Simplified Analytical Models for Progressive Collapse Assessment of Precast RC Beam-Column Assemblies Upgraded with Hybrid NSM/FRP System

Hussein M. Elsanadedy a, b

a Chair of Research and Studies in Strengthening and Rehabilitation of Structures, Dept. of Civil Eng., College of Eng., King Saud University, P.O. Box 800, Riyadh 11421, Saudi Arabia
E-mail: helsanadedy@ksu.edu.sa
b On leave from Helwan Univ., Cairo, Egypt

Abstract—Due to discontinuity at beam-column joints, precast concrete structures are vulnerable to the potential of progressive collapse under the removal of one or more columns. Hence, efficient techniques are needed to strengthen existing precast beam-to-column connections for prohibiting (or diminishing) the risk of progressive collapse under column-removal event. This paper presents simplified analytical models for estimating the progressive collapse resistance of precast beam-column assemblies upgraded with a hybrid system having steel NSM (near-surface mounted) bars combined with FRP (fiber reinforced polymer) sheets. The developed models were validated with the help of available test data on half-scale 2D frame assemblies—comprising of three columns and two beams—tested under the loss of the middle column. Test assemblies included unstrengthened reference precast specimen, cast-in-situ concrete frame, and NSM/FRP-upgraded precast specimen. Parametric studies were also performed for assessing the influence of different variables on the progressive collapse capacity of retrofitted precast assembly in the event of column removal.

Index Terms—Strengthening; FRP sheets; NSM bars; Progressive collapse; Precast beam-column connection; Column-removal scenario, Analytical models.

I. INTRODUCTION

Structures made of precast concrete members are commonly used around the world for several reasons such as speed of construction, better control of concrete quality, and money saving owing to reduced material wastage and minimizing the formwork cost. Nevertheless, connections between precast concrete members, especially beam-to-column joints, are the weakest element in precast buildings, and they may lead to catastrophic events such as progressive collapse [1, 2]. These connections cause discontinuity among structural members, and therefore, they may not be able to redistribute the loads in the event of column loss in extreme events. Accordingly, strengthening of beam-to-column joints in precast buildings is necessitated for prohibiting (or diminishing) the progressive collapse risk.

Many researchers investigated the behavior of beam-to-column joints in precast structures, and the behavior of such joints was compared with their corresponding cast-in-situ connections [3-6]. The scenario of column loss was employed extensively in the literature for checking the vulnerability of multistory buildings against progressive collapse [7-11]. Even though numerous studies were carried out on retrofitting of monolithic RC (reinforced concrete) beam-to-column connections [12-15], research on upgrading of precast concrete beam-column connections is scanty [16-20]. An experimental program was performed by Da Fonseca et al. [16] for investigating the performance of two (unstrengthened and strengthened) precast concrete assemblies. Strengthening of precast connections was achieved using NSM FRP strips. Two-point loading was applied on the beam of the assembly till failure. It was evident that retrofitting of the precast concrete beam-column connections significantly lowered down the displacement of beam in comparison with the unstrengthened assembly. An effective strengthening system was proposed by Al-Salloum et al. [17] to minimize the potential of progressive collapse in buildings having precast beam-column connections. Three 2D assemblies were tested in the event of center column loss. Specimens included unstrengthened reference precast assembly, monolithic concrete frame, and precast frame like the reference specimen; nevertheless, it was retrofitted with steel plates in the beam-column connection zone. The suggested upgrading methodology was found to substantially minimize the progressive collapse risk of the precast specimen in the event of column loss.

More recently, Elsanadedy et al. [20] tested three single-story, two-bay RC beam-column assemblies. Tests included unstrengthened precast control specimen, cast-in-situ RC frame having continuous beam reinforcement, and precast concrete specimen like the control assembly, but it was strengthened with hybrid system composed of NSM steel bars combined with FRP sheets. A vertical quasi-static loading was applied on the center column of test assemblies in order to represent the event of column removal in progressive collapse incidents. The suggested retrofitting system was capable of enhancing the progressive collapse capacity of the upgraded frame significantly.

This study aims to develop simplified analytical models for assessing the progressive collapse resistance of precast RC beam-column assemblies upgraded with steel NSM bars combined with FRP sheets. The developed models were validated with the help of the test data of Elsanadedy et al. [20]. Parametric studies were also performed for assessing the influence of different variables on the progressive collapse capacity of strengthened precast assembly in the event of column removal.
II. EXPERIMENTAL STUDY

The test data of Elsanadedy et al. [20] were employed for validating the analytical models developed in current research. The test specimens included three beam/column assemblies (P-CON, CIS, and P-STR). These assemblies were designed as half-scale of precast non-prestressed prototype frame, which was a portion of existing precast concrete building. Dimension and reinforcement detailing of tested assemblies are presented in Table I and Figs. 1 and 2. The first specimen P-CON was precast control assembly that simulated the existing beam/column connections in precast construction within Saudi Arabia. The second assembly CIS was cast-in-situ frame with continuous longitudinal beam bars. The third assembly P-STR was upgraded precast specimen (Figs. 1 and 2). It was identical to the precast control assembly P-CON, but it was retrofitted in the beam-column connection zones using steel NSM bars combined with CFRP sheets. Fig. 2 illustrates the strengthening scheme for assembly P-STR. The material properties of the three assemblies are shown in Table 2. More details of the test assemblies can be obtained from Elsanadedy et al. [20].

A vertical loading was put on the middle column of the assembly in a displacement-controlled manner at a quasi-static rate of 100 mm/s to represent the progressive collapse event (Fig. 3).

Figs. 4(a) to 4(c), respectively, present the experimentally observed modes of failure of assemblies P-CON, CIS, and P-STR. As seen in Fig. 4(a), unstrengthened precast control assembly P-CON failed owing to crushing of concrete at ends of precast beams. Nevertheless, as displayed in Fig. 4(b), cast-in-situ assembly CIS failed due to development of plastic hinges at ends of monolithic beams (identified by flexural cracking in tension and concrete crushing in compression). It should be mentioned here that the flexural action stage was the only developed phase in specimen CIS, and the catenary action stage was not reached. This was owing to the inadequate bracing offered by the exterior columns, the discontinuity of exterior columns because of considering single story, the discontinuity of RC beams at ends, and the unexposure of columns to axial loads (to represent the worst-case situation in multistory buildings).

The mode of failure of strengthened assembly P-STR is presented in Fig. 4(c). Failure was in the beam-to-column region owing to rupture of horizontal (0°) FRP layer (close to the tension side) combined with fracture of the lowest NSM bars. Failure was ended up with concrete crushing in the beam near face of column.

<table>
<thead>
<tr>
<th>Assembly designation</th>
<th>Dimensions and reinforcement of beams</th>
<th>Dimensions and reinforcement of columns</th>
<th>Dimensions and reinforcement of corbels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L x b x h (mm)</td>
<td>Longitudinal bottom (or top) steel</td>
<td>Stirrups</td>
</tr>
<tr>
<td>Precast specimens P-CON &amp; P-STR</td>
<td>2620 x 350 x 350</td>
<td>4 ( \phi 16 )</td>
<td>2-legged ( \phi 8 @ 100 ) mm/c/c</td>
</tr>
<tr>
<td>Cast-in-situ specimen CIS</td>
<td>2650 x 350 x 350</td>
<td>4 ( \phi 16 )</td>
<td>2-legged ( \phi 8 @ 100 ) mm/c/c</td>
</tr>
</tbody>
</table>

*L = net span of beam/corbel; H = height of column; b = width of member section; h = depth of member section.

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>Compressive strength of standard cylinders (MPa)</td>
<td>37.3</td>
<td>Measured on test date</td>
</tr>
<tr>
<td>Cementitious grout</td>
<td>Compressive strength of standard cubes (MPa)</td>
<td>60.0</td>
<td>Measured on test date</td>
</tr>
<tr>
<td>Steel bars</td>
<td>Diameter (mm)</td>
<td>( \phi 8 )</td>
<td>10</td>
</tr>
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<td></td>
<td>Yield strength (MPa)</td>
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<td>CFRP material</td>
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<td>Based on standard test coupons</td>
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<td>Ultimate tensile strain (MPa)</td>
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<td>bars</td>
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<td>------</td>
<td>----------------------</td>
<td>-----</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Elongation at failure</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bond strength to concrete substrate (MPa)</td>
<td>3.5</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1. Detailing of assemblies P-CON and CIS [20] (Dimensions are measured in mm): (a) Concrete dimensions for assembly P-CON; (b) Concrete dimensions for assembly CIS; (c) Beam section; (d) Column section; (e) Corbel section.
vertical CFRP layer / side to be in the form of U-shape beyond the corbel face

4 ø12 U-shape NSM steel rebars (placed into 30 × 25 mm grooves filled with epoxy adhesive mortar)

horizontal CFRP layer wrapped around column

Support underneath middle column removed just before testing

Laser transducer

Accelerometer

Actuator

Steel stubs

Inclinometer

Concrete crushing at end of precast beams

Plastic hinge formation near connection zone

Wide flexural cracks

Fig. 2. Detailing of upgraded precast assembly P-STR [20] (Dimensions are measured in mm): (a) Beam-column connection at middle column; (b) Beam-column connection at exterior column; (c) Section of upgraded beam; (d) NSM bars for middle connection; (e) NSM bars for exterior connection – plan view.

Fig. 3. Control precast assembly P-CON ready for testing after instrumentation completed [20].
III. ANALYTICAL MODELS

Simple analytical models were proposed to calculate the maximum load of assemblies CIS and P-STR. Then, the calculated maximum loads were validated with the test results. As mentioned previously, specimens were exposed to vertical load at a quasi-static rate of 100 mm/s. Therefore, the dynamic increase factor (DIF) should be calculated for the different materials to represent strain-rate effect. For concrete, Section 2.1.5 of the CEB-FIP model [21] was employed to assess the DIF for both modulus of elasticity and compressive strength. However, for reinforcing bars, the equations proposed by Malvar [22] were used to assess the DIF for both yield and tensile strengths. It is worth mentioning here that as per the test results discussed in [20] for specimens P-CON, CIS, and P-STR, the strain rate employed to estimate the DIF for concrete and steel bars was taken as 0.01 s⁻¹. The peak loads of the two specimens were then calculated as follows:

A. Specimen CIS

1) Strength of Beams in Flexure: Fig. 5 illustrates the distribution of strains and stresses in beam section of specimen CIS. The symbols in the figure are identified as: $A_s$ = area of compression (top) steel bars; $d'$ = depth of top bars (calculated from the top side of beam); $\varepsilon_s$ and $f_s$ = strain and stress in top compression bars; $A_t$ = area of tension (bottom) bars; $d$ = depth of tension steel bars; $\varepsilon_t$ and $f_t$ = strain and stress in tension steel bars. As seen in Fig. 5, a linear distribution of strain was assumed and the top concrete compressive strain was taken as $\varepsilon_{cu} = 0.003$ [23]. At the onset of analysis, strain in the tension steel bars was unidentified, and therefore, the neutral axis depth ($c$) was also unknown. Thus, it was initially taken as $c = 0.2d$, and this value was adjusted upon checking the equilibrium of internal forces. Strains in steel bars were computed with the help of similar triangles (Fig. 5), and the related stresses were then assessed using the bilinear stress versus strain curve of reinforcing bars shown in Fig. 6. Internal forces in top and bottom steel bars were then calculated, respectively, from $C_s = A_s \times f_s$ & $T_s = A_s \times f_s$. Therefore, equilibrium of internal forces in beam section can be written as

$$T_s = C_s + C_{cd} \Rightarrow A_s f_s = 0.85 f_{cd} \beta c b + A_s \varepsilon_s$$  \hspace{1cm} (1)

where $f_{cd}$ = specified concrete strength considering effect of strain rate and $\beta_s = parameter$ for equivalent stress block, and it was estimated using the ACI 318-19 code [23]. Then, the depth of the neutral axis was reassessed using the following formula.

$$c = \frac{T_s - C_s}{0.85 f_{cd} \beta_s b}$$  \hspace{1cm} (2)

The preceding calculations were repeated until the equilibrium of beam section was achieved. The ultimate moment of beam section was thus computed using the following equation.

$$M_s = 0.85 f_{cd} \beta c b \left( d - \frac{\beta_s c}{2} \right) + A_s \varepsilon_s (d - d')$$  \hspace{1cm} (3)
The frame assembly was simulated with the help of CSI SAP2000 package [24] (see Fig. 7(a)). A load of 1.0 kN was applied vertically on the middle column of the frame assembly. Then, the diagrams for moment and shear force were, respectively, outputted as shown in Figs. 7(b) and (c). The bending moment at the face of the center column was estimated as 0.7 kN.m, and therefore, the maximum load owing to the strength of RC beams in flexure was given by

\[ P_{u,f} = 1.43M_u \text{ (Units: m and kN)} \]  

(4)

2) Strength of Beams in Shear: The strength of beams in shear was calculated from

\[ V_u = V_c + V_s \]  

(5)

In the above additive equation, \( V_c \) and \( V_s \) are, respectively, the shear forces resisted by concrete and transverse reinforcement. Due to formation of plastic hinges in beam-column connection zones, shear strength resisted by concrete had to be reduced as per [25, 26] using the following formula.

\[ V_c = 0.05\lambda_1\lambda_2 b(d-c)\sqrt{f_{cd}} \text{ (Units: mm and N)} \]  

(6)

In the above equation, \( \lambda_1 \) is a coefficient accounting for aspect ratio of RC member (assumed as 1.0), and \( \lambda_2 \) is a parameter accounting for ratio of tension steel bars (taken as \( \lambda_2 = 0.5 + 20\rho_s \), where \( \rho_s = A_s / bd \)). As per [25, 26], the shear force carried by transverse steel was estimated from

\[ V_s = \frac{A_s(d-c)f_{shd}}{s} \]  

(7)

where \( A_s \) is the total area of stirrups, \( f_{shd} \) is the yield strength of stirrups (scaled to account for strain rate), and \( s \) is the
spacing of transverse reinforcement (measured on centers). According to the shear diagram presented in Fig. 7(c), the peak load of assembly CIS that corresponds to the strength of beams in shear was given by

$$P_{u,sh} = 2V_{u}$$  (8).

3) Shear Friction at Face of Column: According to the ACI 318-19 [23], the shear transferred at face of column was given by

$$V_{u,sh} = \mu A_f f_s \leq \frac{0.2 f_{c,ld} A_t}{(3.3 + 0.08 f_{yd}) A_t} \text{ (Units: mm and N)}$$  (9)

In the above formula, \(\mu\) is the friction coefficient at the shear friction plane (= 1.4 for cast-in-situ concrete of assembly CIS); \(A_f\) is the total area of bars crossing the shear friction plane = \(A_t + A_i\); \(f_{yd}\) is the yield strength of bars (scaled to account for strain rate); and \(A_i\) is the effective area of beam section (= bd). Then, the peak load that corresponds to shear friction at face of column was given by

$$P_{u,sh} = 2V_{u,sh}$$  (10)

The peak load of assembly CIS was estimated as the minimum of Eqs. (4), (8), and (10).

B. Upgraded Specimen P-STR

1) Strength of Upgraded Part of Beams in Flexure: Fig. 8 illustrates the distribution of strains and stresses in the beam section of assembly P-STR at flexural strength state. The ultimate moment of FRP-upgraded RC beams having steel NSM bars can be calculated using strain compatibility, equilibrium of internal forces, and the governing failure mode as illustrated in Fig. 8. Two modes of failure in flexure were checked for upgraded section (Mode I: concrete crushing before de-bonding of FRP sheets; and Mode II: de-bonding of FRP sheets before crushing of concrete). Then, the ultimate moment of the upgraded section was computed as follows:

**Analysis of Section for Mode I:** The neutral axis depth was initially taken as \(c = 0.2h\) (where \(h\) is the total beam depth), and this value was revised after the equilibrium was checked. The FRP section below the neutral axis (in the tension side of the beam) was divided into 20 slices. For the \(i^{th}\) slice, the effective depth was calculated from

$$d_i = h -(i-1) h_f - 0.5 h_f$$

Strain in the extreme tensile FRP slice was estimated from

$$\varepsilon_{f, \text{max}} = 0.003 \frac{d_{i, \text{max}} - c}{c} \leq \varepsilon_{fy}$$  (11)

where \(d_{i, \text{max}}\) = depth of extreme tensile FRP slice = \(h - 0.5 h_f\) and \(\varepsilon_{fy}\) = FRP de-bonding strain estimated from the following equation [27].

$$\varepsilon_{fy} = \left[ \frac{E_f}{n t_f E_y} \right]^{1/4} \left( 6.5 + \frac{n t_f E_y}{135.000} \right) \frac{f_{yd}}{f_y} \leq \varepsilon_{fy}$$  (12)

where \(\varepsilon_{fy}\) = yield strain of NSM bars; \(n\) = number of FRP layers; \(t_f\) = thickness per layer of FRP material; \(E_f\) = elastic modulus of FRP sheets; and \(\varepsilon_{fy}\) = strain at rupture of FRP sheet. The strain value of each slice (\(\varepsilon_{fy}\)) was then calculated from the following equation.

$$\varepsilon_f = \varepsilon_{f, \text{max}} \frac{d_{i, \text{max}} - c}{d_{i, \text{max}} - c}$$  (13)

The corresponding FRP stress \(f_{fy}\) was then estimated from

$$f_{fy} = E_f \varepsilon_f$$

and the force in each FRP slice was given by

$$T_{fy} = 2n t_f h_f f_{fy}$$

The corresponding stresses \(f_{fy}\) were calculated from the stress versus strain curve of steel bars illustrated in Fig. 6. The force of each NSM layer was then estimated as \(F_{s} = 2 A_t f_{fy}\), where \(A_s\) = area of one bar of NSM reinforcement. The total force in the NSM reinforcement was calculated from \(F_s = \sum_{i=1}^{n} F_{s,i}\). The depth of the neutral axis was recomputed using the following formula.

$$c = \frac{T_{fy} + F_{s}}{0.85 \beta_1 f_{yd} b}$$  (15)

The preceding steps were repeated until the equilibrium of beam section was achieved. Then, the ultimate moment of the upgraded beam section was assessed from the following equation.

$$M_{u} = \sum_{i=1}^{n} T_{fy} \left( d_{i} - \frac{\beta c}{2} \right) + \sum_{i=1}^{n} F_{s} \left( d_{i} - \frac{\beta c}{2} \right)$$  (16)

**Analysis of Section for Mode II:** Similar to Mode I, a preliminary assessment of \(c\) was assumed as \(0.2h\) and the number of FRP slices \(n\) was taken as 20. Strain in the extreme tensile FRP slice was set equal to the FRP de-bonding strain given by Eq. (12). The corresponding extreme compression strain was then given by the following formula (Fig. 8).

$$\varepsilon_c = \varepsilon_{fy} \frac{c}{d_{i, \text{max}} - c} < 0.003$$  (17)

The approximate stress block factors \((\alpha_1\) and \(\beta_1)\) were computed using the parabolic stress-strain relationship for concrete [28]. The total forces in both FRP and NSM reinforcement \((T_f\) and \(F_s)\) were calculated following the same procedure used in Mode I. The depth of the neutral axis was recomputed from

$$c = \frac{T_{fy} + T_{fs}}{\alpha_1 \beta_1 b f_{yd}}$$  (18)

Equilibrium of section was achieved and the ultimate moment of the upgraded beam section was estimated from Eq. (16). The maximum load of assembly P-STR
corresponding to the strength of upgraded portion of beams in flexure was computed using Eq. (4).

2) Strength of Upgraded Part of Beams in Shear:
Owing to the large displacement level in specimen P-STR, accompanied with flexure cracking and concrete crushing near connection region, concrete resistance to shear resistance was neglected. Thus, the strength of strengthened section in shear was given by \( V_u = V_f \), where \( V_f \) is the shear resisted by vertical (90°) FRP layer, estimated as per [28].

The peak load of specimen P-STR considering the strength of upgraded part of beams in shear was given by Eq. (8).

3) Shear Friction at Face of Column:
The horizontal FRP layers that cross the connection zone cannot be accounted for in resisting the direct shear transferred at face of column. Then, the NSM bars were employed for transmitting the shear at face of column, which was calculated from the following formula [23].

\[
V_{u,NSM} = \mu A_{NSM} f_{yd} \left( \frac{0.2f_{yd}A_c}{5.5A_f} \right) \quad \text{(Units: mm and N)}
\]

In the above equation, \( \mu \) is the friction coefficient at beam-column intersection (= 0.6 for the case of cementitious grout placed against unintentionally roughened hardened concrete); \( A_{NSM} \) is the area of NSM bars; \( f_{yd} \) is the yield strength of NSM bars (scaled to account for strain rate); and \( A_c \) is the effective area of upgraded beam section (= \( bh \)). Then, the peak load that corresponds to shear friction at face of column was calculated using Eq. (10).

4) Strength of Unstrengthened Portion of Beams in Flexure and Shear:
For unstrengthened part of beams, the flexural strength was estimated like specimen CIS (Eqs. (1) to (3)). Nevertheless, as presented in Fig. 7(b), a bending moment of 0.37 kN.m was calculated at the critical section (end of FRP layers) of the unstrengthened portion of the beam. Accordingly, the maximum load corresponding to the strength of the unstrengthened beam section in flexure was given by

\[
P_{u,STR} = 2.7M_u \quad \text{(Units: m and kN)}
\]

Also, the shear resistance of the unstrengthened beam section was similar to that computed before for specimen CIS (Eqs. (5) to (7)), and the related maximum load was calculated from Eq. (8).

The maximum load of the strengthened assembly P-STR was estimated as the minimum of the above modes for unstrengthened and strengthened portions of beam. Table 3 lists the peak loads of assemblies CIS and P-STR, computed according to the various possible modes discussed previously, and the experimental and predicted final peak loads. Ratio between tested and predicted maximum loads is also given in Table 3 for the two assemblies. As shown from Table 3, errors in the calculated maximum loads ranged from 5% to 12%.

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![Fig. 8. Section analysis for strengthened precast specimen P-STR.](image)

**TABLE III**

EXPERIMENTAL AND ANALYTICALLY PREDICTED MAXIMUM LOADS FOR ASSEMBLIES CIS AND P-STR

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>Max. load for strengthened part of beam (kN)</th>
<th>Max. load for unstrengthened part of beam (kN)</th>
<th>Max. load (kN)</th>
<th>( P_{u,exp} )</th>
<th>( P_{u,th} )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Strength in flexure</td>
<td>Strength in shear</td>
<td>Shear friction</td>
<td>Strength in flexure</td>
<td>Strength in shear</td>
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<tr>
<td>CIS</td>
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<td>-</td>
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<td>360</td>
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<tr>
<td>P-STR</td>
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<td>313</td>
<td>674</td>
<td>386</td>
<td>360</td>
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IV. PARAMETRIC STUDY

The developed analytical models were utilized for studying the influence of various strengthening parameters on the progressive collapse capacity of strengthened assembly in the event of center column removal. The analysis matrix for the parametric study is shown in Table 4. It has 36 specimens with various parameters such as: thickness of FRP sheet and area of NSM steel bars. It should be stated that the assemblies in Table 4 had the same material properties as the strengthened assembly P-STR.

The analytical results of the 36 assemblies utilized in the parametric study are presented in Table 4 in terms of peak load and mode of failure. For evaluating the efficacy of hybrid NSM/FRP upgrading system at minimizing the progressive collapse risk of existing precast beam-column assemblies in the event of column removal, three parameters were developed in this research. These are strengthening parameters (ωNSM and λs) and the peak load efficiency (ηp). They can be estimated from the following formulas.

\[
\omega_{NSM} = \frac{\rho_{NSM} f_{NSM} s}{\rho f_{NSM} s + f_p E_s / \rho_{m}}
\]

(21)

\[
\lambda_s = \frac{\rho_{NSM} f_{NSM} s + f_p E_s / \rho_{m}}{\rho s f_y}
\]

(22)

\[
\eta_p = \frac{P_{u,m}}{P_{u,m} \times 100}\%
\]

(23)

where ρNSM = ratio of NSM bars (the ratio of total area of NSM bars to the area of strengthened beam section); \( f_{NSM} \) = static (unscaled) yield strength of NSM bars; \( f_p \) = flexural FRP reinforcement ratio (the ratio of area of horizontal FRP sheets to the area of strengthened beam section); \( \rho_s \) = ratio of bottom steel bars in precast beam; \( f_{sy} \) = static (unscaled) yield strength of beam bottom bars; \( P_{u,m} \) = maximum load of upgraded precast assembly; and \( P_{u,STR} \) = maximum load of equivalent cast-in-situ assembly with continuous beam bars (specimen CIS). The two strengthening parameters (ωNSM and λs) were computed for upgraded specimens of Table 4 and are then scatter plotted versus each other as shown in Fig. 9. Shear friction failure at inner column face was predicted for specimens with low ratio of NSM reinforcement, which was evident in Fig. 9 for specimens with ωNSM ≤ 0.1. However and as clear from Fig. 9, for assemblies with high ratio of NSM bars (with ωNSM > 0.1), failure was due to either flexural failure of strengthened part of beam (for specimens with λs < 9.0) or shear failure of unstrengthened beam portion (for specimens with λs > 9.0).

Taking only the analytical results for the 24 specimens with flexural failure of strengthened section, the peak load efficiency (ηp) was plotted versus the strengthening parameter λs, as seen in Fig. 10. Also, the data point for λs = 0 was added in Fig. 10 by considering the maximum experimental load of unstrengthened precast assembly P-CON. Fig. 10 also shows the best-fit curve of the data points with the following equation.

\[
\eta_p = -2.3\lambda_s^2 + 38.5\lambda_s + 6.3
\]

(24)

Considering all 25 data points, the above equation has \( R^2 = 0.98 \). It is clear from the above equation and Fig. 10 that in designing of hybrid NSM/FRP system for upgrading the progressive collapse capacity of precast beam-column connections, the parameter λs has to be ≥ 3.0 for achieving a peak load efficiency of 100%. Conclusively, it is suggested to design the hybrid NSM/FRP upgrading technique with ωNSM > 0.1 and 3.0 ≤ λs < 6.0, for achieving flexural failure with peak load efficiency ranging from 100% to 150%.

![Fig. 9. Relationship between strengthening parameters ωNSM and λs.](image)

**TABLE IV**

**ANALYSIS MATRIX AND RESULTS OF THE PARAMETRIC STUDY**

<table>
<thead>
<tr>
<th>Specimen designation</th>
<th>NSM bars per side</th>
<th>No. of horizontal (or vertical) FRP layers per side</th>
<th>Max. load for strengthened part of beam (kN)</th>
<th>Max. load for unstrengthened part of beam (kN)</th>
<th>( P_{u,STR} ) (kN)</th>
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*P_{n0} = theoretical maximum load; FF-FR = flexural failure of upgraded part of beam due to rupture of FRP sheets; FF-CC = flexural failure of upgraded part of beam due to concrete crushing; FF-DB = flexural failure of upgraded part of beam due to debonding of FRP sheets; SFR = shear friction failure of strengthened beam at column face; SF-U = shear failure of unstrengthened part of beam.
V. CONCLUSIONS

The simplified analytical models detailed in this research were efficient in estimating the progressive collapse capacity (in the flexural action stage) for both cast-in-situ and NSM/FRP-upgraded precast beam-column assemblies in the event of column loss. In these models, different possible flexural and shear failure modes were investigated at critical beam sections, and the peak load was taken as the minimum of all possible modes. Errors in the estimation of peak loads ranged from 5% to 12%. Three parameters were developed in this research. These are strengthening parameters (\(\alpha_{NSM}\) and \(\lambda_s\)) and the maximum load efficiency parameter (\(\eta_p\)). They were utilized to compare the progressive collapse capacity of NSM/FRP-upgraded precast beam/column assemblies with their cast-in-situ counterparts. In the design of the hybrid NSM/FRP system for minimizing the progressive collapse risk of precast beam-column joints, it is suggested to have \(\alpha_{NSM} > 0.1\) and \(3.0 \leq \lambda_s < 6.0\), for achieving flexural failure with peak load efficiency ranging from 100% to 150%.

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