)^{\frac{1}{2}} \\
&+ \left(\frac{\rho_{eq}}{\rho_g h_o + \rho_p h_p + \rho_s h_i} \right)^{\frac{1}{2}} + \left(\frac{\rho_{eq}}{\rho_g h_o + \rho_p h_p + \rho_s h_i} \right)^{\frac{1}{2}} = \left(\frac{\rho_{eq}}{\rho_g h_o + \rho_p h_p + \rho_s h_i} \right)^{\frac{1}{2}}
\end{aligned}$$
Laminated glass plates are widely used in many engineering applications. The simple applications of this system have the shape of a glass plate. In case of impact under a small hard impactor, dynamic responses are considered to occur in the impact zone in which direct contact of the impactor and the glass takes place. Therefore it is important to estimate accurately the history of contact force and deflection. The relaxation modulus $G(t)$ for a viscoelastic material like this system is commonly given as

$$G(t) = G_p + (G_0 - G_p)e^{-t/\beta}$$

where $G_p$, $G_0$, $\beta$ and $t$ are the long time shear modulus, the short time shear modulus and the decay factor and time, respectively.

Because the impact is in the range of millisecond duration, the relaxation modulus $G(t)$ of PVB interlayer changes very little during impact of hard impactor. In the range of this short time, PVB interlayer behaves like a solid glassy material. The most papers have shown that PVB interlayer can be considered as a linear elastic material using the short term shear modulus for a transient behavior. $E_p$ and $\nu_p$ of the PVB interlayer by Dharani’s PVB interlayer model [3,4,11] are given according to $G = G_0$ and $K$ as

$$E_p = 9KG_0/(3K + G_0)$$

$$\nu_p = (3K - 2G_0)/(6K + 2G_0)$$

where $E_p$, $\nu_p$ and $K$ are the Young’s modulus, the Poisson’s ratio of the PVB interlayer and bulk modulus, respectively.

For the equation of motion of this glass plate, we derive the Hamilton’s energy principle as

$$\delta \int_{t_1}^{t_2} (U - V - T) dt = 0$$

where $U$, $V$ and $T$ are the strain energy, the work of external forces and the kinetic energy of the glass plate, respectively. Applying the principle of minimum total energy leads to the general equation of motion and boundary conditions.

Further simulating processes are described in Refs. [5, 6] in detail. HSDT in consideration of Hertzian contact law and Dharani’s PVB interlayer model that a round-trip process is conducted for the study of the dynamic prediction of the LG plates with the same total glass thickness under small mass impact. The thicknesses of the MG plate considered is 0mm and those of the LG plates are 12mm and 12.76, 13.52, 14.28mm, respectively. In other words, PVB interlayer of MG plate does not exist and that of LG plates have 2, 4 and 6 interlayers (thickness of 1 interlayer=0.38mm).

3. Numerical Results

Fig. 3 shows contact forces in various PVB interlayer at 30μs subjected to impact loading. In Fig. 3, the maximum contact forces for various plates occur at around 15μs after the initial impact. We can see that the maximum contact force in MG plate is a little (2-3%) larger than those of LG plate, but PVB thickness in LG plate does not affect so much on contact force under small mass impact. And we can see that the deflections of plate in LG plates are larger than that of the MG plate, and the larger the thickness of PVB interlayer, the higher the magnitude of deflection. These results present the similar trend with those of simulation by FSDT but very small differences (2%) in its magnitudes.

Fig. 4 depicts the relationship of contact force and deflection in LG with various PVB interlayer by impact loading and then shows the similar results with Fig. 3 because of low flexure stiffness of PVB interlayer. We can see that the maximum contact force of y-axis does not occur at the maximum deflection of x-axis. We can see that this result show a typical wave-controlled impact that the plate deflection is localized to the region around the impact point and the contact force and deflection are never in phase [12]. These results present the similar trend with those of simulation by FSDT [5, 6] but small difference in magnitudes of deflection.
Fig. 4. Relation of contact force and deflection in various PVB interlayer

Fig. 5. Velocity histories in various PVB interlayer

Fig. 6. Kinetic energy histories in various PVB interlayer

Fig. 7. Relation of kinetic energy and ball displacement in various PVB interlayer

Velocity history in LG with various PVB interlayer is shown in Fig. 5 and kinetic energy history in Fig. 6. The velocity at the time zero is the initial velocity at which the impactor hits the LG plate. Velocity curves in Fig. 5 decrease and take negative values and remain constant by passing time. These negative values present rebound velocity of the small mass impactor. Minimum kinetic energies in Fig. 6 occur when velocities in Fig. 5 are zero. The lowest tip of the curve depicts minimum kinetic energy, and the end of curve that remains constant depicts the rebound kinetic energy. And the difference between initial kinetic energy and rebound kinetic energy becomes absorbed kinetic energy by LG plate. In Fig. 5, we can see that velocity histories for various plates are independent of the thickness of PVB interlayer and it can be seen that from Fig. 6 the thickness of PVB interlayer between the various plates does not so affect so much on rebound energy but the larger the thickness of PVB interlayer is, the larger the rebound energy become. In Fig. 7, all closed loops of kinetic energy-displacement curve for various plates have approximately same area and it can be seen that PVB interlayer number affect a little on kinetic energy.

Fig. 8. Wavefronts in x-direction at impact point in various PVB interlayer

Fig. 8 shows wavefronts in x-direction at impact point for various PVB interlayer obtained from higher-order finite element simulation by wave propagation model. We can see that from Fig. 8 the larger the thickness of PVB interlayer is, the
larger the wavefront becomes because of the bending rigidity of plate, but the difference of wavefront due to various thickness of PVB interlayer is very small. It can be seen that the thickness of PVB interlayer does not affect so much on the magnitude of wavefront.

Fig. 9. The in-plane stresses in glass plates with various thicknesses on S4 at 15μs

Fig. 9 depicts the in-plane stresses for glass plates with various thicknesses on S4 (opposite side of impact point) at 15μs. We can see that the larger the thickness of PVB interlayer in various plates is, the lower the magnitude of in-plane stress becomes.

Fig. 10. The in-plane stresses in glass plates with various thickness on S3 at 15μs

Fig. 10 depicts the in-plane stresses for glass plates with various thicknesses on S3 (under side of PVB interlayer) at 15μs. We can see that the larger the thickness of PVB interlayer in various plates is, the higher the magnitude of in-plane stress becomes unlike result shown in Fig. 9.

4. CONCLUSION
A powerful and refined finite element formulation based on Reddy’s Higher-order Shear Deformation Theory (HSDT) in conjunction with Hertzian contact law and Dharani’s PVB interlayer model for the dynamic prediction of glass plates due to drop weight impact is considered in this paper. Dynamic predictions like deflection, kinetic energy, wave fronts and in-plane stress in monolithic glass plate are a few sensitive than those of laminated glass plate and prone to more fracture risk. But we can see that the variation of PVB thickness of laminated glass plate does not affect so much on dynamic responses. These results present a similar trend with those of simulation by using First-order Shear Deformation Theory (FSDT) and very small differences in its magnitudes of every simulation. That is, we can’t see so much difference in application of macroscopic behaviors for laminated glass plate between HSDT and FSDT. Therefore both simple FSDT and refined HSDT can be permitted for this macroscopic predictions but a study for microscopic behaviors like in-plane stress distributions of each layer by two theories will be studied from now on. This simulation may be supplied a reference in making some preliminary prediction considering macroscopic behaviors for laminated glass plate involving thin film like PVB with different material property.

REFERENCES